# A Program to Estimate Resolution for Charged Particles in GlueX

GlueX-doc-1015-v1

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**Abstract**

This note describes a set of FORTRAN routines (`REZEST`) that calculates estimates of charged track resolution in transverse momentum and direction for the GlueX detector geometry. Parameters of that geometry can be varied to quickly obtain estimates for new configurations. Since no Monte Carlo is used in the calculations, results are returned immediately. Instructions on how to obtain and use the software are provided and a description of the approximations used is given.

## Contents

# 1 Introduction

We often use "back of the envelope" estimates of detector performance to inform detailed design decisions. These calculations have the advantage of simplicity and provide quick feedback for new design ideas. The FORTRAN routines described here provide a facility for making such calculations for GlueX.

The fixed detector geometry concept used is the standard one for GlueX: a central cylindrical drift chamber (CDC) in a solenoidal field with a set of planar drift chambers in the forward direction (FDC). The CDC is assumed to have both axial and stereo layers. The parameters that can be varied include:

- $z$ position[1] of the target
- inner and outer radius of the CDC
- length of the CDC
- front and back $z$-positions of the FDC
- amount of material in the FDC and CDC tracking volume
- amount of material in front of the FDC and CDC
- magnetic field strength
- number of axial and stereo position measurements in the CDC
- stereo angle for stereo layers in the CDC
- number of position measurements in the FDC

Many approximations are made and generic formulae applied in the calculations. The numbers that are produced are not a substitute for a full-scale, hit-based Monte Carlo simulation of the detector. That having been said, the relative variation of resolution when a particular parameters are varied should give a good feeling for the effect of parameter change.

The following assumptions are made:

- The magnetic field is uniform everywhere.
- Particles travel in straight lines, independent of momentum.
- Measurements of track parameters in the FDC are statistically independent from those made in the CDC.
- All position measurements within a detector (FDC or CDC) are statistical independent of one another.
- All positions measurements within a detector are made at locations uniformly spaced along the trajectory.
- All positions measurements within a detector have the same resolution.

A note on notation: in the following, $\sigma_x$ and $\delta x$ will be used interchangeable to represent the root mean square deviation of the quantity $x$.

[1]The $z$-direction is along the incident photon beam.

## 2 Using the Software

The principal subroutine is documented in the source code file `rezest_fdc_cdc.F`:

```
      SUBROUTINE REZEST_FDC_CDC(P, LAMBDA, M,
     X     DP_OVER_P, DPHI_TOT, DTHETA_TOT)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C
C This routine estimates the resolution in GlueX for charged particles
C in tranverse momentum, azimuthal angle, and polar angle.
C
C Input arguments, all REAL*4
C
C     P        Magnitude of total momentum (GeV/c)
C     LAMBDA   Dip angle, difference in polar angle in lab between track
C              and pi/2 (i. e., 90 degrees) (radians)
C     M        Mass of the particle (GeV/c^2)
C
C Output arguments, all REAL*4
C
C     DP_OVER_P  Relative resolution in transverse momentum
C                ("sigma_{p_t}/p_t")
C     DPHI_TOT   Resolution in azimuthal angle ("sigma_phi")
C     DTHETA_TOT Resolution in polar angle ("sigma_theta")
C
C The routine combines the measurements in the FDC and CDC where
C appropriate. Parameters describing the geometry and materials are
C defined in the routine REZEST_COMPONENTS which appears below.
C
```

### 2.1 Getting the code

Two methods:

1. Get the tar ball from

   `http://www.jlab.org/~marki/misc/rezest.tar`

2. Check it out from the subversion repository with the command

   `svn checkout https://halldsvn.jlab.org/repos/trunk/home/marki/gluex/rezest`

### 2.2 Building the files

There is a simple makefile in the directory:

```
> cd <rezest directory>
> make
gfortran -g   -c -o rezest.o rezest.F
gfortran -g   -c -o rezest_fdc_cdc.o rezest_fdc_cdc.F
ar rcv librezest.a rezest.o rezest_fdc_cdc.o
a - rezest.o
a - rezest_fdc_cdc.o
gfortran -g   -c -o rezest_point.o rezest_point.F
gfortran -o rezest_point rezest_point.o -L. -lrezest
gfortran -g   -c -o rezest_point_comp.o rezest_point_comp.F
gfortran -o rezest_point_comp rezest_point_comp.o -L. -lrezest
```

This creates three files that you care about:

1. `librezest.a`: the object library
2. `rezest_point`: a binary
3. `rezest_point_comp`: a binary

## 2.3 Using the files

### 2.3.1 `librezest.a`

Link this into your own program to retrieve resolution values. It contains the routine `rezest_fdc_cdc` described in Section 2.

