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Calculations of magnetic shield effectiveness for long μ-metal cylinders

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1 Overview

There are several Hall D systems that require photomultipliers which will be operated at various locations in the fringe field of the solenoid. Both the pulse height and timing resolution suffers when photomultiplier tubes are operated in magnetic fields as low as a few tenths of Gauss. Therefore most photomultiplier tubes will require some amount of passive magnetic shielding in our detector. Ultimately the configuration of the shield must be optimized for the specific operating conditions and particular orientation of the phototube in the magnetic field. However, analytical estimates of the shield effectiveness are very useful during the design stage in order to select alternatives for more detailed study. In this paper we work out the details of some specific examples for the case of a long cylindrical shield which is placed in a uniform magnetic field perpendicular to the tube axis.

2 Formalism

The magnetic shield effectiveness of an infinitely long cylinder with magnetic permeability μ, which is placed in a magnetic field perpendicular to the axis of the tube, is a solved magnetostatic problem and worked out in various E&M texts [1]. The tube has an inner radius a and outer radius b and is placed with its axis along the z-direction in a uniform magnetic field with an undisturbed magnet strength H_0 along the x-direction.

This problem is solved as a boundary-valued problem for the magnetic scalar potential V_m, where $\vec{H} = -\nabla V_m$. The three regions of the problem (1, 2 and 3) and the chosen coordinate system are shown in Fig. 1. The general solution is given by

$$V_1 = -H_0\, r\cos\theta + \frac{1}{r} B_1 \cos\theta \tag{1}$$

$$V_2 = A_2\, r\cos\theta + \frac{1}{r} B_2 \cos\theta \tag{2}$$

$$V_3 = A_3\, r\cos\theta \tag{3}$$

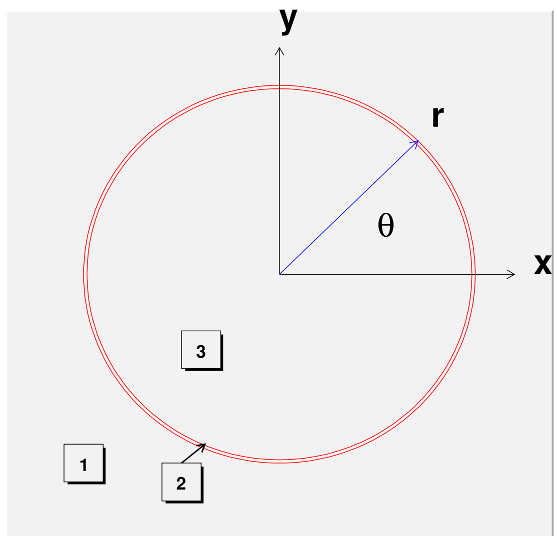

Figure 1: Geometry and coordinate system for calculation. The axis of the magnetic shield is oriented along the z-axis.The three regions are indicated by the boxes: region 1 outside the shield, region 2 in the μ-metal shield itself, and region 3 constituting the area inside the shield.

Using the continuity conditions of the normal component of magnetic field B and the parallel component of the magnetic field strength H at both boundaryies of the magnetic shield, the following conditions are obtained:

$$\begin{aligned} A_3 &= -4\mu H_0 b^2/\Delta, \quad \Delta = b^2(1+\mu)^2 - a^2(1-\mu)^2 &(4)\\ -H_0\, b^2 + B_1 &= A_2\, b^2 + B_2 &(5)\\ A_2\, a^2 + B_2 &= -\mu\,(-A_2\, a^2 + B_2) &(6)\\ -H_0\, b^2 - B_1 &= \mu\,(A_2\, b^2 - B_2), &(7)\end{aligned}$$

where the first equation is solved explicitly in the references. In order to extract the other constants, the simultaneous equations can be solved with the following solution:

$$\begin{aligned} B_1 &= H_0 b^2(\mu^2-1)(b^2-a^2)/\Delta &(8)\\ B_2 &= -2H_0 b^2 a^2(\mu-1)/\Delta &(9)\\ A_2 &= -2H_0 b^2(\mu+1)/\Delta &(10)\end{aligned}$$

The solutions for the magnetic field strength in the three regions are obtained by differentiating V_m:

$$H_1 = H_0\,\hat{i} + \frac{1}{r^2} B_1 \left(\cos 2\theta\,\hat{i} + \sin 2\theta\,\hat{j}\right) \qquad (11)$$

$$H_2 = -A_2\,\hat{\imath} + \frac{1}{r^2} B_2 \left(\cos 2\theta\,\hat{\imath} + \sin 2\theta\,\hat{\jmath}\right) \tag{12}$$

$$H_3 = -A_3\,\hat{\imath} \tag{13}$$

It is interesting to note that the magnetic field strength inside the shield only has an x-component, i.e. is in the same direction as the unperturbed field, but reduced in intensity by a factor g called the shield effectiveness. The shield effectiveness can be computed readily as

$$g = \frac{H_0}{H_3} = \frac{b^2(1+\mu)^2 - a^2(1-\mu)^2}{4\mu b^2} \tag{14}$$

This reduces to the common expression for g in the limit of large μ of

$$g = \left(\frac{\mu}{4}\right)\left(1 - \frac{a^2}{b^2}\right) \simeq \frac{\mu}{2}\left(\frac{t}{b}\right), \tag{15}$$

where we have introduced the thickness $t = (b - a)$ of the shield.

