Scintillating Fiber Trapping Efficiency

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Abstract

Analytical calculations of the trapping efficiency of scintillating fibers are presented. These verify manufacturers' claims and clarify the use of trapping efficiency towards the estimation of the number of photons reaching each end of the fiber. The analytical calculations yield upper and lower limits of the trapping efficiency resulting from the consideration of both meridional and skew optical rays and meridional rays alone, respectively. These limits agree with Monte Carlo simulations.

Key words: scintillating fiber, trapping efficiency, capture ratio
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1 Introduction

The electro-magnetic barrel calorimeter (BCAL) for the GlueX Project consists of alternating layers of thin (0.5 mm) lead sheets and 1-mm-diameter scintillating fibers (SciFi). It is important to understand the light production and transmission through the fibers towards ensuring that the BCAL readout chosen is adequate for the task at hand.

In this document, analytical calculations are presented showing the *trapping efficiency* (also referred to as *capture ratio*) of the produced light in scintillating fibers, which may be a result of the stimulation of the fiber by a traversing charged particle.

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2 Analytical Formulae

The solid angle corresponding to a finite element can be written as:

$$d\Omega = \sin\theta d\theta d\phi \tag{1}$$

using the standard nomenclature in spherical coordinates (θ is the polar angle and ϕ is the azimuthal angle). The total solid angle of a unit sphere can be expressed as:

$$\Omega = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta d\theta d\phi = -2\pi cos\theta|_0^{\pi} \tag{2}$$

owing to the symmetry of the fiber geometry with respect to ϕ, and which gives $\Omega = 4\pi$ when integrated. The last part of the formula can be generalized to give:

$$\Omega' = -2\pi cos\theta|_{\theta_1}^{\theta_2} \tag{3}$$

Finally, we use Snell's Law:

$$n_1 sin\theta_1 = n_2 sin\theta_2 \tag{4}$$

and the equation relating the numerical aperture (NA) of a fiber to the emission (exit) angle, α, of light from the end of the fiber:

$$NA = \sin\alpha = \sqrt{n_1^2 - n_2^2} \tag{5}$$

where n_1 and n_2 are the indices of refraction of on either side of an optical interface.

3 Calculations and Comparisons to Manufacturers' Numbers

Kuraray [1] claims an exit angle of $\alpha = 45.7^o$ and a NA=0.72 for a double-clad fiber. Bicron [2] states $\alpha = 35.7^o$ and NA=0.58 for a single-clad fiber and $\alpha = 47.5^o$ and NA=0.74 for a double-clad fiber. These numbers reflect a simple calculation employing *meridional* optical rays. A graphical representation of the cross section of a double-clad fiber is shown in Figure 1. The comparison of properties between the two manufacturers are shown in Table 1.

Fig. 1. Cross sectional expansion of a double-clad fiber, showing the core, first and second claddings.

When *skew rays* are included as well, the trapping efficiency increases considerably [3]. For purposes of reference, the trapping efficiency equations from that publication are reproduced here:

$$\epsilon_m = \frac{1}{2}(1 - \cos\theta_c) \approx \frac{\theta_c^2}{4} \tag{6}$$

$$\epsilon_s = \frac{1}{2}(1 - \cos\theta_c)\cos\theta_c \tag{7}$$

$$\epsilon_t = \frac{1}{2}(1 - \cos\theta_c^2) \approx \frac{\theta_c^2}{2} \tag{8}$$

where the subscripts m, s, and t reflect the trapping efficiency due to meridional, skew, and total (meridional and skew) rays, respectively. Equation (6) is equivalent to equation (3) when $\theta_1 = 0^o$ and $\theta_2 = \theta_c$.

Only meridional rays need to be considered to compare with manufacturers' brochures, and these provide a *lower limit* of the trapping efficiency. However, the realistic trapping efficiency of a fiber is much closer to the *upper limit* which results from the inclusion of both meridional and skew rays. The precise number for the trapping efficiency can be obtained only by Monte Carlo simulations and is slightly less than ϵ_t, owing to the large path length of skew rays that results in losses.

The optical rays leading to total internal reflection at the core-first fiber and first-to-second fiber are showing schematically in Figure 2. Using equation (4) for a Bicron single-clad fiber we obtain $\theta_c = 68.6^o$ and $\theta_t = 90^o - \theta_c = 21.4^o$, where the subscript t refers to the trapping half-angle. Reapplying equation (4) for a Bicron double-clad fiber between the first and second cladding interfaces, we obtain $\theta_c = 72.4^o$ and $\theta_t = 27.44^o$. When the trapping angles are substituted in equation (3), we obtain 6.9% and 11.3% of 2π, ie *per side*, or, more properly denoted, trapping efficiencies of 3.4% and 5.6% (*trapping efficiency is defined versus* 4π*)*.

Finally, for a Bicron double-clad fiber, $\theta_t = 27.4^o$ leads to $\alpha = 47.5^o$, in agreement with Table 1.

Property	**Bicron**		**Kuraray**	
Fiber Type (Claddings)	Single	Double	Single	Double
Index of refraction of core	1.60	1.60	1.59	1.59
Index of refraction of first cladding	1.49	1.49	1.49	1.49
Index of refraction of second cladding	N/A	1.42	N/A	1.42
Numerical Aperture	0.58	0.74	0.55	0.72
Exit Half-Angle	35.7^o	47.5^o	33.7^o	45.7^o
Trapping Half-Angle	21.4^o	27.4^o	20.3^o	26.6^o
ϵ_m (meridional)	3.4%	5.6%	3.1%	5.4%
ϵ_s (skew)	3.2%	5.0%	2.9%	4.7%
ϵ_t (total)	6.7%	10.6%	6.0%	10.0%

Table 1
Properties of single and double-clad fibers from two manufacturers.

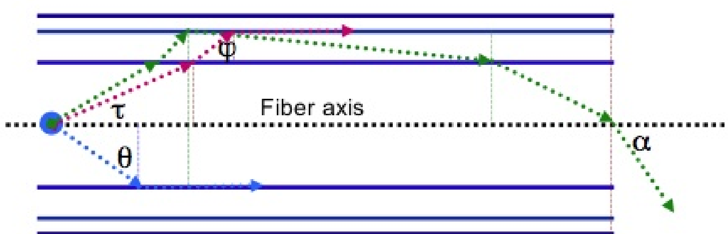

Fig. 2. Ray traces used for the calculation of angles from Snell's Law. The blue and red lines represent follow the critical angle for the core to first cladding and first to second cladding with the corresponding critical angles labeled as θ and ϕ, respectively. The green line portrays a ray that undergoes total internal reflection at the first to second cladding and exits from the fiber with an exit half-angle α with respect to the fiber axis; for this ray the trapping half-angle is denoted as τ.

4 Conclusions

The trapping efficiency of a fiber is defined as the ratio (or percentage) of the solid angle of a unit sphere (4π), and this is the number quoted by fiber manufacturers. When needing to calculate the number of photons arriving at *one* end of a fiber so as to properly correct for attenuation, one must start with the number of 8,000/MeV/per side, which represents the total number of photons traveling to *both* ends of the fiber and then apply the trapping efficiency *per side*.

Bicron fibers were used for the two prototype BCAL modules (Module 1 and Module 2). From Table 1, the actual trapping efficiency, ϵ_{real}, for these modules is in the range $\epsilon_m < \epsilon_{real} < \epsilon_t$, or $\epsilon_{real} \in [6.7\%, 10.6\%]$. Finally, the numbers are consistent with simulated $\epsilon_{real} = 9.3\%$ reported in reference [4], as extracted from GuideIt Monte Carlo simulations.

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