### 2.3.2 `rezest_point`

This stand-alone binary will accept three arguments on standard input (space separated on a single line) and will write the resolution values on standard output.

**Input:**

1. total magnitude of momentum in GeV/$c$
2. dip angle in radians (polar angle referenced from 90° in the lab; $\theta_{\rm dip} = \pi/2 - \theta$ there $\theta$ is the standard polar angle)
3. mass in GeV/$c^2$

**Output:**

1. relative resolution in transverse momentum ($\delta p_t/p_t$)
2. resolution in azimuthal angle ($\delta\phi$)
3. resolution in polar angle ($\delta\theta$)

For example:

```
> rezest_point
1.0 1.22 0.139
  2.8597742E-02  1.6185496E-02  8.1181811E-04
```

says that a 1 GeV/$c$ particle traveling in a direction 1.22 radians forward of the transverse direction with a mass of 139 MeV has an estimated $\delta p_t/p_t = 2.9\%$, $\delta\phi = 16$ mR, and $\delta\theta = 0.8$ mR.

### 2.3.3 `rezest_point_comp`

Same as `rezest_point` except that the individual components of the resolution are listed. Note that the first three numbers being output are the same as for `rezest_point`.

Output:

1. relative resolution in transverse momentum
2. resolution in azimuthal angle
3. resolution in polar angle
4. resolution in curvature ($k = 1/R$) due to multiple scattering in the CDC in inverse meters
5. same for FDC
6. resolution in curvature due to position resolution in the CDC

7. same for FDC
8. total resolution in curvature in the CDC
9. same for the FDC
10. resolution in azimuthal angle ($\phi$) due to multiple scattering in the CDC in radians
11. same for FDC
12. resolution in azimuthal angle due to position resolution in the CDC
13. same for FDC
14. resolution in azimuthal angle due to curvature resolution in the CDC
15. same for FDC
16. total resolution in azimuthal angle in the CDC
17. same for the FDC
18. resolution in polar angle ($\theta$) due to multiple scattering in the CDC in radians
19. same for FDC
20. resolution in polar angle due to position resolution in the CDC in radians
21. same for FDC
22. total resolution in polar angle in the CDC
23. same for the FDC

For example:

```
> rezest_point_comp
1.0 1.22 0.139
  2.8597742E-02  1.6185496E-02  8.1181811E-04  4.2887099E-02
  2.2642065E-02  1.3919008E-02  0.1605156      4.5089267E-02
  0.1621047      2.5197803E-03  3.1950074E-04  5.0111138E-04
  2.4516270E-03  1.7020071E-02  4.7496673E-02  1.7212879E-02
  4.7560975E-02  2.5197803E-03  3.1950074E-04  7.7741774E-04
  7.9118297E-04  2.6369814E-03  8.5325917E-04
```

(All values in the output actually appear on one line.)

# 3 How the Estimates Are Done

In this section the various formulae and concepts used to produce the estimates are described.

## 3.1 Transverse Momentum Resolution

The formulae used to estimate transverse are taken from the Particle Data Group's Review of Particle Physics[2]. Using (for the most part) their notation and descriptions, for a particle with charge $q$ of momentum $p$ in a uniform magnetic field $B$ with a pitch angle (dip angle) $\lambda$

$$p\cos\lambda = (0.3)qBR \tag{1}$$

where $r$ is the radius of curvature in the projection of the trajectory onto the bend plane, $p$ is in GeV/c, $B$ is in Tesla, and $R$ is in meters. In the remainder of this note it will be assumed that $q = 1$. The curvature $k = \frac{1}{R}$. The variance of $k$ has two contributions,

$$(\delta k)^2 = (\delta k_{\text{res}})^2 + (\delta k_{\text{ms}})^2. \tag{2}$$

$$\delta k_{\text{res}} = \frac{\epsilon}{L'^2}\sqrt{\frac{720}{N+4}} \tag{3}$$

where $\epsilon$ is the position resolution in meters, $L'$ is the projected length of the track onto the bending plane in meters and $N$ is the number of measurements.

$$\delta k_{\text{ms}} = \frac{(0.016\ \text{GeV}/c)z}{Lp\beta\cos^2\lambda}\sqrt{N_{\text{rl}}} \tag{4}$$

where $N_{\text{rl}}$ is the number of radiation lengths in the detector and $L$ is the total track length in the detector.

