

Parity Relations in the Reflectivity Basis

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Parity Relations of Production Strengths

We want to consider the photo-production process $\gamma N \rightarrow X N'$ and to derive parity relations between the production strengths, V , as seen in the reflectivity basis. These will then be combined with the corresponding decay amplitudes A as computed in the reflectivity basis to allow us to compute event weights for our PWA analysis. We start initially with conservation of parity in the helicity frame. For the process $\gamma N \rightarrow X N'$, we know that the following relationship holds.

$$V_{-\lambda_x - \lambda_{N'}; -\lambda_\gamma - \lambda_N} = P_\gamma P_x P_N P_{N'} (-1)^{(J_x + J_{N'} - J_\gamma - J_N)} (-1)^{[(\lambda_\gamma - \lambda_N) - (\lambda_x - \lambda_{N'})]} V_{\lambda_x \lambda_{N'}; \lambda_\gamma \lambda_N}$$

The λ_i refer to the helicities of the indicated particles, the J_i refer to the spins of the particles, and the P_i refer to the parity of the particles. This can be simplified by noting first that three of the parities can be eliminated:

$$P_\gamma P_N P_{N'} = -1.$$

Next, we can eliminate a lot of the spin factors as:

$$(-1)^{J_{N'} - J_\gamma - J_N} = (-1)^{-J_\gamma} = -1.$$

and then we know that the naturality of X is given as:

$$\eta_x = P_x (-1)^{J_x}.$$

Finally, we can also simplify the term involving λ_i . For the case of photo-production we know that $\lambda_\gamma = \pm 1$ and also that $\lambda_x = \pm 1$. This means that $\lambda_\gamma - \lambda_x = (-2, 0, 2)$, and therefore $(-1)^{(\lambda_\gamma - \lambda_x)} = +1$. We can now simplify the parity relationship to:

$$V_{-\lambda_x - \lambda_{N'}; -\lambda_\gamma - \lambda_N} = \eta_x (-1)^{(\lambda_{N'} - \lambda_N)} V_{\lambda_x \lambda_{N'}; \lambda_\gamma \lambda_N}$$

For the case of spin-flip, $\lambda_{N'} - \lambda_N = \pm 1$, while for the case of spin-non-flip, $\lambda_{N'} - \lambda_N = 0$ which yields:

$$\begin{aligned} V_{-\lambda_x - \lambda_{N'}; -\lambda_\gamma - \lambda_N} &= -\eta_x V_{\lambda_x \lambda_{N'}; \lambda_\gamma \lambda_N} \text{ spin flip} \\ V_{-\lambda_x - \lambda_{N'}; -\lambda_\gamma - \lambda_N} &= \eta_x V_{\lambda_x \lambda_{N'}; \lambda_\gamma \lambda_N} \text{ spin nonflip} \end{aligned}$$

We can now write these production strengths for the $m_x = \lambda_x$ states in the reflectivity basis for a produced particle of naturality η_x in a reflectivity state ϵ_x . In this case,

$${}^\epsilon V_{\lambda_N \lambda_{N'} \epsilon_x} = \frac{1}{\sqrt{2}} \left[V_{[\lambda_N \lambda_{N'} \lambda_\gamma = +1 \lambda_x = +|m_x|]} + \epsilon_x \eta_x (-1)^{|m_x|} V_{[\lambda_N \lambda_{N'} \lambda_\gamma = -1 \lambda_x = -|m_x|]} \right]$$

Let us now take the case where we only have a spin-flip contributing. In addition, helicity conservation will limit the values of λ_x to be λ_γ . In this case $\lambda_{N'} = -\lambda_N$, and we can write that:

$$\begin{aligned} {}^\epsilon V_{\lambda_N - \lambda_N \epsilon_x^\pm} &= \frac{1}{\sqrt{2}} [V_{[\lambda_N - \lambda_N \lambda_\gamma = +1 \lambda_x = +1]} - \epsilon_x \eta_x V_{[\lambda_N - \lambda_N \lambda_\gamma = -1 \lambda_x = -1]}] \\ {}^\epsilon V_{-\lambda_N \lambda_N \epsilon_x^\pm} &= \frac{1}{\sqrt{2}} [V_{[-\lambda_N \lambda_N \lambda_\gamma = +1 \lambda_x = +1]} - \epsilon_x \eta_x V_{[-\lambda_N \lambda_N \lambda_\gamma = -1 \lambda_x = -1]}] \\ &= \frac{-\eta_x}{\sqrt{2}} [V_{[\lambda_N - \lambda_N \lambda_\gamma = -1 \lambda_x = -1]} - \epsilon_x \eta_x V_{[\lambda_N - \lambda_N \lambda_\gamma = +1 \lambda_x = +1]}] \\ &= \frac{+\epsilon_x}{\sqrt{2}} [V_{[\lambda_N - \lambda_N \lambda_\gamma = +1 \lambda_x = +1]} - \epsilon_x \eta_x V_{[\lambda_N - \lambda_N \lambda_\gamma = -1 \lambda_x = -1]}] \\ {}^\epsilon V_{-\lambda_N \lambda_N \epsilon_x^\pm} &= \epsilon_x^\epsilon V_{\lambda_N - \lambda_N \epsilon_x^\pm} \end{aligned}$$

For the case of only spin-non-flip contributions, we have that $\lambda_{N'} = \lambda_N$, and we can write that:

$$\begin{aligned} {}^\epsilon V_{\lambda_N \lambda_N \epsilon_x^\pm} &= \frac{1}{\sqrt{2}} [V_{[\lambda_N \lambda_N \lambda_\gamma = +1 \lambda_x = +1]} - \epsilon_x \eta_x V_{[\lambda_N \lambda_N \lambda_\gamma = -1 \lambda_x = -1]}] \\ {}^\epsilon V_{-\lambda_N - \lambda_N \epsilon_x^\pm} &= \frac{1}{\sqrt{2}} [V_{[-\lambda_N - \lambda_N \lambda_\gamma = +1 \lambda_x = +1]} - \epsilon_x \eta_x V_{[-\lambda_N - \lambda_N \lambda_\gamma = -1 \lambda_x = -1]}] \\ &= \frac{\eta_x}{\sqrt{2}} [V_{[\lambda_N \lambda_N \lambda_\gamma = -1 \lambda_x = -1]} - \epsilon_x \eta_x V_{[\lambda_N \lambda_N \lambda_\gamma = +1 \lambda_x = +1]}] \\ &= \frac{-\epsilon_x}{\sqrt{2}} [V_{[\lambda_N \lambda_N \lambda_\gamma = +1 \lambda_x = +1]} - \epsilon_x \eta_x V_{[\lambda_N \lambda_N \lambda_\gamma = -1 \lambda_x = -1]}] \\ {}^\epsilon V_{-\lambda_N - \lambda_N \epsilon_x^\pm} &= -\epsilon_x^\epsilon V_{\lambda_N \lambda_N \epsilon_x^\pm} \end{aligned}$$

The Photon Spin–Density Matrix

The last part is to rotate the spin–density matrix of the photon from the helicity frame, $\rho_{\lambda_\gamma \lambda'_\gamma}$ to the reflectivity basis $\rho_{\epsilon_\gamma \epsilon'_\gamma}$. Where $\rho_{\lambda_\gamma \lambda'_\gamma} = |\lambda_\gamma\rangle \langle \lambda'_\gamma|$. To do this, we recall that the basis states can be transformed as:

$$\begin{aligned} \langle \epsilon_\gamma | &= \frac{1}{\sqrt{2}} [\langle \lambda = +1 | + \epsilon_\gamma \eta_\gamma (-1)^{|m_\gamma|} \langle \lambda = -1 |] \\ &= \frac{1}{\sqrt{2}} [\langle \lambda = +1 | - \epsilon_\gamma \langle \lambda = -1 |] \\ \langle \epsilon_\gamma = +1 | &= \frac{1}{\sqrt{2}} [\langle \lambda = +1 | - \langle \lambda = -1 |] \\ \langle \epsilon_\gamma = -1 | &= \frac{1}{\sqrt{2}} [\langle \lambda = +1 | + \langle \lambda = -1 |]. \end{aligned}$$

There is also a simple relation between the reflectivity states and the linear polarization states, $|\epsilon_\gamma = +1\rangle = |x\rangle$ and $|\epsilon_\gamma = -1\rangle = -i|y\rangle$. We can now multiply

out to obtain the four elements of the spin-density matrix in the reflectivity basis.