3 Applications

In order to apply the results from the previous section, we need the magnetic permeability of specific materials. Typical properties of μ-metal which is used in the fabrication of magnetic shields for photomultiplier tubes are given in Table 1 [2]. The permeability is a strong function of the magnetic field in the material and its typical behavior is shown in Fig. 2 as a function of the magnetic field strength H [3]. However, the problem has been solved assuming a constant value for the permeability μ. For the purpose of this study we make the assumption that the minimum value of the

Table 1: Magnetic properties of Ad-Vance Magnetics magnetic alloys from http://www.advancemag.com/Materials.htm.

Magnetic Property	AD-MU-80	AD-MU-78	AD-MU-48	AD-MU-00
Initial Permeability at 40 G	75,000	60,000	11,500	300
Permeability at 100-200 G	100,000	43,000	20,000	13,000
Max. permeability	300,000	250,000	130,000	3,000
Saturation Induction (G)	8,000	7,600	15,500	22,000
Coersive Force (Oe)	0.015	0.010	0.050	1.000

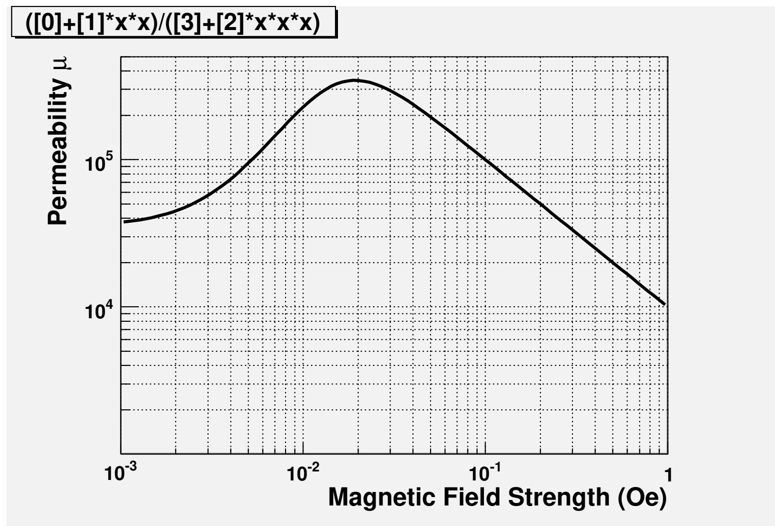

Figure 2: Parameterization of the permeability of typical high permeability material, such as AD-MU-78, as a function of the magnetic field strength.

permeability within the shield will determine the effectiveness of the shield. This is because the magnetic flux lines must be continuous and any limitation in the strength of the field at some point in the material will reduce the intensity in the rest of the volume. We note that the minimum value of μ is typically close to the quoted value at 40 G (H$\sim 10^{-3}$ Oe), unless the field strength exceeds about 0.2 Oe, which is well into saturation.

3.1 3" PMT Shields

We first compute the residual field in a 3.25" magnetic shield which can be compared to existing measurements [4, 5]. The shield was a 32P80 CO-NETIC AA tube purchased from Perfection Mica Corporation [6], 8.0" long and 1 mm thick, The components of the magnetic field strength and intensity are show in Fig. 3 and the total magnetic flux density is shown in Fig. 4. The calculated residual field inside the shield is 0.07 G compared to the measured value of 0.25 G. We note that this tube had a relatively small length-to-diameter ratio for effective pmt shielding, but long enough that the field in the center of the tube (fairly uniform over the middle 2 inches of the tube) should be representative of a longer tube. This comparison suggests that the calculation underestimates the residual field by a factor of 3.5. These estimates can be compared to the "magnetic shield calculator" from the company which computes the much higher shield effectiveness of 5694 and a residual field of 0.0053 G.

This calculation is consistent with using the maximum value of μ=450,000 for this material. The discrepancy could be due to the actual value of the permeability of the tested shield, but I suspect lower values of μ in parts of the shield are also limiting the measured effectiveness, as suggested by the closer agreement between our calculation and the measurements.

3.2 2" PMT Shields

Many applications require shielding of 2" pmts, and a standard shield is 2.25" in diameter and a thickness of 1 mm. We have calculated the residual field for $H_0 = 50$ (Figs. 5 and 6) and $H_0 = 200$ (Figs. 7 and 8). The calculated residual fields for 50 G is 0.08 G and for 200 G it is 0.41 G. Scaling up by the empirical factor of 3.5 from the previous section, we estimate these simple shields will reduce the fields in inside the shield to 0.28 G (50 G Field) and 1.4 G (200 G field).

For comparison we also plot the residual fields at 50 G and the same outer diameter as before, but with a shield thickness of 0.5 mm. The results are shown in Fig. 9 and Fig. 10. Comparison between this calculation and previous figures shows that this estimate essentially scales with the thickness of the shield as indicated by Eq. 15.

References

[1] A. Nussbaum. *Electromagnetic Theory for Engineers and Scientists.* Prentice-Hall, Inc., Englewood Cliffs, NJ, 1965. In *Classical Electrodynamics* by J.D. Jackson, the problem is solved for a spherical shell.

[2] Ad-Vance Magnetics. Procurement catalog / engineering manual no.90. Ad-Vance Magnetics, Inc., 625 Monroe St., Rochester, IN, 46975.

[3] Hamamatsu Photonics K.K. *Photomultiplier Tube, Principle to Application: Photon is our Business*, 1994. p. 155.

[4] C.E. Gliniewicz. Magnetic fields surrounding a shield for the photomultiplier tubes in the CLAS TOF. CLAS-NOTE 93-08.

[5] J. Flint and E.S. Smith. Tests of phillips XP4312B/D1 pmt in magnetic field. CLAS-NOTE 94-08.

