BCAL Radiation Length Calculations

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Abstract

Analytical calculations of the radiation length for the Barrel Calorimeter of the GlueX Project are presented. These show that the full thickness radiation length ranges from $18-20 X_0$ depending on the thickness of the aluminum backing plate and the analytical formalism used.

Key words: radiation length, scintillating fiber, barrel calorimeter
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1 Introduction

The electro-magnetic barrel calorimeter (BCAL) for the GlueX Project [1–3] consists of alternating layers of thin ($0.5\,mm$) lead sheets and 1-mm-diameter scintillating fibers (SciFi). The lead sheets are grooved (plastically deformed or swaged) after passing through a swaging machine. The fibers are glued in the resulting grooves by using an optical epoxy. The resulting matrix has a fiber pitch of $1.35\,mm$ in the horizontal direction and $1.18\,mm$ in the vertical. The BCAL is segmented into 48 modules with each module comprised of approximately 18,300 4-m-long fibers, thus requiring a total of over 3,500 km of fibers.

The dominant energy loss mechanism for photons in a calorimeter is the $\gamma \rightarrow e^+e^-$ reaction, via consecutive pair production and bremsstrahlung processes. The quantity that best characterizes the penetration depth of a photon shower is the radiation length, X_0. This quantity is expressed in units of cm or g/cm^2. The radiation length [4]:

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- describes the mean distance over which a high energy electron loses all but $1/e$ of its energy;
- represents 7/9 of the mean free path for pair production by a high energy photon;
- is an appropriate scale for describing high energy electro-magnetic cascades; and
- is a scaling variable for the probability of occurrence of bremsstrahlung and pair production as well as for the variance of the angle of multiple scattering.

Three formulae can be used to determine X_0 (see references [5], [6], and [7] respectively). The second and third of these provide a clearer view of the functional dependence on Z, and they include screening effects of the nuclear field by the atomic electrons. The third equation is an approximation while the second a result of a compact fit to the data with an accuracy of better than 2.5% for all elements except He.

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A} \left(Z^2 [L_{rad} - f(Z)] + Z L'_{rad} \right) \tag{1}$$

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A} \left(Z(Z+1) \ln \frac{287}{\sqrt{Z}} \right) \tag{2}$$

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A} \left(Z(Z+1) \ln \frac{183}{Z^{\frac{1}{3}}} \right) \tag{3}$$

In these, A=1 gmol^{-1}, $4\alpha r_e^2 \frac{N_A}{A} = (716.408 \; g \; cm^{-2})^{-1}$, and

$$f(Z) \approx a^2 \left[(1+a^2)^{-1} + 0.20206 - 0.0369a^2 + 0.0083a^4 - 0.002a^6 \right] \tag{4}$$

where $a = \alpha Z$ [8]. The function $f(Z)$ is an infinite sum, but for elements up to uranium it is accurate to four significant figures. For $Z > A$ we have $L_{rad} = \ln \left(184.15 Z^{-\frac{1}{3}} \right)$ and $L'_{rad} = \ln \left(1194 Z^{-\frac{2}{3}} \right)$. Finally, the quantity $r_e = \frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2}$ represents the classical electron radius and is equal to $2.8 \times 10^{-13} \; cm$.

As an example of the use of equation (3) we look at lead (Pb) that has $A = 207.2$, $Z = 82$, $\rho = 11.35 \; gcm^{-3}$, and of course $N_A = 6.02 \times 10^{23} (gmol)^{-1}$. The result is $X_0 = 5.91 \; g \; cm^{-2}$ or $X_0 = 0.52 \; cm$. On the other hand, equation (1) yields $6.45 \; g \; cm^{-2}$ or $0.57 \; cm$, and equation (2) yields $6.39 \; g \; cm^{-2}$ or $0.56 \; cm$, in closer agreement with the RPP values of $6.37 \; g \; cm^{-2}$ or $0.56 \; cm$.

2 PBSciFi Matrix

Next, it is of interest to apply these formulae to the lead scintillating fiber (PbSciFi) matrix. The volume ratio of lead:fibers:glue is 42:50:8 and the relevant chemical formulae for the determination of the effective mass and atomic numbers are C_8H_9 for the scintillating fibers and $C_{60}H_{79}O_3N_2$ for the Bicron [1] BC-600 two-component optical epoxy. Finally, we require the following:

$$A_{mol} = \sum_i n_i A_i, \quad Z_{mol} = \sum_i n_i Z_i, \quad p_i = \frac{n_i A_i}{A_{mol}} \tag{5}$$

$$A_{eff} = \sum_i p_i A_i, \quad Z_{eff} = \sum_i p_i Z_i \tag{6}$$

where n_i represents that number of atoms of the ith component of the compound and p_i is the weight of each element in the compound. The A and Z of each element is given in Table 1, whereas the densities of the sub-compounds are presented in Table 2. The A_{eff} and Z_{eff} in the latter table were calculated using equations (5) and (6). A third use of these equations provides the final values for the PbSciFi matrix listed in the last row of Table 2.

Element	A	Z
H	1.01	1
C	12.01	6
N	14.01	7
O	16.00	8
Pb	207.2	82

Table 1
 Mass and atomic numbers of the PbSciFi elements.

Compound or Element	ρ ($g\ cm^{-3}$)	A_{eff}	Z_{eff}
SciFi	1.032	11.064	5.57
Glue	1.2	11.08	5.62
Pb	11.35	207.2	82
PbSciFi Matrix	**5.5**	**193.5**	**76.6**

Table 2
 Densities and effective A and Z of the PbSciFi compounds.

[1] Bicron, Newbury, Ohio, http://www.detectors.saint-gobain.com/.

Using the PbSciFi numbers and formulae (1)-(3) results in the figures in Table 3. The full thickness of the BCAL, d, is equal to $25\,cm$ minus the thickness of the aluminum backing plate. For the latter, two values are assumed (1" and $\frac{1}{2}$") for illustration purposes. The prototype 4-m-long (full-sized) module has a 1" Al plate.

Formula	X_0 $(g\ cm^{-2})$	X_0 (cm)	$n = \frac{d}{X_0}$ with 1" Al	$n = \frac{d}{X_0}$ with $\frac{1}{2}$" Al
1	6.7	1.24	18.11	19.14
2	6.4	1.16	19.36	20.46
3	5.9	1.07	21.00	22.18

Table 3
 Radiation length of PbSciFi matrix and number of radiation lengths contained in the full thickness of the BCAL.

3   Conclusions

The BCAL calorimeter will have a radiation length thickness of $18\text{-}20X_0$, depending on the thickness of the aluminum backing plate and the formula used. This is more than adequate to contain the bulk of the electro-magnetic showers and compares favourably to the KLOE calorimeter's thickness of $\sim 15X_0$ [9].

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