Note on ρ exchange production amplitudes

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Charge ρ- exchange, (ORE) production amplitudes of $Y^0\pi^+$ (Y is the isobar) via a_1, a_2, π_2 and $\hat{\rho}$.

$$\gamma p \to X^+ n \to Y^0 \pi^+ n (\to \pi^+\pi^-\pi^+ n) \tag{1}$$

$$\frac{d^2\sigma}{dt dM_{\rho\pi} d\Omega_k} = \frac{1}{2}\sum_{\lambda_N \lambda_{N'} \lambda_Y} \frac{389.3\,\mu\text{b GeV}^2}{64\pi m_N^2 E_\gamma^2}\frac{|\mathbf{k}_Y|}{2(2\pi)^3}|A(s,t,M_{Y\pi},\Omega_k,\lambda_\gamma,\lambda_N,\lambda'_N,\lambda_Y)|^2 \tag{2}$$

As before, *i.e.* in the note on OPE, all spin projections (with the exception of the nucleon which is defined by its helicity) correspond to spin quantization along the photon momentum.

The amplitude A is written as a product of the production amplitude A_{ORE} and the amplitude T for the resonance (assumed BW) to decay into $Y\pi$.

$$A = \sum_{X,J^P,\lambda_X} A_{ORE}(s,t,\lambda_N,\lambda_\gamma,\lambda'_N,J^P,\lambda_X) T(M_{Y^0\pi^+},\Omega_k,J^P,\lambda_X,\lambda_Y) \tag{3}$$

The amplitude T is the same as in Eq. 6 in the OPE note.

$$\begin{aligned} A_{ORE}(s,t,\lambda_N,\lambda_\gamma,\lambda'_N,J^P,\lambda_X) &= \sum_{\lambda_\rho=\pm1,0} R_\rho(s,t)\beta_\rho(t)\left[\frac{G_T}{m_N}\sqrt{\frac{t'}{4m_N^2}}\sigma^z_{\lambda'_N,-\lambda_N} + \frac{G_V}{m_N}\delta_{\lambda'_N,\lambda_N}\right] \\ &\times \left[\left(\frac{m_X^2+t}{m_X^2-t}\right)\delta_{\lambda_\rho,0} + \sqrt{\frac{t'}{4m_N^2}}\left(\frac{2m_N^2}{m_X^2-t}\right)\left(\delta_{\lambda_\rho,+1}-\delta_{\lambda_\rho,-1}\right)\right] \\ &\times \sum_{L\gamma S_\gamma}\sqrt{\frac{2L_\gamma+1}{4\pi}}\langle 1\lambda_\gamma, 1\lambda_\rho|S_\gamma\lambda_X\rangle\langle S_\gamma\lambda_X, L_\gamma 0|J_X\lambda_X\rangle g^{L_\gamma S_\gamma}_{X\gamma\rho}\left(\frac{\omega}{m_X}\right)^{L_\gamma}. \end{aligned} \tag{4}$$

Here σ^z is the Pauli matrix, $\sigma^z_{\lambda'_N,-\lambda_N} = 1$ if $\lambda'_N = -\lambda_N = +1/2$ and $\sigma^z_{\lambda'_N,-\lambda_N} = -1$ if $\lambda'_N = -\lambda_N = -1/2$, $t' = |t - t_{min}|$ and the Regge factor is given by

$$R_\rho(s,t) = \sin\left(\frac{\pi\alpha_\rho(t)}{2}\right)\Gamma(1-\alpha_\rho(t))\left(\frac{s}{s_0}\right)^{\alpha_\rho(t)} \tag{5}$$

with $\alpha_\rho(t) = 1 + 0.9\text{GeV}^{-2}(t - m_\rho^2)$. The vector and tensor couplings are approximately given by $G_V \sim 2-5$ and $G_T \sim 18-30$.

The above expression can be simplified by redefining the photocouplings $g^{L_\gamma S_\gamma}_{X\gamma\rho}$ so that they absorb the CG coefficients in a manner which explicitly display helicity dependence,

$$A_{ORE}(s,t,\lambda_N,\lambda_\gamma,\lambda'_N,J^P,\lambda_X) = R_\rho(s,t)\left[\frac{G_T}{m_N}\sqrt{\frac{t'}{4m_N^2}}\sigma^z_{\lambda'_N,-\lambda_N} + \frac{G_V}{m_N}\delta_{\lambda'_N,\lambda_N}\right]$$
$$\times\left[\left(\frac{m_X^2+t}{m_X^2-t}\right)\beta_{1,X}(t)\left(\delta_{\lambda_\gamma,+1}\delta_{\lambda_X,+1}+\tau_X\delta_{\lambda_\gamma,-1}\delta_{\lambda_X,-1}\right)\right.$$
$$\left.+\sqrt{\frac{t'}{4m_N^2}}\left(\frac{2m_N^2}{m_X^2-t}\right)\beta_{2,X}(t)\left(\delta_{\lambda_\gamma,+1}\delta_{\lambda_X,0}-\tau_X\delta_{\lambda_\gamma,-1}\delta_{\lambda_X,0}\right)\right]. \tag{6}$$