[6] *Magnetic Shield Corporation, Perfection Mica Company.* 740 North Thomas Dr, Bensenville, IL 60106. http://www.magnetic-shield.com.

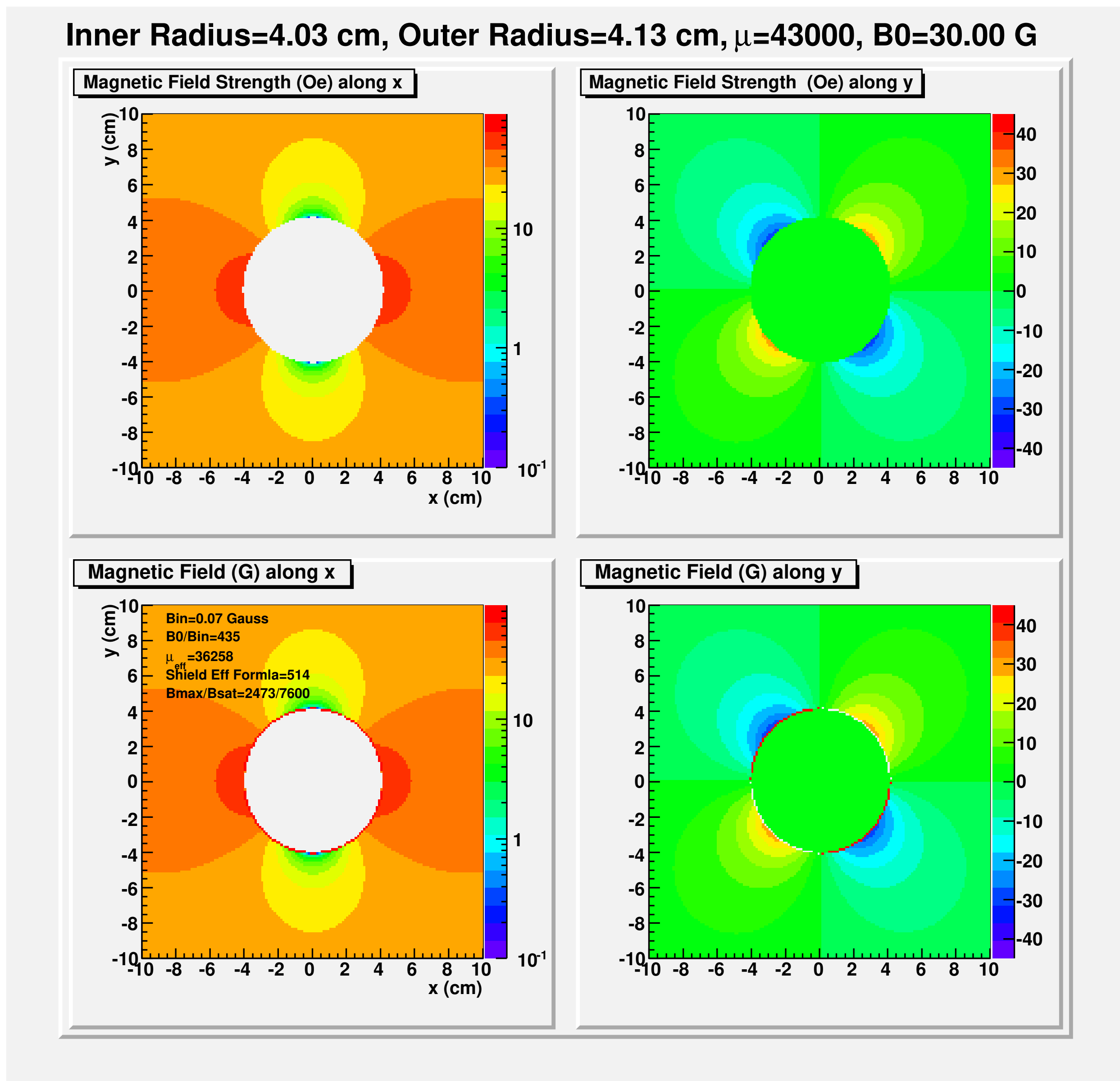

Figure 3: Plots of the magnetic field strength (top) and magnetic flux density (bottom) along the x and y-axis inside a 3.25 in. diameter shield. The unperturbed magnetic field is 30 G in the x-direction.

