Note on OPE production amplitudes [1]

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Charge exchange, OPE production amplitudes of $Y^0\pi^+$ (Y is the isobar) via a_1, a_2, π_2 and $\hat{\rho}$.

$$\gamma p \to X^+ n \to Y^0 \pi^+ n (\to \pi^+\pi^-\pi^+ n) \tag{1}$$

$$\frac{d^2\sigma}{dt dM_{\rho\pi} d\Omega_k} = \frac{1}{2}\sum_{\lambda_N \lambda_{N'} \lambda_Y} \frac{389.3\,\mu\text{b GeV}^2}{64\pi m_N^2 E_\gamma^2} \frac{|\mathbf{k}_Y|}{2(2\pi)^3} |A(s,t,M_{Y\pi},\Omega_k,\lambda_\gamma,\lambda_N,\lambda'_N,\lambda_Y)|^2 \tag{2}$$

Spin projections of the isobar, λ_Y and resonances, λ_X, $M_{Y\pi}$, (see below) are defined in the GJ frame *i.e.* resonance at rest, photon momentum along the z axis, (λ_γ=photon helicity). In contrast, nucleon will be parametrized by its helicity.

The amplitude A is written as a product of the production amplitude A_{OPE} (production as defined by Dennis) and the amplitude T for the resonance (assumed BW) to decay into $Y\pi$. (I think Dennis has that in his code, I'm giving it here to completeness),

$$A = \sum_{X, J^P, \lambda_X} A_{OPE}(s,t,\lambda_N,\lambda_\gamma,\lambda'_N,J^P,\lambda_X) T(M_{Y^0\pi^+},\Omega_k,J^P,\lambda_X,\lambda_Y) \tag{3}$$

$$\begin{aligned} A_{OPE}(s,t,\lambda_N,\lambda_\gamma,\lambda'_N,J^P,\lambda_X) &= \sqrt{2} g_{\pi NN} \left(\frac{s}{s_0}\right)^{\alpha_\pi(t)} \frac{\sqrt{t'}}{(t-m_\pi^2)} \beta(t) \delta_{\lambda_N,-\lambda'_N} \\ &\times \sum_{L_\gamma} \sqrt{\frac{2L_\gamma+1}{4\pi}} \langle J,\lambda_X | L_\gamma 0, 1\lambda_\gamma\rangle \left(\frac{\omega}{m_X}\right)^{L_\gamma} g^{L_\gamma}_{X\gamma\pi}. \end{aligned} \tag{4}$$

Here $t' = |t - t_{min}|$, $\omega = \lambda(m_X, 0, t)$ is the breakup momentum of the resonance X with mass m_X decaying into photon and an off-shell pion with mass t, $g^2_{\pi NN}/(4\pi) \sim 14.4$, $s = 2E_\gamma m_n + m_n^2$, and $\alpha_\pi = 0.9\text{GeV}^{-2}(t - m_\pi^2)$. The form factor $\beta(t)$ was fitted to the $\gamma p \to a_2^+ n$ data from Ref. [2], [3] and is given by

$$\beta(t) = (a_1 e^{b_1 t} + a_2 e^{b_2 t})^{1/2} \tag{5}$$

with $b_1 \sim 35\text{GeV}^{-2}$, $b_2 \sim 3\text{GeV}^{-2}$, $a_1 \sim 30$, $a_2 \sim 0.05$ for $s_0 = 10\text{GeV}^2$.

$$\begin{aligned} T(M_{Y^0\pi^+},\Omega_k,J^P,\lambda_X,\lambda_Y) &= \sum_{L_{Y\pi} M_{Y\pi}} \frac{m_X g^{L_{Y\pi}}_{XY^0\pi^+}}{2(M_{Y^0\pi^+} - m_x + i\frac{\Gamma_{X,J^P}}{2})} \left(\frac{k_{Y\pi}(m_X)}{m_x}\right)^{L_{Y\pi}} \\ &\times \langle s_Y \lambda_Y, L_{Y\pi} M_{Y\pi} | J_X \lambda_X \rangle Y_{L_{Y\pi},M_{Y\pi}}(\Omega_k) \end{aligned} \tag{6}$$