Here η_X is the naturality of the produced resonance. In terms of the photocouplings the, functions $\beta_{i,X}(t)$ are given by

$$\beta_{1,X}(t) = \beta_\rho(t)\sum_{L\gamma S_\gamma}\sqrt{\frac{2L_\gamma+1}{4\pi}}\langle 11,10|S_\gamma 1\rangle\langle S_\gamma 1, L_\gamma 0|J_X 1\rangle g^{L_\gamma S_\gamma}_{X\gamma\rho}\left(\frac{\omega}{m_X}\right)^{L_\gamma}$$
$$\beta_{2,X}(t) = \beta_\rho(t)\sum_{L\gamma S_\gamma}\sqrt{\frac{2L_\gamma+1}{4\pi}}\langle 11,1-1|S_\gamma 0\rangle\langle S_\gamma 0, L_\gamma 0|J_X 0\rangle g^{L_\gamma S_\gamma}_{X\gamma\rho}\left(\frac{\omega}{m_X}\right)^{L_\gamma} \tag{7}$$

The analogous expression for OPE looks like (rewriting the A_{OPE} from the OPE note)

$$A_{OPE}(s,t,\lambda_N,\lambda_\gamma,\lambda'_N,J^P,\lambda_X) = R_\pi(s,t)\left[\bar{g}_{\pi NN}\sqrt{\frac{t'}{4m_N^2}}\delta_{\lambda'_N,-\lambda_N}\right]$$
$$\times\,\beta_\pi(t)\left[\delta_{\lambda_\gamma,+1}\delta_{\lambda_X,+1}-\tau_X\delta_{\lambda_\gamma,-1}\delta_{\lambda_X,-1}\right] \tag{8}$$

where

$$\beta_\pi(t) = \beta(t)\sum_{L_\gamma}\sqrt{\frac{2L_\gamma+1}{4\pi}}\langle L_\gamma 011|J_X 1\rangle g^{L_\gamma}_{X\gamma\pi}\left(\frac{\omega}{m_X}\right)^{L_\gamma}$$
$$= \beta(t)\sum_{L_\gamma}\left[\sqrt{\frac{J+1}{2}}\delta_{J,L+1}-\sqrt{\frac{2J+1}{2}}\delta_{J,L}+\sqrt{\frac{J}{2}}\delta_{J,L-1}\right]\left(\frac{\omega}{m_X}\right)^{L_\gamma}\frac{g^{L_\gamma}_{X\gamma\pi}}{\sqrt{4\pi}} \tag{9}$$

and β given by Eq. 5 in the OPE note, $\bar{g}_{\pi NN} = \sqrt{2}g_{\pi NN}2m_N = 35.7\text{GeV}$ and

$$R_\pi(s,t) = \alpha'\cos\left(\frac{\pi\alpha_\pi(t)}{2}\right)\Gamma(-\alpha_\pi(t))\left(\frac{s}{s_0}\right)^{\alpha_\pi(t)} \sim \frac{1}{t-m_\pi^2}\left(\frac{s}{s_0}\right)^{\alpha_\pi(t)} \tag{10}$$

with $\alpha_\pi(t) = \alpha'(t-m_\pi^2)$, $\alpha' = 0.9\text{GeV}^{-2}$

¿From Table 2. in the OPE note it follows that $g^{L_\gamma}_{X\gamma\pi}/\sqrt{4\pi}\sim 1$. Since I don't know the corresponding, $X\to\gamma\rho$ couplings, I would use something similar, say $g^{L_\gamma S_\gamma}_{X\gamma\rho}/\sqrt{4\pi} = 0.1-1$. Furthermore, for the exercise I would ignore the angular momentum factor (ω/m_X). The final expression to use is therefore given by Eq. 6 with

$$\beta_{1X}(t)\sim\beta_{2X}(t)\sim N_\rho\beta(t) \tag{11}$$

with

$$\beta(t) = (a_1 e^{b_1 t} + a_2 e^{b_2 t})^{1/2} \tag{12}$$

$b_1 \sim 35\mathrm{GeV}^{-2}$, $b_2 \sim 3\mathrm{GeV}^{-2}$, $a_1 \sim 30$, $a_2 \sim 1.5$. N_ρ fixes the relative normalization between OPE and ORE, $N_\rho \sim 1$.