## 3.2 Curvature vs. Momentum

The estimates in the previous section are given in terms of the error on curvature. These must be converted to momentum. Rewriting Eq. 1

$$p_t = (0.3)BR = \frac{(0.3)B}{k} \tag{5}$$

So

$$\frac{dp_t}{dk} = -\frac{(0.3)B}{k^2} = -(0.3)BR^2 \tag{6}$$

and since

$$\delta p_t = \left|\frac{dp_t}{dk}\right|\delta k, \tag{7}$$

we have

$$\frac{\delta p_t}{p_t} = R\delta k = \frac{\delta k}{k}. \tag{8}$$

## 3.3 Error on Slope and $y$-intercept of a Straight-Line Fit, Equally Spaced Measurements

To estimate the error due to position resolution on the direction of a fitted track, we use the error on the slope of a straight line fitted to the same number of measurements as the fundamental input. This approach ignores the fact that the trajectory is in fact curved, but is being used to estimate the error in the angle and not the angle itself.

From Ref. [1]

$$\chi^2 = \sum\left[\frac{1}{\sigma_i^2}(y_i - a - bx_i)^2\right] \tag{9}$$

For equal errors ($\sigma_i = \sigma\ \forall i,$), $\chi^2$ is minimized for $a$ and $b$ when

$$a = \frac{1}{\Delta'}\left(\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i\right) \tag{10}$$

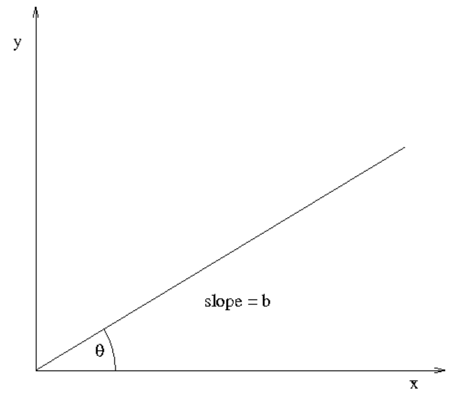


Figure 1: Generic straight line through origin.

$$b = \frac{1}{\Delta'} \left( n \sum x_i y_i - \sum x_i \sum y_i \right) \tag{11}$$

where

$$\Delta' = n \sum x_i^2 - \left( \sum x_i \right)^2 \tag{12}$$

The variance of the parameters $a$ and $b$ are

$$\sigma_a^2 \approx \frac{\sigma^2}{\Delta'} \sum x_i^2 \tag{13}$$

and

$$\sigma_b^2 \approx \frac{n\sigma^2}{\Delta'}. \tag{14}$$

For $n$ equally spaced measurements spanning the interval $[0, L]$,

$$x_i = \frac{L(i-1)}{n-1} \tag{15}$$

and using expressions from Appendix A, we get

$$\sigma_a^2 = \frac{2\sigma^2(2n-1)}{n(n+1)} \tag{16}$$

$$\sigma_b^2 = \frac{12\sigma^2(n-1)}{L^2 n(n+1)} \tag{17}$$

We need to translate an error in slope to an error in angle. In Fig. 1, $\theta = \tan^{-1} b$ so

$$\delta\theta = \left| \frac{d\theta}{db} \right| \delta b = \frac{\delta b}{\sec^2 \theta}. \tag{18}$$

## 3.4 Angular Error Due to Multiple Coulomb Scattering

Again from Ref. [2], we define

$$\theta_0 = \theta_{\text{plane}}^{\text{rms}} = \frac{1}{\sqrt{2}} \theta_{\text{space}}^{\text{rms}}. \tag{19}$$

The central angular distribution is approximately Gaussian with a width given by

$$\theta_0 = \frac{(13.6 \text{ MeV})}{\beta c p} z \sqrt{x/X_0} \left[ 1 + 0.038 \ln(x/X_0) \right]. \tag{20}$$

