Angular distributions for the PWA study

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June 14, 2000

1 Introduction

In this document, we want to look at the photo–produced $a_2(1320)$, and examine the effects of photon polarization on various observables. We start with the assumption that the a_2 is produced via π exchange. This implies that for the a_2, the only allowed values of m_j are ± 1. We will look at the reaction:

$$a_2^+ \to \rho^\circ \pi^+ \to (\pi^+\pi^-)\pi^+$$

In the Gottfried–Jackson, (GJ), frame, the a_2 decays with decay angles θ and ϕ. We then move into the ρ helicity frame, where the ρ decays with angles θ' and ϕ'.

Under these assumptions, the most general density matrix for the initial a_2, $\rho_{[a_2 a_2']}$ can be written as in 1 in terms of four parameters. There are also two relations between these, $\alpha + \beta = 1$ and $\delta = \gamma^*$. We can simplify this by writeing $\eta = \mathrm{Real}(\gamma)$ and $\xi = \mathrm{Imaginary}(\gamma)$.

$$\rho_{[a_2 a_2']} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & \gamma & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \delta & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & \eta + i\xi & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \eta - i\xi & 0 & \beta & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \quad (1)$$

The density matrix for the ρ can be obtained from the a_2 via the operation in 2.

$$\rho_{[\rho\rho']} = \sum_{a_2, a_2'} f_{[a_2 a_2'; \rho\rho']} \rho_{[a_2 a_2']} f^\dagger_{[a_2 a_2'; \rho\rho']} \quad (2)$$

The a_2 decays to $\rho\pi$ with relative orbital angular momentum of $L = 2$, which means that the transition amplitudes, f depend on the T's in equation 3.

$$T_{[\lambda_1 \lambda_2]} = \langle J\lambda \mid LS0\lambda \rangle \langle S\lambda \mid S_1 S_2 \lambda_1 - \lambda_2 \rangle \quad (3)$$

There are three possible non–zero terms, of which two turn out to be non–zero. These are given below.

$$\begin{aligned} T_{[10]} &= \langle 21 \mid 2101 \rangle \langle 11 \mid 1010 \rangle &&= -\frac{1}{\sqrt{2}} \\ T_{[00]} &= \langle 20 \mid 2100 \rangle \langle 10 \mid 1000 \rangle &&= 0 \\ T_{[-10]} &= \langle 2-1 \mid 210-1 \rangle \langle 1-1 \mid 10-10 \rangle &&= \frac{1}{\sqrt{2}} \end{aligned}$$

The transition amplitude, f can then be written as in equation 4. We have simplified f by explicitly setting to zero all terms that cannot contribute the $\rho_{[\rho\rho']}$. In particular, $T_{[00]}$ removes all of the d^2_{m0} terms, and the density matrix of the a_2 means that only the $d^2_{\pm1\pm1}$ terms will remain.

$$f_{[a_2 a_2'; \rho\rho']} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} d^2_{11}(\theta) e^{i\phi} & 0 & -\frac{1}{\sqrt{2}} d^2_{1-1}(\theta) e^{-i\phi} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} d^2_{-11}(\theta) e^{i\phi} & 0 & \frac{1}{\sqrt{2}} d^2_{-1-1}(\theta) e^{-i\phi} & 0 \end{pmatrix} \quad (4)$$

The d functions are given as:

$$\begin{aligned} d^2_{11}(\theta) &= d^2_{-1-1}(\theta) &= \frac{1}{2}(1+\cos\theta)(2\cos\theta-1) == A(\theta) \\ d^2_{1-1}(\theta) &= d^2_{-11}(\theta) &= \frac{1}{2}(1-\cos\theta)(2\cos\theta+1) == B(\theta) \end{aligned}$$

We can now use equation 2 to find that the density matrix for the ρ. We take $\eta = \text{Real}(\gamma)$ and $\xi = \text{Imaginary}(\gamma)$, and find that $\rho_{[\rho\rho']}$ is given as:

$$\begin{pmatrix} \frac{1}{2}(\alpha A^2+\beta B^2)+AB\left[\eta\cos 2\phi-\xi\sin 2\phi\right] & 0 & -\frac{1}{2}AB-\frac{1}{2}(A^2+B^2)\left[\eta\cos 2\phi-\xi\sin 2\phi\right] \\ 0 & 0 & 0 \\ -\frac{1}{2}AB-\frac{1}{2}(A^2+B^2)\left[\eta\cos 2\phi-\xi\sin 2\phi\right] & 0 & \frac{1}{2}(\alpha B^2+\beta A^2)+AB\left[\eta\cos 2\phi-\xi\sin 2\phi\right] \end{pmatrix}$$