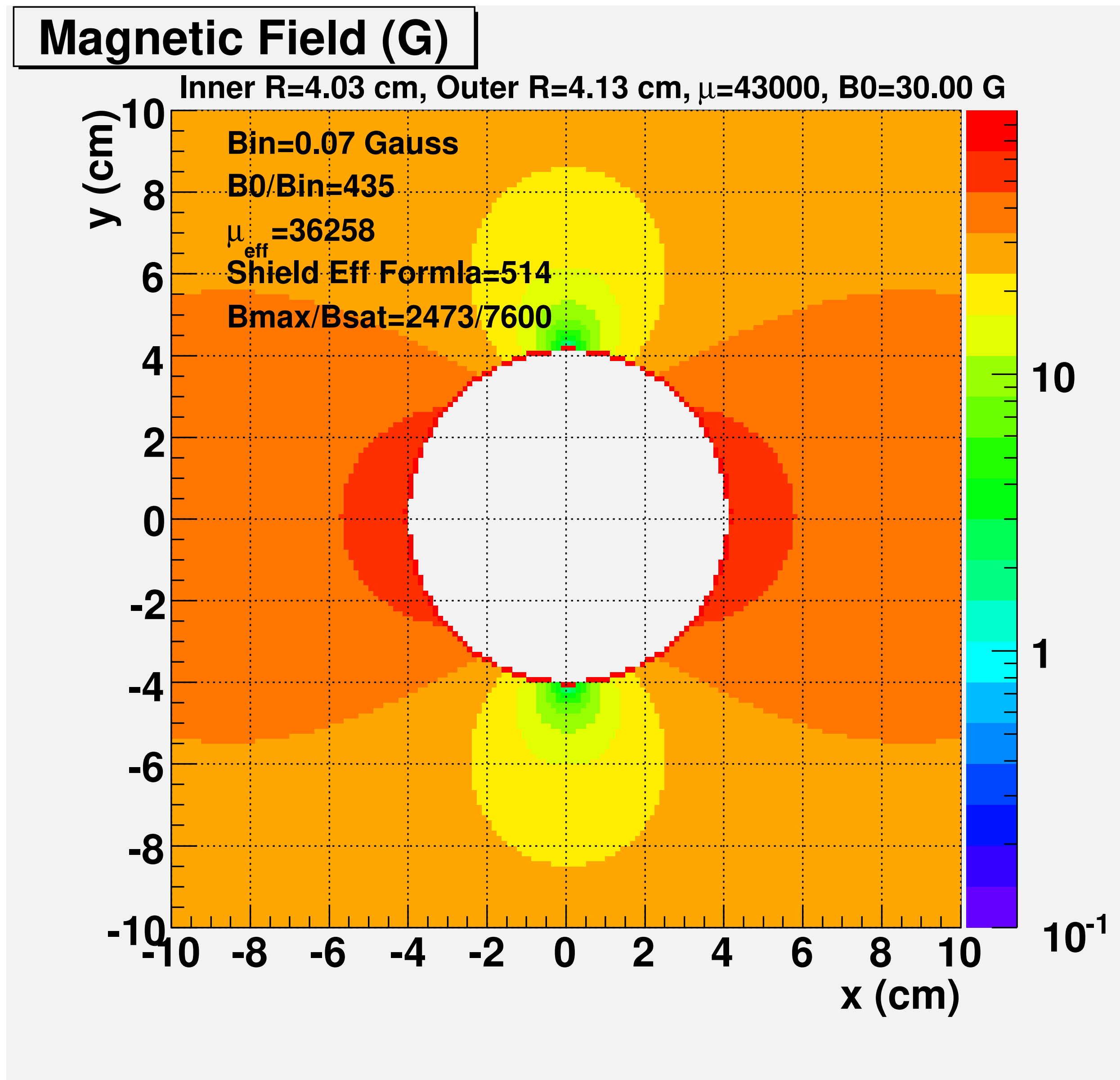

Figure 4: Plots of the total magnetic flux density in a 3.25 in. diameter shield. The unperturbed magnetic field is 30 G in the x-direction. The maximum field in the μ-metal shield is 2473 G below the saturation value of 7600 G. The shield effectiveness is 515 and results in a magnetic field inside the shield of about 0.06 G.This value is to be compared to a measured value of 0.22 G [?].