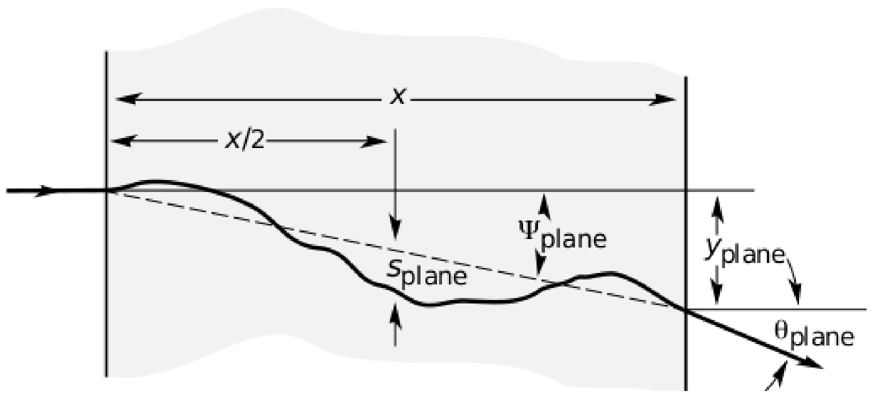


Figure 2: Quantities used to describe multiple Coulomb scattering. The particle is incident in the plane of the figure.

As seen in Fig. 2The angle $\Psi_{\mathrm{plane}}$ is the angle between an unscattered trajectory and a line drawn from the entrance point of the detector to the exit point. We use this as an approximation to the contribution of multiple scattering to both the azimuthal and polar angles.

$$\Psi^{\mathrm{rms}}_{\mathrm{plane}} = \frac{1}{\sqrt{3}}\theta_0. \tag{21}$$

## 3.5 Contribution to Azimuthal Angle Resolution from Curvature Resolution

Curvature $k = 1/R$ and direction in plane is measured at some position rotated from the vertex by an angle $\alpha$ about the center of curvature, not at the vertex. To infer the azimuthal angle $\phi$ at the vertex, track must be swum backward through angle $\alpha$. Determination of $\alpha$ depends on $R$ and thus on $k$. An error in $\alpha$ translates directly into an error in $\phi$.

$$\sin\frac{\alpha}{2} = \frac{r_{\mathrm{mid}}}{2R} = \frac{r_{\mathrm{mid}}k}{2} \tag{22}$$

so

$$\delta\alpha = r_{\mathrm{mid}}\delta k \sec\frac{\alpha}{2} \tag{23}$$

where $\alpha = 2\sin^{-1}(r_{\mathrm{mid}}/2R)$ We take as an approximation $r_{\mathrm{mid}} = (r_{\mathrm{in}} + r_{\mathrm{out}})/2$.

Note that a side effect of using a straight-line approximation for the trajectory is that in some cases $r_{\mathrm{out}}$ can be greater than $2R$, *i. e.*, the curving track never reaches the outer radius obtained from the straight-line approximation. In those cases, the outer radius is set to $2R$. This avoids arguments to the inverse sine greater than one.

## 3.6 Geometry of the CDC

Fig. 4 shows a schematic of the CDC chamber displaying some of the parameters that serve as input to the estimate:

$r_{\mathrm{min}}$ minimum radius of the CDC tracking volume

$r_{\mathrm{max}}$ maximum radius of the CDC tracking volume

$z_{\mathrm{CDC}}$ $z$-coordinate distance from center of target to downstream end of the CDC

Other parameters are:

$r_{\mathrm{min,stereo}}$ minimum radius of the CDC stereo layers

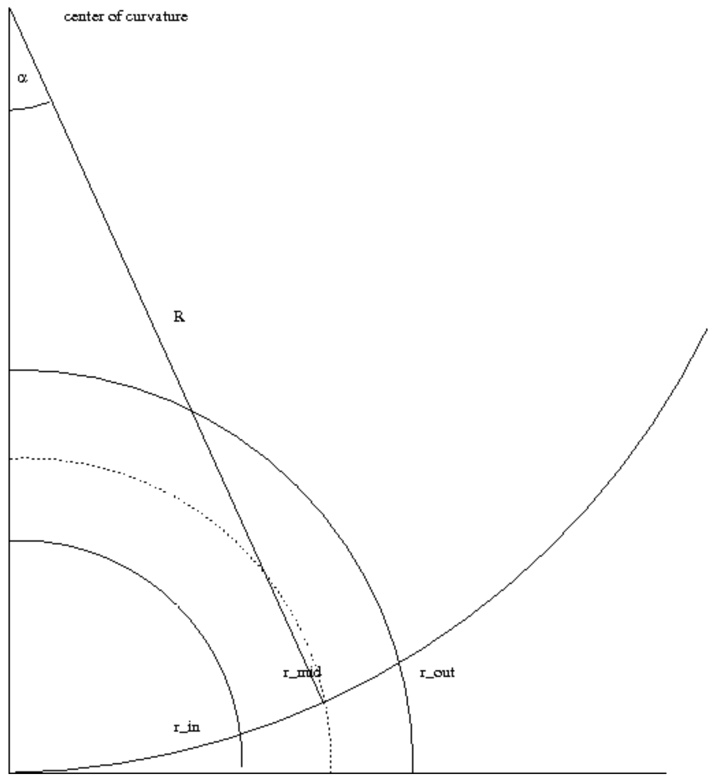


Figure 3: The concept for estimating the effect of curvature resolution on azimuthal angle resolution.

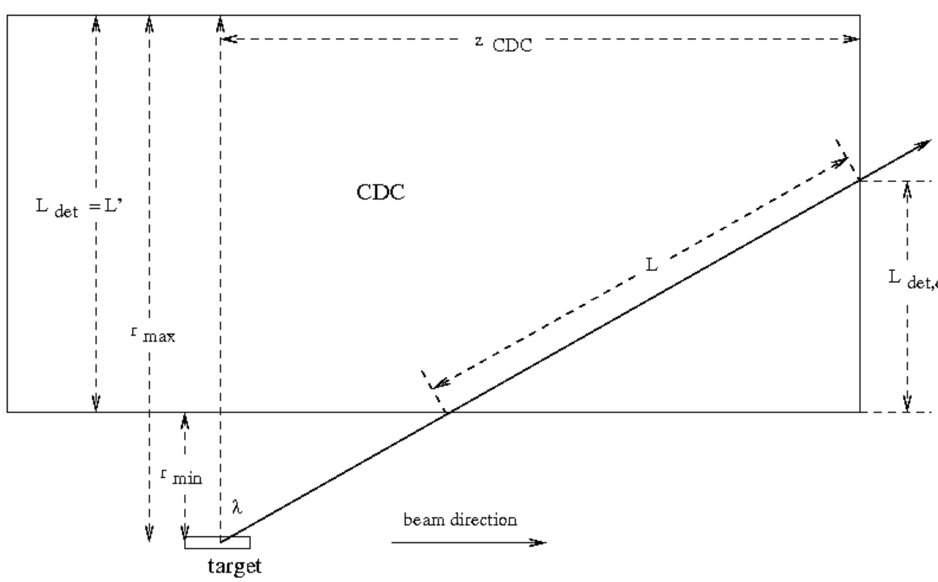


Figure 4: Geometry of the CDC

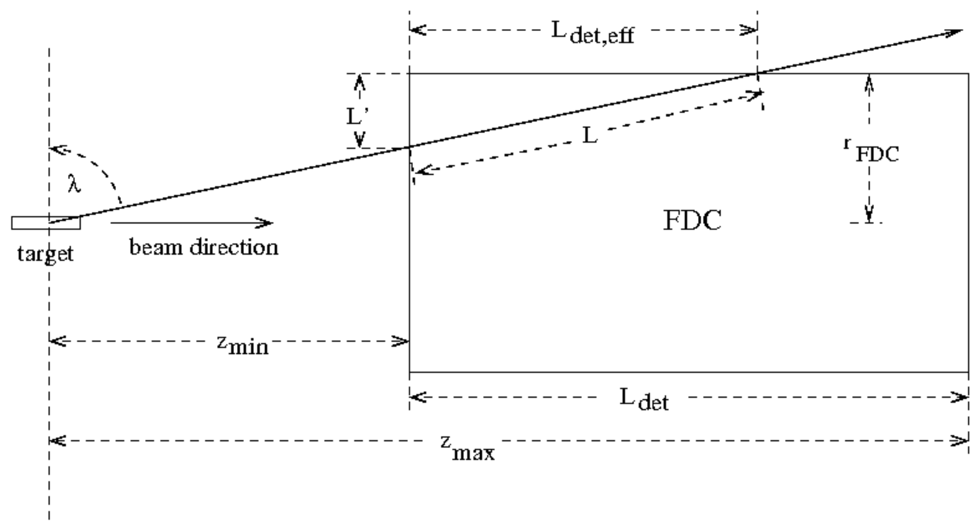


Figure 5: Geometry of the FDC.

