Note on angular distributions in $\pi^+\pi^-\pi^+$ photoproduction

Adam Szczepaniak, 10/04/99

For the process $\vec{\gamma}p \to X^+ n \to \rho^0\pi^+ n \to \pi^+\pi^-\pi^+ n$ define $\mathbf{k} = (\sin\theta_k\cos\phi_k, \sin\theta_k\sin\phi_k, \cos\theta_k)$ to be the direction of flight of the π^- in the $\pi^+(-\mathbf{k})\pi^-(\mathbf{k})$ c.m. frame corresponding to the ρ^0 at rest. Similarly let $\mathbf{p} = (\sin\theta_p\cos\phi_p, \sin\theta_p\sin\phi_p, \cos\theta_p)$ be the direction of the momentum of the π^+ from the decay of the resonance X at rest.

The amplitude $A_i(\theta_p, \phi_p, \theta_k, \phi_k)$ describing the distribution of the 3π system coming form a photon linearly polarized along $i = \mathbf{x}, \mathbf{y}$ is given by

$$A_i(\Omega_p, \Omega_k) = A_i(\theta_p, \phi_p, \theta_k, \phi_k) = \sum_X \sum_{L_X} a_{X,L_X} A_i^{X,L_X}(\theta_p, \phi_p, \theta_k, \phi_k), \tag{1}$$

where the sum extends over all resonances, $X = a_1, a_2, \pi_2, \hat{\rho}$ and all contributing partial waves. The individual amplitudes are given by $(i = \mathbf{y})$,

$$\begin{aligned}
A_{\mathbf{y}}^{a_1,S} &= \frac{1}{4\pi}\sqrt{3}k^y \\
A_{\mathbf{y}}^{a_1,D} &= \frac{1}{4\pi}\sqrt{\frac{27}{2}}(p^y(\mathbf{p}\cdot\mathbf{k}) - \frac{1}{3}k^y) \\
A_{\mathbf{y}}^{a_2,D} &= \frac{1}{4\pi}\sqrt{\frac{15}{2}}(p^z(p^y k^z - p^z k^y) + p^x(p^x k^y - p^y k^x)) \\
A_{\mathbf{y}}^{\pi_2,P} &= \frac{1}{4\pi}\frac{3}{\sqrt{2}}(k^y p^z + k^z p^y) \\
A_{\mathbf{y}}^{\pi_2,F} &= \frac{1}{4\pi}\sqrt{75}(p^z p^y(\mathbf{p}\cdot\mathbf{k}) - \frac{1}{5}p^z k^y - \frac{1}{5}k^z p^y) \\
A_{\mathbf{y}}^{\hat{\rho},P} &= \frac{1}{4\pi}\frac{3}{\sqrt{2}}(k^y p^z - k^z p^y),
\end{aligned} \tag{2}$$

and the amplitudes $A_{\mathbf{x}}^{X,L_X}$ are obtained from the above by interchanging all x and y components. The angular distributions are normalized so that

$$\begin{aligned}
&\int d\Omega_k d\Omega_p W_{\mathbf{x}}(\Omega_p, \Omega_k) = \int d\Omega_k d\Omega_p W_{\mathbf{y}}(\Omega_p, \Omega_k) \\
&= \int d\Omega_k d\Omega_p |A_{\mathbf{x}}(\Omega_p, \Omega_k)|^2 = \int d\Omega_k d\Omega_p |A_{\mathbf{y}}(\Omega_p, \Omega_k)|^2 = \sum_{X,L_X} |a_{X,L_X}|^2.
\end{aligned} \tag{3}$$

In terms of photon helicity (circular polarization) the corresponding amplitudes, $A_\lambda(\Omega_p, \Omega_k)$, $\lambda = \pm$ are given by

$$A_+(\Omega_p,\Omega_k) = -\frac{1}{\sqrt{2}}(A_{\mathbf{x}}(\Omega_p,\Omega_k) + iA_{\mathbf{y}}(\Omega_p,\Omega_k))$$
$$A_-(\Omega_p,\Omega_k) = +\frac{1}{\sqrt{2}}(A_{\mathbf{x}}(\Omega_p,\Omega_k) - iA_{\mathbf{y}}(\Omega_p,\Omega_k)). \tag{4}$$

In general if $\vec{P}_\gamma$ represents the direction of photon linear polarization

$$\vec{P}_\gamma = P_\gamma(-\cos 2\Phi, -\sin 2\Phi, 0) \tag{5}$$

with $0 \leq P_\gamma \leq 1$ and Φ being the angle between the polarization vector of the photon and the production plane (x,z plane), $\epsilon = (\cos\Phi, \sin\Phi, 0)$ the intensity is given by

$$W(\Omega_p,\Omega_k) = \sum_{\lambda',\lambda=\pm} A^*_{\lambda}(\Omega_p,\Omega_k)\left[\frac{1}{2}\mathbf{I} + \frac{1}{2}\mathbf{P}_\gamma\cdot\boldsymbol{\sigma}\right] A_\lambda(\Omega_p,\Omega_k)$$
$$= |A_{\mathbf{x}}(\Omega_p,\Omega_k)|^2\frac{(1+P_\gamma\cos 2\Phi)}{2} + |A_{\mathbf{y}}(\Omega_p,\Omega_k)|^2\frac{(1-P_\gamma\cos 2\Phi)}{2} - P_\gamma \sin 2\Phi \Re(A^*_{\mathbf{x}}(\Omega_p,\Omega_k)A_{\mathbf{y}}(\Omega_p,\Omega_k)). \tag{6}$$