BCAL Radiation Length Calculations

Z. Papandreou[a,b,*], R. Hakobyan[b], N. Kolev[b]

[a] *Department of Physics, University of Regina, Regina, SK, S4S 0A2, Canada*
[b] *Prairie Particle Physics Institute, Regina, SK, S4S 0A2, Canada*

Abstract

Analytical calculations of the radiation length for the Barrel Calorimeter of the GlueX Project are presented, using updated (measured) volume ratios for the lead, scintillating fibers and optical epoxy. These show that the full thickness radiation length ranges from ∼16-17X_0 depending on the thickness of the aluminum backing plate and the analytical formalism used.

Key words: radiation length, scintillating fiber, barrel calorimeter
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1 Introduction

The electro-magnetic barrel calorimeter (BCAL) for the GlueX Project [1–3] consists of alternating layers of thin ($0.5\,mm$) lead sheets and 1-mm-diameter scintillating fibers (SciFi). The lead sheets are grooved (plastically deformed or swaged) after passing through a swaging machine. The fibers are glued in the resulting grooves by using an optical epoxy. The resulting matrix has a fiber pitch of $1.35\,mm$ in the horizontal direction and $1.18\,mm$ in the vertical. The BCAL is segmented into 48 modules with each module comprised of approximately 18,300 4-m-long fibers, thus requiring a total of over 3,500 km of fibers.

The dominant energy loss mechanism for photons in a calorimeter is the $\gamma \rightarrow e^+e^-$ reaction, via consecutive pair production and bremsstrahlung processes. The quantity that best characterizes the penetration depth of a photon shower is the radiation length, X_0. This quantity is expressed in units of cm or g/cm^2. The radiation length [4]:

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[*] Corresponding author's e-mail: zisis@uregina.ca

- describes the mean distance over which a high energy electron loses all but 1/e of its energy ($E(x) = E_0 e^{-x/X_0}$);
- represents 7/9 of the mean free path for pair production by a high energy photon ($I(x) = I_0 e^{-7x/9X_0}$);
- is an appropriate scale for describing high energy electro-magnetic cascades; and
- is a scaling variable for the probability of occurrence of bremsstrahlung and pair production as well as for the variance of the angle of multiple scattering.

Three formulae can be used to determine X_0 (see references [5], [6], and [7] respectively). The second and third of these provide a clearer view of the functional dependence on Z, and they include screening effects of the nuclear field by the atomic electrons. The third equation is an approximation while the second a result of a compact fit to the data with an accuracy of better than 2.5% for all elements except He.

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A}\left(Z^2[L_{rad} - f(Z)] + Z L'_{rad}\right) \tag{1}$$

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A}\left(Z(Z+1)\ln\frac{287}{\sqrt{Z}}\right) \tag{2}$$

$$\frac{1}{X_0} = 4\alpha r_e^2 \frac{N_A}{A}\left(Z(Z+1)\ln\frac{183}{Z^{\frac{1}{3}}}\right) \tag{3}$$

For $A = 1\ g\ mol^{-1}$ we obtain $4\alpha r_e^2 \frac{N_A}{A} = (716.408\ g\ cm^{-2})^{-1}$, and

$$f(Z) \approx a^2\left[(1+a^2)^{-1} + 0.20206 - 0.0369a^2 + 0.0083a^4 - 0.002a^6\right] \tag{4}$$

where $a = \alpha Z$ [8], $\alpha = \frac{1}{137}$, and of course $N_A = 6.022 \times 10^{23}\ (mol)^{-1}$. The function $f(Z)$ is an infinite sum, but for elements up to uranium it is accurate to four significant figures. For $Z > 4$ we have $L_{rad} = \ln\left(184.15 Z^{-\frac{1}{3}}\right)$ and $L'_{rad} = \ln\left(1194 Z^{-\frac{2}{3}}\right)$. Finally, the quantity $r_e = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mc^2}$ represents the classical electron radius and is equal to $2.818 \times 10^{-13}\ cm$.