The photocouplings $g^{L_\gamma}_{X\gamma\pi}$ are calculated from the radiative widths $\Gamma_{X\gamma\pi}$ using

$$\Gamma_{X\to\gamma\pi}(E) = m_X \sum_{L_\gamma, L'_\gamma} \frac{g^{L_\gamma}_{X\gamma\pi} g^{L'_\gamma}_{X\gamma\pi}}{32\pi^2} \left(\frac{q}{m_x}\right)^{L_\gamma + L'_\gamma + 1}$$
$$\times \left[\delta_{L_\gamma, L'_\gamma}\left(\delta_{L_\gamma,J} + \delta_{L_\gamma,J+1}\frac{J}{2J+1} + \delta_{L_\gamma,J-1}\frac{J+1}{2J+1}\right) + \left(\delta_{L_\gamma,J+1}\delta_{L'_\gamma,J-1} + \delta_{L'_\gamma,J+1}\delta_{L_\gamma,J-1}\right)\frac{\sqrt{J(J+1)}}{2J+1}\right], \tag{7}$$

with $q = \lambda(m_X, 0, m_\pi)$. Note that, if there is more then one partial amplitude, L_γ contributing, $\Gamma_{X\gamma\pi}$ determines only a product of different couplings. In such a case, to completely determine all $g^{L_\gamma}_{X\gamma\pi}$'s we assumed that the amplitude ratios are the same as for $X \to \rho\pi$.

The strong couplings are computed from

$$\Gamma^{L_{Y\pi}}_{X,J^P} = m_X \frac{(g^{L_{Y\pi}}_{XY\pi})^2}{32\pi^2} \left(\frac{k_{Y\pi}}{m_X}\right)^{2L_{Y\pi}+1}. \tag{8}$$

Here $k_{Y\pi} = \lambda(m_X, m_Y, m_\pi)$ is the resonance breakup momentum. Photocouplings and strong couplings to selected $Y\pi$ channels are summarized in Table 2. These are calculated from Eqs. 7, 8 using, where known, the PDG values * for $\Gamma_{X\gamma\pi}$ and $\Gamma^{L_{Y\pi}}_{X,J^P}$ and "reasonable guesses" otherwise, (all listed in Table. 1).

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*See caption for Table.1

REFERENCES

[1] A. Afanasev and A.P. Szczepaniak almost finished.

[2] Y. Eisenberg *et al.*, Phys. Rev. Lett. **23**, 1322 (1969).

[3] G.T. Condo *et al.*, Phys. Rev. **D**48, 3045 (1993).

TABLES

Resonance decay channel	J^{PC}	Mass[GeV]	Partial waves (L^{AB}, $L_\gamma = L^{\rho\pi}$)	$\Gamma^{L_{AB}}_{XAB}/\Gamma$	Γ[MeV]	Γ^e[keV]
a_1	1^{++}	1.26			400	640
$\rho\pi$			S	0.99		
			D	0.01		
a_2	2^{++}	1.32			110	295
$\rho\pi$			D	0.70		
$\eta\pi$			D	0.30		
π_2	2^{-+}	1.67			258	300
$\rho\pi$			P	0.98*0.31		
			F	0.02*0.31		
$f_2\pi$			S	0.69		
$\hat{\rho}$	1^{-+}	1.6			400	170
$\rho\pi$			P	0.50		
$b_1\pi$			S	0.50		

TABLE I. Resonance parameters. $\rho\pi$ widths are taken from the PDG. Γ is the total hadronic width, $\Gamma = \sum_{AB,L_{AB}} \Gamma^{L_{AB}}_{XAB}$ and Γ^e is the total radiative width to $\gamma\pi$ Here I have assumed that resonances can be saturated by the (two body) decay channels listed above. This is close to reality, however in principle on can recalculate the couplings in Table 2 for the "true" PDG values". (This assumption was made to make the theoretical model of Ref. [1] simpler).

Resonance decay channel	Partial wave	$\frac{(g^{L_{AB}}_{XAB}(k_{AB}(m_X)))^2}{4\pi}$	$\frac{(g^{L_\gamma}_{X\gamma\pi})^2}{4\pi}$
a_1			
$\rho\pi$	S	26.28	1.03
	D	32.54	1.04×10^{-2}
a_2			
$\rho\pi$	D	442.4	2.37
$\eta\pi$	D	57.08	72.71
π_2			
$\rho\pi$	P	20.01	1.74
	F	20.15	4.01×10^{-2}
$f_2\pi$	S	13.72	
$\hat{\rho}$			
$\rho\pi$	P	24.59	1.51
$b_1\pi$	S	7.12	

TABLE II. Strong and electromagnetic couplings corresponding to (BW) widths given in Table 1. Strong couplings, $g^{L_{Y\pi}}_{XY^0\pi^+}$ to the charged state $Y^0\pi^+$ when Y^0 is part of an isovector should be reduced by $\sqrt{2}$.