Scintillating Fiber Trapping Efficiency

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Abstract

Analytical calculations of the trapping efficiency of scintillating fibers are presented. These verify manufacturers' claims and clarify the use of trapping efficiency towards the estimation of the number of photons reaching each end of the fiber.

Key words: scintillating fiber, trapping efficiency, capture ratio
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1 Introduction

The electro-magnetic barrel calorimeter (BCAL) for the GlueX Project consists of alternating layers of thin (0.5 mm) lead sheets and 1-mm-diameter scintillating fibers (SciFi). It is important to understand the light production and transmission through the fibers towards ensuring that the BCAL readout chosen is adequate for the task at hand.

In this document, analytical calculations are presented showing the *trapping efficiency* (also referred to as *capture ratio*) of the produced light in scintillating fibers, which may be a result of the stimulation of the fiber by a traversing charged particle.

2 Analytical Formulae

The solid angle corresponding to a finite element can be written as:

$$\Delta\Omega = \sin\theta d\theta d\phi \tag{1}$$

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using the standard nomenclature in spherical coordinates (θ is the polar angle and ϕ is the azimuthal angle). The total solid angle of a unit sphere can be expressed as:

$$\Omega = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta d\theta d\phi = -2\pi cos\theta|_0^\pi \tag{2}$$

owing to the symmetry of the fiber geometry with respect to ϕ, and which gives $\Omega = 4\pi$ when integrated. The last part of the formula can be generalized to give:

$$\Omega' = -2\pi cos\theta|_{\theta_1}^{\theta_2} \tag{3}$$

Finally, we use Snell's Law:

$$n_1 sin\theta_1 = n_2 sin\theta_2 \tag{4}$$

and the equation relating the numerical aperture (NA) of a fiber to the emission (exit) angle, α, of light from the end of the fiber:

$$NA = \sin\alpha = \sqrt{n_1^2 - n_2^2} \tag{5}$$

where n_1 and n_2 are the indices of refraction of the core and cladding, respectively.

3 Calculations and Comparisons to Manufacturers' Numbers

Kuraray [1] claimes an exit angle of $\alpha = 45.7^o$ and a NA=0.72 for a double-clad fiber. Bicron [2] states $\alpha = 35.7^o$ and NA=0.58 for a single-clad fiber and $\alpha = 47.5^o$ and NA=0.74 for a double-clad fiber. All these numbers are correct. The comparison of properties between the two manufacturers are show in Table 1 and a graphical representation of the cross section of a double-clad fiber is shown in Figure 1.

Fig. 1. Cross sectional expansion of a double-clad fiber, showing the core, first and second claddings.

Property	**Bicron**		**Kuraray**	
	Single	Double	Single	Double
Index of refraction of core	1.60	1.60	1.59	1.59
Index of refraction of first cladding	1.49	1.49	1.49	1.49
Index of refraction of second cladding	N/A	1.42	N/A	1.42
Numerical Aperture	0.58	0.74	0.55	0.72
Exit Half-Angle	35.7^o	47.5^o	33.7^o	45.7^o
Trapping Half-Angle	21.4^o	27.4^o	20.3^o	26.6^o
Trapping Efficiency	3.44%	5.6%	3.1%	5.4%

Table 1
Properties of a double-clad fiber from two manufacturers.

It should be emphasized that the calculation of the trapping efficiency in this manner reflects the consideration of *meridional rays* only. When *skew rays* are included as well, the trapping efficiency increases considerably. For purposes of reference, the trappng efficiency equations are reproduced here [3]:

$$\epsilon_m = \frac{1}{2}(1 - \cos\theta_c) \approx \frac{\theta_c^2}{4} \tag{6}$$

$$\epsilon_s = \frac{1}{2}(1 - \cos\theta_c)\cos\theta_c \tag{7}$$

$$\epsilon_t = \frac{1}{2}(1 - \cos\theta_c^2) \approx \frac{\theta_c^2}{2} \tag{8}$$

Nevertheless, for the discussion that follows, only meridional rays are considered in order to facilitate comparisons with manufacturers' brochures.

The optical rays leading to total internal reflection at the core-first fiber and first-to-second fiber are showing schematically in Figure 2. Using equation (4) for a Bicron single-clad fiber we obtain $\theta_c = 68.6^o$ and $\theta_t = 90^o - \theta_c = 21.4^o$, where the subscript t refers to the trapping half-angle. Reapplying equation (4) for a Bicron double-clad fiber between the first and second cladding interfaces, we obtain $\theta_c = 72.4^o$ (denoted as ϕ in the figure) and $\theta_t = 27.44^o$. When the trapping angles are substituted in equation (3), we obtain 6.9% and 11.3% of 2π, ie *per side*, or, more properly denoted, trapping efficiencies of 3.4% and 5.6% (*trapping efficiency is defined versus* 4π).

Finally, for a Bicron double-clad fiber, $\theta_t = 27.4^o$ leads to $\alpha = 47.5^o$, in agreement with Table 1.

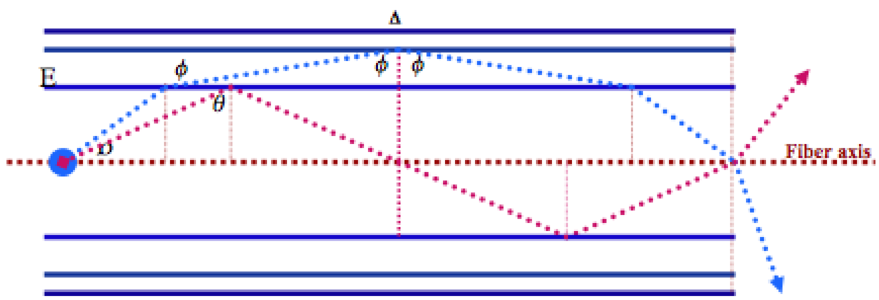

Fig. 2. Ray traces used for the calculation of angles via Snell's Law.

4 Conclusions

The trapping efficiency of a fiber is defined as the ratio (or percentage) of the solid angle of a unit sphere (4π), and this is the number quoted by fiber manufacturers. When needing to calculate the number of photons arriving at *one* end of a fiber so as to properly correct for attenuation, one must start with the number of 4,000/MeV/per side, instead of the number quoted by industry of 8,000/MeV, which represents the total number of photons, traveling to *both* ends of the fiber. Finally, the numbers are consistent with those reported in reference [4], as extracted from GuideIt Monte Carlo simulations.

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