$$\begin{aligned}\rho_{++} &= \frac{1}{2} [\rho_{11} - \rho_{1-1} - \rho_{-11} + \rho_{-1-1}] \\ \rho_{+-} &= \frac{1}{2} [\rho_{11} + \rho_{1-1} - \rho_{-11} - \rho_{-1-1}] \\ \rho_{-+} &= \frac{1}{2} [\rho_{11} - \rho_{1-1} + \rho_{-11} - \rho_{-1-1}] \\ \rho_{--} &= \frac{1}{2} [\rho_{11} + \rho_{1-1} + \rho_{-11} + \rho_{-1-1}]\end{aligned}$$

We also know that $\rho_{11} + \rho_{-1-1} = +1$ and that $\rho_{-1+1} = \rho_{+1-1}$. Using this, we can simplify the spin-density matrix as follows:

$$\rho_{\epsilon_\gamma \epsilon'_\gamma} = \begin{pmatrix} \left[\frac{1}{2} - \rho_{1-1} \right] & \left[\frac{1}{2} - \rho_{-1-1} \right] \\ \left[\frac{1}{2} - \rho_{-1-1} \right] & \left[\frac{1}{2} + \rho_{1-1} \right] \end{pmatrix}$$

For the case of an unpolarized photon, we know that $\rho_{11} = \rho_{-1-1} = \frac{1}{2}$ and that $\rho_{1-1} = \rho_{-11} = 0$. This means that the spin-density matrix is as expected:

$$\rho_{\epsilon_\gamma \epsilon'_\gamma} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}.$$

For the case of linearly polarized photons, we know that $\rho_{1-1} = \frac{1}{2} \cos 2\alpha$ where α is the angle between the polarization vector and the normal to the production plane as seen in the Gottfried Jackson frame. In addition, $\rho_{11} = \frac{1}{2}(1 + \sin 2\alpha)$ and $\rho_{-1-1} = \frac{1}{2}(1 - \sin 2\alpha)$. This allows us to simplify the spin-density matrix to:

$$\rho_{\epsilon_\gamma \epsilon'_\gamma} = \begin{pmatrix} \sin^2 \alpha & \frac{1}{2} \sin 2\alpha \\ \frac{1}{2} \sin 2\alpha & \cos^2 \alpha \end{pmatrix}.$$

Specific Examples

Now, for the case of a_2 production via π exchange, there are four complex production amplitudes, $V_{[\lambda_N - \lambda_N \epsilon_x]}$. There are two possible spin-flips for each of the two reflectivity. However, parity will reduce this down to two complex production strengths, $V_{[-1,1,+]} = V_{[1,-1,+]}$ and $V_{[-1,1,-]} = -V_{[1,-1,-]}$. If we write $A_{|m|=1}^+ [a_2 \rightarrow \rho\pi]$ and $A_{|m|=1}^- [a_2 \rightarrow \rho\pi]$ as the decay amplitudes for the two reflectivity states of the a_2 , then the weight for a particular event is given as:

$$\begin{aligned} w &= 2 \left\{ \rho_{++} | V_{[-1,1,+]} A_{|m|=1}^+ [a_2 \rightarrow \rho\pi] |^2 + \rho_{--} | V_{[-1,1,-]} A_{|m|=1}^- [a_2 \rightarrow \rho\pi] |^2 \right. \\ &\quad + \rho_{+-} (V_{[-1,1,+]} A_{|m|=1}^+ [a_2 \rightarrow \rho\pi])^* (V_{[-1,1,-]} A_{|m|=1}^- [a_2 \rightarrow \rho\pi]) \\ &\quad \left. + \rho_{-+} (V_{[-1,1,-]} A_{|m|=1}^- [a_2 \rightarrow \rho\pi])^* (V_{[-1,1,+]} A_{|m|=1}^+ [a_2 \rightarrow \rho\pi]) \right\} \end{aligned}$$

Using the density matrix from above and noting the fact that $a^*b + b^*a = 2\text{Re}(ab)$, we can somewhat simplify the above expression to be proportional to:

$$w = \sin^2 \alpha | R^+ |^2 + \cos^2 \alpha | R^- |^2 + \sin \alpha \cos \alpha \text{Re}(R^+ R^-)$$

Where R^\pm is the product of V^\pm times A^\pm .