The angular distribution of the spectator π^+ as seen in the GJ frame is given as the trace of ρ. We can extract this and find the angular distribution as is 5. We note that if the a_2 is unpolarized, then $\alpha = \beta = \frac{1}{2}$ and $\eta = \xi = 0$. This then yields the distribution in equation 6 which is independent of ϕ.

$$w(\theta,\phi) = \frac{1}{2}(A^2+B^2) + 2AB\left[\left[\eta\cos 2\phi - \xi\sin 2\phi\right]\right. \tag{5}$$

$$w_{\text{unpolarized}}(\theta,\phi) = \frac{1}{2}(A^2+B^2) \tag{6}$$

We can now continue this by looking at the angular distributions of the π^+ and π^- in the helicity frame, $w(\theta',\phi')$. We can rewrite $\rho_{[\rho\rho']}$ in terms of three real parameters:

$$\begin{aligned} C(\theta,\phi) &= \frac{1}{2}\left[\alpha A^2(\theta)+\beta B^2(\theta)\right] + A(\theta)B(\theta)\left[\eta\cos 2\phi-\xi\sin 2\phi\right] \\ D(\theta,\phi) &= \frac{1}{2}\left[\alpha B^2(\theta)+\beta A^2(\theta)\right] + A(\theta)B(\theta)\left[\eta\cos 2\phi-\xi\sin 2\phi\right] \\ E(\theta,\phi) &= -\frac{1}{2}A(\theta)B(\theta) - \frac{1}{2}\left[A^2(\theta)+B^2(\theta)\right]\left[\eta\cos 2\phi-\xi\sin 2\phi\right] \end{aligned}$$

These can then be used with equation 7 to determine the angular distributions in the helicity frame.

$$w(\theta,\phi;\theta',\phi') = \sum_{\rho\rho'} f_{[\rho\rho']}\rho_{[\rho\rho']}f^\dagger_{[\rho\rho']} \tag{7}$$

The transition amplitudes for $\rho \to \pi\pi$ are given as follows:

$$f_{[\rho\rho']} = \begin{pmatrix} d^1_{01}(\theta')e^{i\phi'} & d^1_{00}(\theta') & d^1_{0-1}(\theta')e^{-i\phi'} \end{pmatrix}$$

which when combined with the density matrix, and using the fact that $d^1_{01}(\theta' = \frac{1}{2}\sin\theta'$ and $d^1_{0-1}(\theta' = -\frac{1}{2}\sin\theta'$, we find equation 8.

$$w(\theta,\phi;\theta',\phi') = \sin^2\theta'\left\{\frac{1}{2}\left[C(\theta,\phi)+D(\theta,\phi)\right] - E(\theta,\phi)\cos 2\phi'\right\} \tag{8}$$

It is interesting to look carefully at this weight. The ϕ' dependence is given entirely by the $\cos 2\phi'$ term. Note that the parameter E is non zero in both the case of polarized and unpolarized a_2. As such, the ϕ' dependence is *independent* of the polarization. What is true is that the size of the C, D and E terms do depend on the polarization, the the size of the $\cos 2\phi'$ piece relative to the flat piece can vary with the polarization. We can now replace C, D and E with their expansions in terms of A and B. We now find that equation can be written as:

$$\begin{aligned} w(\theta,\phi;\theta',\phi') &= \sin^2\theta' \left\{ \frac{1}{4}\left(A(\theta)+B(\theta)\right)^2 - A(\theta)B(\theta)\sin^2\phi' \right. \\ &+ \left. \left[\frac{1}{2}\left(A(\theta)+B(\theta)\right)^2 - \left(A^2\theta)+B^2(\theta)\right)\sin^2\phi'\right]\left[\eta\cos 2\phi - \xi\sin 2\phi\right]\right\} . \end{aligned}$$

This says that the π^+ and π^- from the ρ decay have the same ϕ dependence in th GJ frame as the π^+ recoiling against the ρ. In fact, becuase of symmetrization of the two π^+'s, the clearest place to see this distribution will be the ϕ distribution of the π^- in the GJ frame!