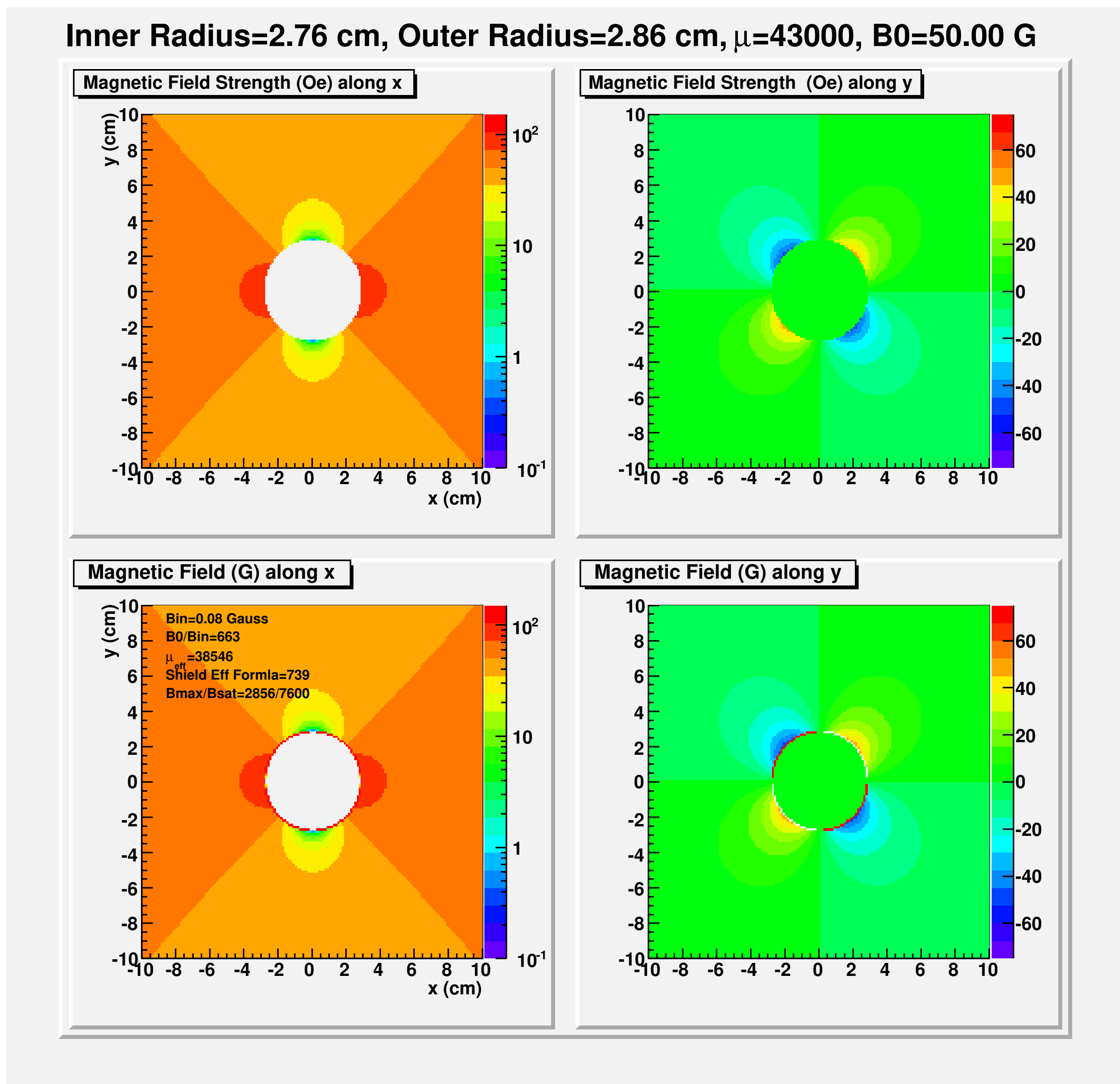

Figure 5: Plots of the magnetic field strength (top) and magnetic flux density (bottom) along the x and y-axis. The unperturbed magnetic field is 50 G in the x-direction.

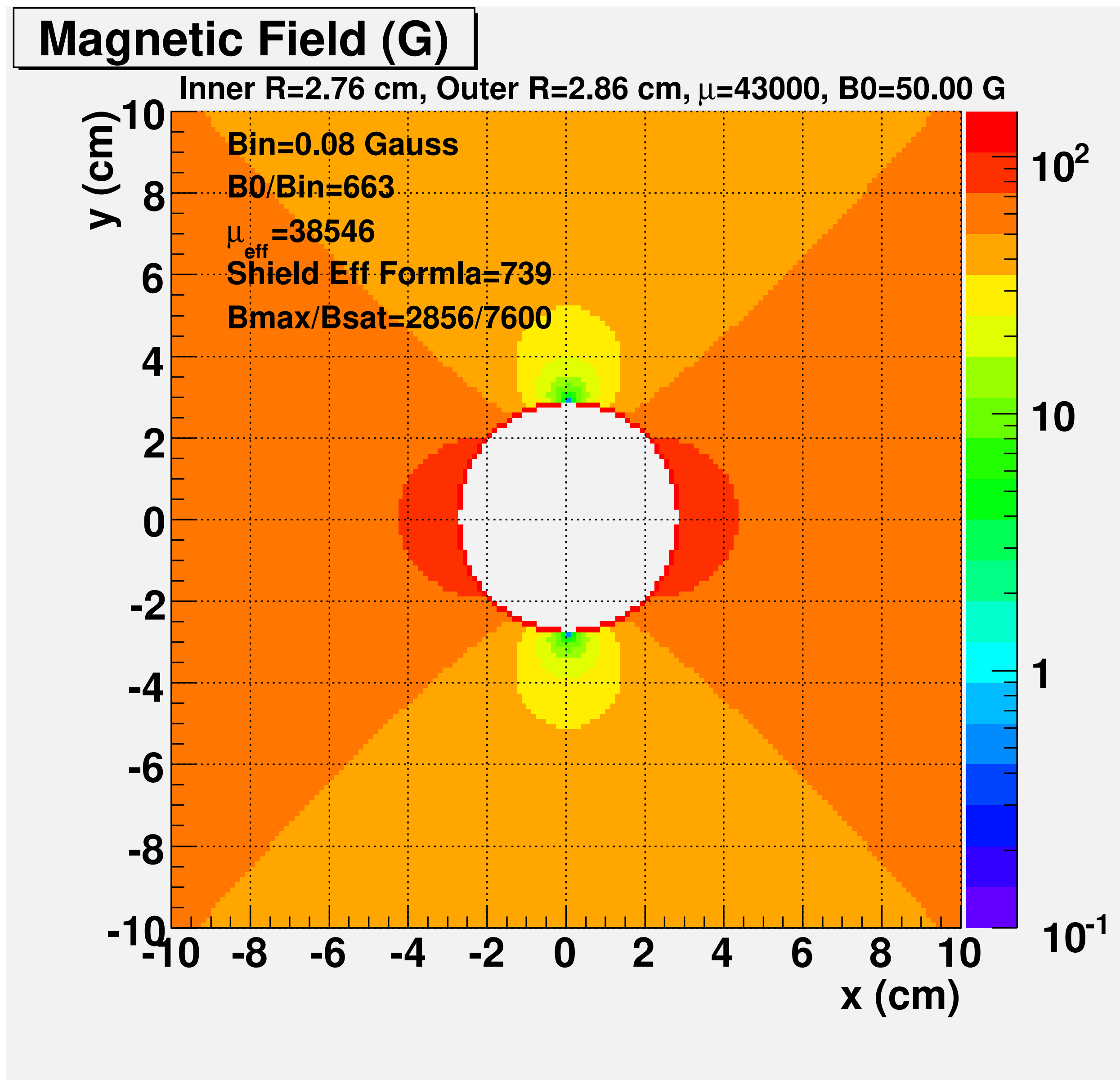

Figure 6: Plots of the total magnetic flux density. The unperturbed magnetic field is 50 G in the x-direction. The maximum field in the μ-metal shield is 2856 G, well below the saturation value of 7600 G. The shield effectiveness is 740 and results in a magnetic field inside the shield of less than 0.1 G.

