# Parity Relations in the Reflectivity Basis

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# Parity Relations of Production Strengths

We want to consider the photo-production process $\gamma N \to XN'$ and to derive partity relations between the production strengths, $V$, as seen in the reflectivity basis. These will then be combined with the coresponding decay amplitudes $A$ as cmputed in the reflectivity basis to allow us to compute event weigths for our PWA analysis. We start initially with conservation of parity in the helicity frame. For the process $\gamma N \to XN'$, we know that the following relation ship holds.

$$V_{-\lambda_x-\lambda_{N'};-\lambda_\gamma-\lambda_N} = P_\gamma P_x P_N P_{N'}(-1)^{(J_x+J_{N'}-J_\gamma-J_N)}(-1)^{[(\lambda_\gamma-\lambda_N)-(\lambda_x-\lambda_{N'})]}V_{\lambda_x\lambda_{N'};\lambda_\gamma\lambda_N}$$

The $\lambda_i$ refer to the helicities of the indicated particles, the $J_i$ refer to the spins of the particles, and the $P_i$ refer to the parity of the particles. This can be simplified by noting first that three of the parities can be eliminated:

$$P_\gamma P_N P_{N'} = -1.$$

Next, we can eliminate a lot of the spin factors as:

$$(-1)^{J_{N'}-J_\gamma-J_N} = (-1)^{-J_\gamma} = -1.$$

and then we know that the naturality of $X$ is given as:

$$\eta_x = P_x(-1)^{J_x}.$$

Finally, we can also simplify the term involving $\lambda_i$. For the case of photo-production we know that $\lambda_\gamma = \pm 1$ and also that $\lambda_x = \pm 1$. This means that $\lambda_\gamma - \lambda_x = (-2, 0, 2)$, and therefore $(-1)^{(\lambda_\gamma-\lambda_x)} = +1$. We can now simplify the parity relationship to:

$$V_{-\lambda_x-\lambda_{N'};-\lambda_\gamma-\lambda_N} = \eta_x(-1)^{(\lambda_{N'}-\lambda_N)}V_{\lambda_x\lambda_{N'};\lambda_\gamma\lambda_N}$$

For the case of spin–flip , $\lambda_{N'} - \lambda_N = \pm 1$, while for the case of spin–non–flip , $\lambda_{N'} - \lambda_N = 0$ which yields:

$$\begin{aligned} V_{-\lambda_x-\lambda_{N'};-\lambda_\gamma-\lambda_N} &= -\eta_x V_{\lambda_x\lambda_{N'};\lambda_\gamma\lambda_N} \text{ spin flip} \\ V_{-\lambda_x-\lambda_{N'};-\lambda_\gamma-\lambda_N} &= \eta_x V_{\lambda_x\lambda_{N'};\lambda_\gamma\lambda_N} \text{ spin nonflip} \end{aligned}$$

We can now write these production strengths for the $m_x = \lambda_x$ states in the reflectivity basis for a produced particle of naturality $\eta_x$ in a reflectivity state $\epsilon_x$. In this case,

$$^{\epsilon}V_{\lambda_N\lambda_{N'}\epsilon_x} = \frac{1}{\sqrt{2}}\left[V_{[\lambda_N\lambda_{N'}\lambda_\gamma=+1\lambda_x=+|m_x|]} + \epsilon_x\eta_x(-1)^{|m_x|}V_{[\lambda_N\lambda_{N'}\lambda_\gamma=-1\lambda_x=-|m_x|]}\right]$$

Let us now take the case where we only have a spin-flip contributing. In addition, helicity conservation will limit the values of $\lambda_x$ to be $\lambda_\gamma$. In this case $\lambda_{N'} = -\lambda_N$, and we can write that:

$$
\begin{aligned}
{}^{\epsilon}V_{\lambda_N -\lambda_N \epsilon_x^\pm} &= \frac{1}{\sqrt{2}}\left[V_{[\lambda_N -\lambda_N \lambda_\gamma=+1 \lambda_x=+1]} - \epsilon_x\eta_x V_{[\lambda_N -\lambda_N \lambda_\gamma=-1 \lambda_x=-1]}\right] \\
{}^{\epsilon}V_{-\lambda_N \lambda_N \epsilon_x^\pm} &= \frac{1}{\sqrt{2}}\left[V_{[-\lambda_N \lambda_N \lambda_\gamma=+1 \lambda_x=+1]} - \epsilon_x\eta_x V_{[-\lambda_N \lambda_N \lambda_\gamma=-1 \lambda_x=-1]}\right] \\
&= \frac{-\eta_x}{\sqrt{2}}\left[V_{[\lambda_N -\lambda_N \lambda_\gamma=-1 \lambda_x=-1]} - \epsilon_x\eta_x V_{[\lambda_N -\lambda_N \lambda_\gamma=+1 \lambda_x=+1]}\right] \\
&= \frac{+\epsilon_x}{\sqrt{2}}\left[V_{[\lambda_N -\lambda_N \lambda_\gamma=+1 \lambda_x=+1]} - \epsilon_x\eta_x V_{[\lambda_N -\lambda_N \lambda_\gamma=-1 \lambda_x=-1]}\right] \\
{}^{\epsilon}V_{-\lambda_N \lambda_N \epsilon_x^\pm} &= \epsilon_x^{\epsilon} V_{\lambda_N -\lambda_N \epsilon_x^\pm}
\end{aligned}
$$

For the case of only spin-non-flip contributions, we have that $\lambda_{N'} = \lambda_N$, and we can write that:

$$
\begin{aligned}
{}^{\epsilon}V_{\lambda_N \lambda_N \epsilon_x^\pm} &= \frac{1}{\sqrt{2}}\left[V_{[\lambda_N \lambda_N \lambda_\gamma=+1 \lambda_x=+1]} - \epsilon_x\eta_x V_{[\lambda_N \lambda_N \lambda_\gamma=-1 \lambda_x=-1]}\right] \\
{}^{\epsilon}V_{-\lambda_N -\lambda_N \epsilon_x^\pm} &= \frac{1}{\sqrt{2}}\left[V_{[-\lambda_N -\lambda_N \lambda_\gamma=+1 \lambda_x=+1]} - \epsilon_x\eta_x V_{[-\lambda_N -\lambda_N \lambda_\gamma=-1 \lambda_x=-1]}\right] \\
&= \frac{\eta_x}{\sqrt{2}}\left[V_{[\lambda_N \lambda_N \lambda_\gamma=-1 \lambda_x=-1]} - \epsilon_x\eta_x V_{[\lambda_N \lambda_N \lambda_\gamma=+1 \lambda_x=+1]}\right] \\
&= \frac{-\epsilon_x}{\sqrt{2}}\left[V_{[\lambda_N \lambda_N \lambda_\gamma=+1 \lambda_x=+1]} - \epsilon_x\eta_x V_{[\lambda_N \lambda_N \lambda_\gamma=-1 \lambda_x=-1]}\right] \\
{}^{\epsilon}V_{-\lambda_N -\lambda_N \epsilon_x^\pm} &= -\epsilon_x^{\epsilon} V_{\lambda_N \lambda_N \epsilon_x^\pm}
\end{aligned}
$$

# The Photon Spin–Density Matrix

The last part is to rotate the spin–density matrix of the photon from the helicity frame, $\rho_{\lambda_\gamma\lambda'_\gamma}$ to the reflectivity basis $\rho_{\epsilon_\gamma\epsilon'_\gamma}$. Where $\rho_{\lambda_\gamma\lambda'_\gamma} = \mid \lambda_\gamma >< \lambda'_\gamma \mid$. To do this, we recall that the basis states can be transformed as:

$$
\begin{aligned}
< \epsilon_\gamma \mid &= \frac{1}{\sqrt{2}}\left[< \lambda = +1 \mid + \epsilon_\gamma\eta_\gamma(-1)^{|m_\gamma|} < \lambda = -1 \mid\right] \\
&= \frac{1}{\sqrt{2}}\left[< \lambda = +1 \mid - \epsilon_\gamma < \lambda = -1 \mid\right]
\end{aligned}
$$

$$
\begin{aligned}
< \epsilon_\gamma = +1 \mid &= \frac{1}{\sqrt{2}}\left[< \lambda = +1 \mid - < \lambda = -1 \mid\right] \\
< \epsilon_\gamma = -1 \mid &= \frac{1}{\sqrt{2}}\left[< \lambda = +1 \mid + < \lambda = -1 \mid\right].
\end{aligned}
$$

There is also a simple relation between the reflectivity states and the linear polarization states, $\mid \epsilon_\gamma = +1 >=\mid x >$ and $\mid \epsilon_\gamma = -1 >= -i \mid y >$. We can now multiply out to obtain the four elements of the spin–density matrix in the reflectivity basis.