As an example of the use of these equations we look at lead (Pb) that has $A = 207.2$, $Z = 82$, $\rho = 11.35\ g\ cm^{-3}$. The result for equation (3) is $X_0 = 5.83\ g\ cm^{-2}$ or $X_0 = 0.51\ cm$. On the other hand, equation (1) yields $6.37\ g\ cm^{-2}$ or $0.56\ cm$, and equation (2) yields $6.31\ g\ cm^{-2}$ or $0.56\ cm$, in excellent agreement with the RPP values of $6.37\ g\ cm^{-2}$ or $0.56\ cm$.

2 PBSciFi Matrix

Next, it is of interest to apply these formulae to the lead scintillating fiber (PbSciFi) matrix. The volume ratio of lead:fibers:glue is 37 : 49 : 14 and the approximate relevant chemical formulae for the determination of the effective mass and atomic numbers are $C_6H_1N_7O_8$ for the scintillating fibers and $C_{60}H_{79}O_3N_2$ for the Bicron[1] BC-600 two-component optical epoxy (for more detail see Table 1). Finally, we require the following:

$$A_{eff} = \sum_i p_i A_i, \quad Z_{eff} = \sum_i p_i Z_i \tag{5}$$

$$A_{mol} = \sum_i n_i A_i, \quad Z_{mol} = \sum_i n_i Z_i \tag{6}$$

$$p_i = \frac{n_i A_i}{A_{mol}} \tag{7}$$

where n_i represents that number of atoms of the ith component of the compound and p_i is the weight of each element in the compound. Alternatively, p_i can be expressed as follows:

$$p_i = \frac{\rho_i \cdot V_i}{\Sigma(\rho_i \cdot V_i)} \tag{8}$$

and this is the formalism used in the calculations below.

The A, Z and proportion by weight of each element is given in Table 1. The effective A, Z, and densities of the sub-compounds are presented in Table 2. The A_{eff} and Z_{eff} in the latter table were calculated using equations (5), (6) and (8). A third use of these equations provides the final values for the PbSciFi matrix listed in the last row of Table 2.

Element	A	Z	Fraction **SciFi**	Fraction **Glue**
H	1.01	1	0.077	0.091
C	12.01	6	0.822	0.822
N	14.01	7	0.001	0.032
O	16.00	8	0.001	0.055

Table 1
Mass, atomic numbers and proportions by weight of the PbSciFi elements.

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[1] Bicron, Newbury, Ohio, http://www.detectors.saint-gobain.com/.

Compound or Element	ρ ($g\ cm^{-3}$)	A_{eff}	Z_{eff}	Fraction by weight
SciFi	1.049	11.163	5.615	0.860781
Glue	1.180	11.291	5.686	0.105358
Pb	11.35	207.2	82	0.033861
PbSciFi Matrix	**4.88**	**179.91**	**71.37**	

Table 2

Densities, effective A and Z and proportion by weight of the PbSciFi compounds.

Using the PbSciFi numbers and formulae (1)-(3) results in the figures in Table 3. The full thickness of the BCAL, d, is equal to $25\,cm$ minus the thickness of the aluminum backing plate. For the latter, two values are assumed: 1" (design value) and $\frac{3}{4}$" (minimum thickness for structural rigidity). The prototype 4-m-long (full-sized) module has a 1" Al plate.

Formula	X_0 ($g\ cm^{-2}$)	X_0 (cm)	$n = \frac{d}{X_0}$ with 1" Al	$n = \frac{d}{X_0}$ with $\frac{3}{4}$" Al
1	7.06	1.45	15.5	16.4
2	7.08	1.45	15.5	16.4
3	6.59	1.35	16.6	17.6

Table 3

Radiation length of PbSciFi matrix and number of radiation lengths contained in the full thickness of the BCAL.

3 Conclusions

The BCAL calorimeter will have a radiation length thickness of 15.5-16.4X_0, depending on the thickness of the aluminum backing plate. This is more than adequate to contain the bulk of the electro-magnetic showers and compares favourably to the KLOE calorimeter's thickness of $\sim 15X_0$ [9].

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