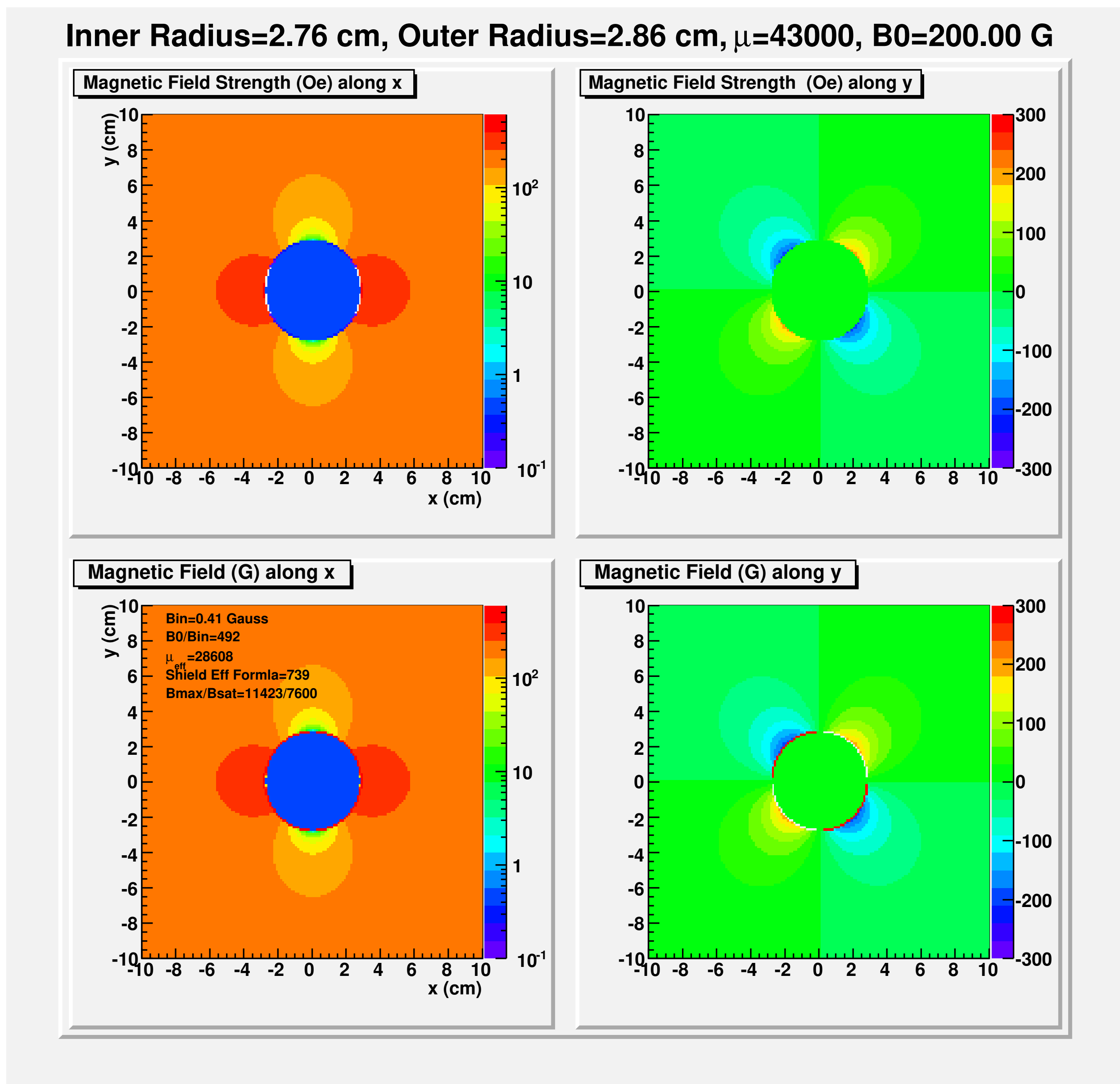

Figure 7: Plots of the magnetic field strength (top) and magnetic flux density (bottom) along the x and y-axis. The unperturbed magnetic field is 200 G in the x-direction.