$$\begin{aligned}
\rho_{++} &= \frac{1}{2}\left[\rho_{11} - \rho_{1-1} - \rho_{-11} + \rho_{-1-1}\right] \\
\rho_{+-} &= \frac{1}{2}\left[\rho_{11} + \rho_{1-1} - \rho_{-11} - \rho_{-1-1}\right] \\
\rho_{-+} &= \frac{1}{2}\left[\rho_{11} - \rho_{1-1} + \rho_{-11} - \rho_{-1-1}\right] \\
\rho_{--} &= \frac{1}{2}\left[\rho_{11} + \rho_{1-1} + \rho_{-11} + \rho_{-1-1}\right]
\end{aligned}$$

We also know that $\rho_{11} + \rho_{-1-1} = +1$ and that $\rho_{-1+1} = \rho_{+1-1}$. Using this, we can simplify the spin-density matrix as follows:

$$\rho_{\epsilon_\gamma \epsilon'_\gamma} = \begin{pmatrix} \left[\frac{1}{2} - \rho_{1-1}\right] & \left[\frac{1}{2} - \rho_{-1-1}\right] \\ \left[\frac{1}{2} - \rho_{-1-1}\right] & \left[\frac{1}{2} + \rho_{1-1}\right] \end{pmatrix}$$

For the case of an unpolarized photon, we know that $\rho_{11} = \rho_{-1-1} = \frac{1}{2}$ and that $\rho_{1-1} = \rho_{-11} = 0$. This means that the spin–density matrix is as expected:

$$\rho_{\epsilon_\gamma \epsilon'_\gamma} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}.$$

For the case of linearly polarized photons, we know that $\rho_{1-1} = \frac{1}{2}\cos 2\alpha$ where $\alpha$ is the angle between the polarization vector and the normal to the production plane as seen in the Gottfried Jackson frame. In addition, $\rho_{11} = \frac{1}{2}(1 + \sin 2\alpha)$ and $\rho_{-1-1} = \frac{1}{2}(1 - \sin 2\alpha)$. This allows us to simplify the spin–density matrix to:

$$\rho_{\epsilon_\gamma \epsilon'_\gamma} = \begin{pmatrix} \sin^2\alpha & \frac{1}{2}\sin 2\alpha \\ \frac{1}{2}\sin 2\alpha & \cos^2\alpha \end{pmatrix}.$$

## Specific Examples

Now, for the case of $a_2$ production via $\pi$ exchange, there are four complex production amplitudes, $V_{[\lambda_N - \lambda_N \epsilon_x]}$. There are two possible spin-flips for each of the two reflectivity. However, parity will reduce this down to two complex production strengths, $V_{[-1,1,+]} = V_{[1,-1,+]}$ and $V_{[-1,1,-]} = -V_{[1,-1,-]}$. If we write $A^{+}_{|m|=1}[a_2 \to \rho\pi]$ and $A^{-}_{|m|=1}[a_2 \to \rho\pi]$ as the decay amplitudes for the two reflectivity states of the $a_2$, then the weight for a particular event is given as:

$$
\begin{aligned}
w &= 2\Big\{\rho_{++} \mid V_{[-1,1,+]}A^{+}_{|m|=1}[a_2 \to \rho\pi] \mid^2 + \rho_{--} \mid V_{[-1,1,-]}A^{-}_{|m|=1}[a_2 \to \rho\pi] \mid^2 \\
&+ \rho_{+-}(V_{[-1,1,+]}A^{+}_{|m|=1}[a_2 \to \rho\pi])^{*}(V_{[-1,1,-]}A^{-}_{|m|=1}[a_2 \to \rho\pi]) \\
&+ \rho_{-+}(V_{[-1,1,-]}A^{-}_{|m|=1}[a_2 \to \rho\pi])^{*}(V_{[-1,1,+]}A^{+}_{|m|=1}[a_2 \to \rho\pi])\Big\}
\end{aligned}
$$

Using the density matrix from above and noting the fact that $a^*b + b^*a = 2\mathrm{Re}(ab)$, we can somewhat simplify the above expression to be proportional to:

$$
w = \sin^2\alpha \mid R^{+} \mid^2 + \cos^2\alpha \mid R^{-} \mid^2 + \sin\alpha\cos\alpha \mathrm{Re}(R^{+}R^{-})
$$

Where $R^{\pm}$ is the product of $V^{\pm}$ times $A^{\pm}$.