$r_{\mathrm{max,stereo}}$ maximum radius of the CDC stereo layers

$n_{\mathrm{rl,FDC}}$ number of radiation lengths measured transverse to the tracking layers, *i. e.*, at 90° in the lab ($n_{\mathrm{rl}} = x/X_0$)

$n_{\mathrm{m,FDC}}$ number of position measurements for a track which passes through all layers of the CDC (sum of number of axial and stereo layers)

$n_{\mathrm{m,stereo}}$ number of position measurements in stereo layers

For the CDC, $L_{\mathrm{det}} = r_{\mathrm{max,CDC}} - r_{\mathrm{min,CDC}}$ is the radial thickness of the active volume. Then $L' = L_{\mathrm{det}}$ and $L = L_{\mathrm{det}}/\cos\lambda$.

For tracks that exit the end of the CDC, the straight-line track approximation is used to scale the number of measurements, the number of radiation lengths and the transverse length of the track ($L'$). In fact the straight-line approximation is used to determine whether this scaling is necessary or not.

## 3.7 Geometry of the FDC

Fig. 5 shows a schematic of the FDC chamber displaying some of the parameters that serve as input to the estimate:

$z_{\mathrm{min}}$ minimum $z$ of the FDC tracking volume

$z_{\mathrm{max}}$ maximum $z$ of the FDC tracking volume

$r_{\mathrm{FDC}}$ outer radius of the FDC

Other parameters are:

$n_{\mathrm{rl,FDC}}$ number of radiation lengths measured transverse to the tracking layers, *i. e.*, in the forward direction ($n_{\mathrm{rl}} = x/X_0$)

$n_{\mathrm{m,FDC}}$ number of one-dimensional position measurements for a track which passes through all layers of the FDC[2]

[2] Since not all layers of the FDC will yield position measurements in a particular chosen dimension, the number of effective measurement planes will be only a fraction of the total measurement planes present in the chamber. While this is true of the CDC as well, the stereo angle is assumed to be so small that stereo layers are assumed to contribute fully to position determination in the $r$-$\phi$ plane (an approximation).

For the FDC $L_{\rm det} = z_{\rm max} - z_{\rm min}$ is the length of the active volume along the beam direction. Then $L' = L_{\rm det}/\tan\lambda$ and $L = L_{\rm det}/\sin\lambda$.

For tracks that exit the side of the FDC, like the CDC, the straight-line track approximation is used both to determine if the track is "exiting early" and to scale the number of measurements, the number of radiation lengths and the transverse length of the track ($L'$).

### 3.8 Combining the CDC and the FDC

For each quantity, the measurements in the CDC and FDC are statistically independent. They are therefore combined using:

$$\sigma_{\rm total} = \frac{1}{\sqrt{\frac{1}{\sigma^2_{\rm CDC}} + \frac{1}{\sigma^2_{\rm FDC}}}} \tag{24}$$

to get the overall resolution.

## 4 Results

Table 1 is a guide to plots which show results of estimates of resolutions for the parameters in the design at this writing. In each figure, the upper left graph shows resolution contributions from the CDC, the upper right shows those from the FDC, and the lower left shows the combination of the CDC and FDC. The lower right contains a legend. In each of the upper plots (of the CDC and FDC alone), the contribution from position resolution is shown in yellow and that from multiple scattering is shown in blue. In addition, for the azimuthal resolution figures, there is a contribution from curvature error shown in magenta. CDC total resolution is displayed in red. This red curve is the same data for the CDC alone and in the combined. Likewise, the blue curve in the FDC alone graph and in the combined is the total resolution from the FDC and displays the same data in both cases. In addition, in each of the combined graphs at lower left there is a comparison with the results of HDGEANT for the same angle or momentum as appropriate.

| | $\delta p_t/p_t$ | $\delta\theta$ | $\delta\phi$ |
|---|---|---|---|
| resolution as a function total momentum | Fig. 6 | Fig. 7 | Fig. 8 |
| resolution as a function of polar angle | Fig. 9 | Fig. 10 | Fig. 11 |

Table 1: Figures displaying resolution estimates.