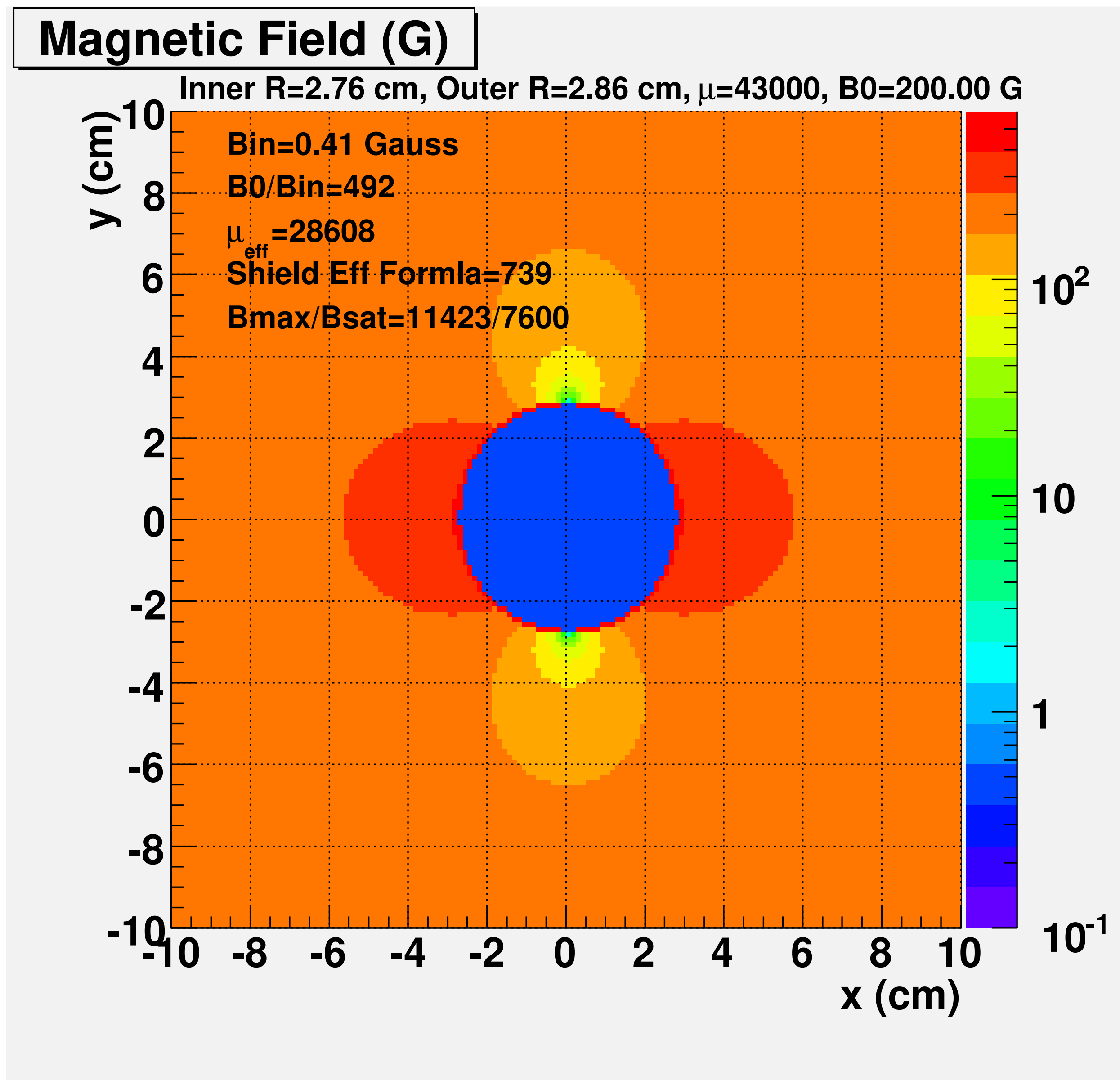

Figure 8: Plots of the total magnetic flux density. The unperturbed magnetic field is 200 G in the x-direction. The maximum field in the μ-metal shield is 11423 G above the saturation value of 7600 G. The shield effectiveness is 492 and results in a magnetic field inside the shield of about 0.4 G.

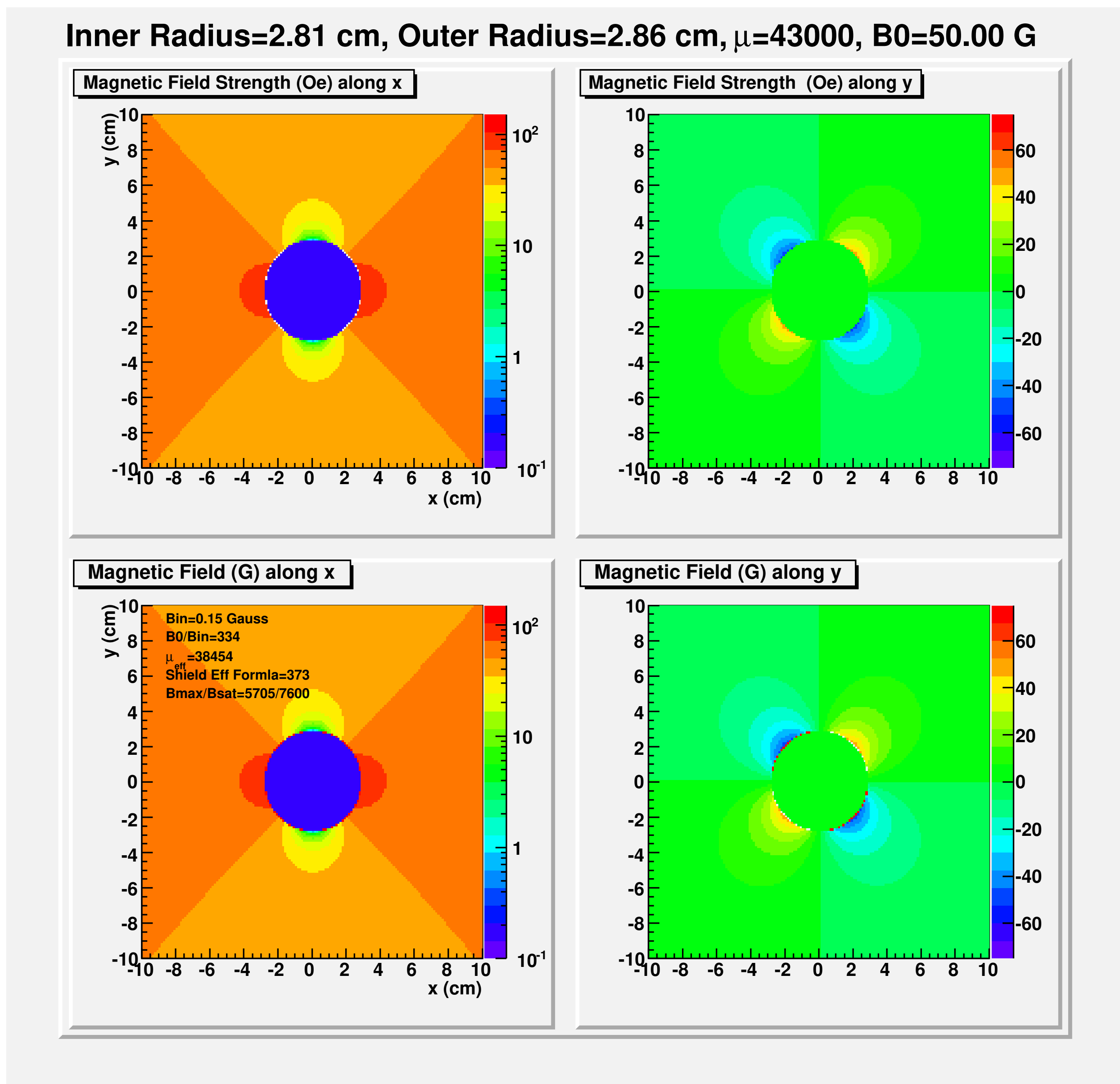

Figure 9: Plots of the magnetic field strength (top) and magnetic flux density (bottom) along the x and y-axis. The unperturbed magnetic field is 50 G in the x-direction.

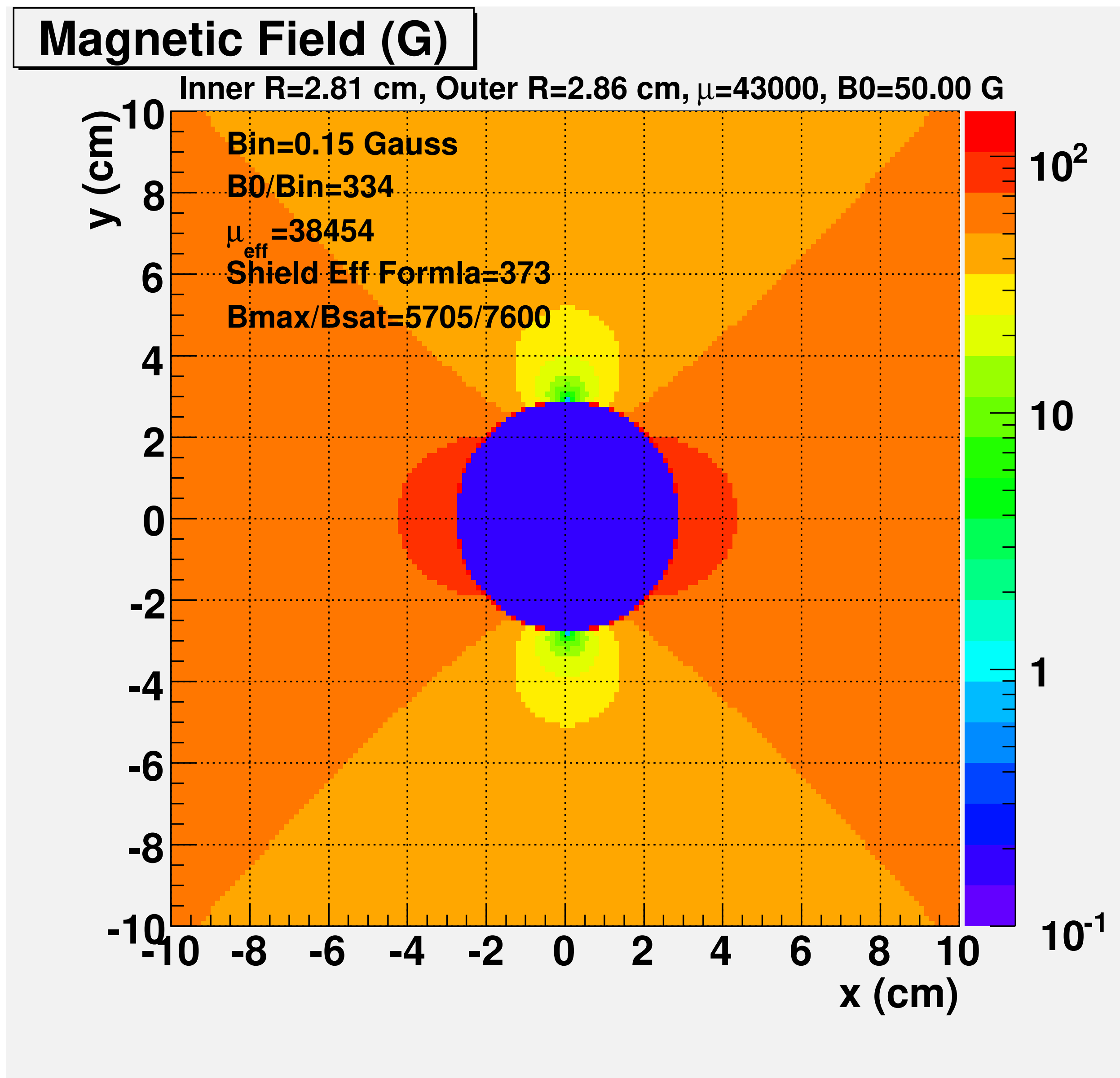

Figure 10: Plots of the total magnetic flux density. The unperturbed magnetic field is 50 G in the x-direction. The maximum field in the μ-metal shield is 5705 G, close to the saturation value of 7600 G. The shield effectiveness is 334 and results in a magnetic field inside the shield of about 0.15 G.