The plots show reasonable agreement with the HDGEANT results.

## A Some useful equations

This appendix contains some intermediate results in the calculation of the error on the slope in the case of equally spaced measurements.

The well-known expressions

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \tag{25}$$

and

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \tag{26}$$

allow us to obtain the following results for equally space measurements:

$$\sum x_i = \frac{nL}{2} \tag{27}$$

$$\sum x_i^2 = \frac{L^2 n(2n-1)}{6(n-1)} \tag{28}$$

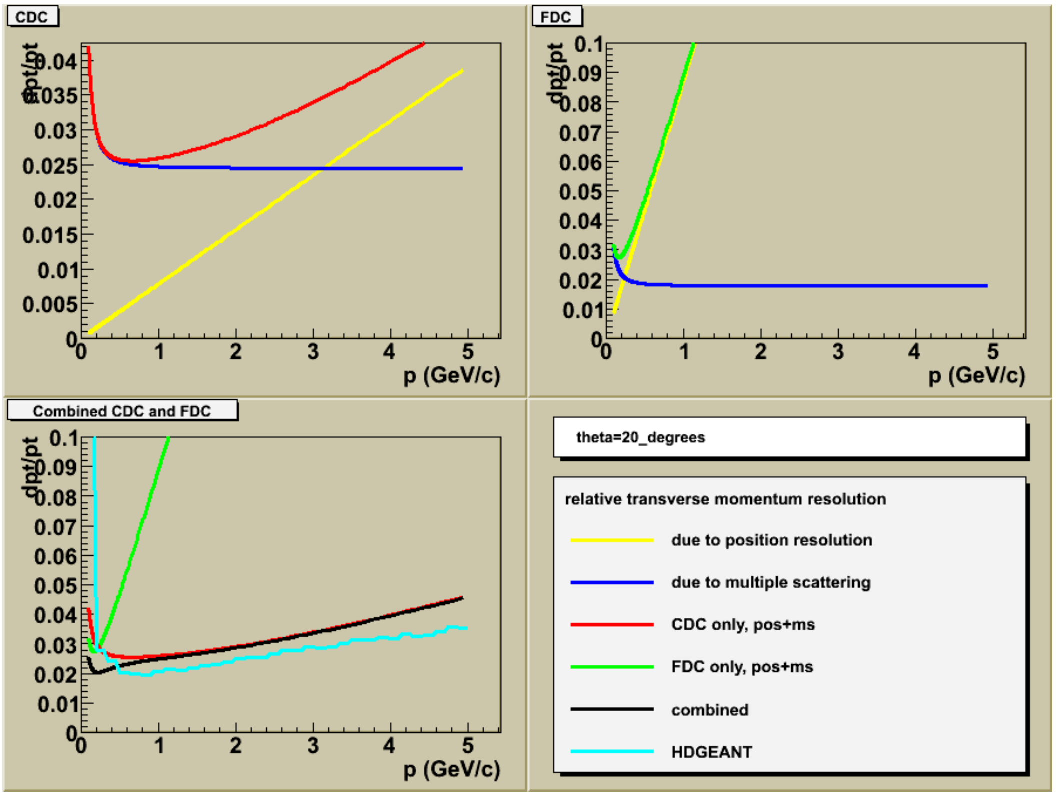


Figure 6: Estimates for resolution in relative transverse momentum as a function of total momentum at 20°.

and for the parameter in the straight-line-fit error expressions we get

$$\Delta' = \frac{L^2 n^2 (n+1)}{12(n-1)}. \tag{29}$$

# References


[1] Philip R. Bevington and D. Keith Robinson. *Data Reduction and Error Analysis for the Physical Sciences*. McGraw-Hill, New York, 2nd edition, 1992.

[2] W. M. Yao et al. Review of particle physics. *J. Phys.*, G33:1–1232, 2006.

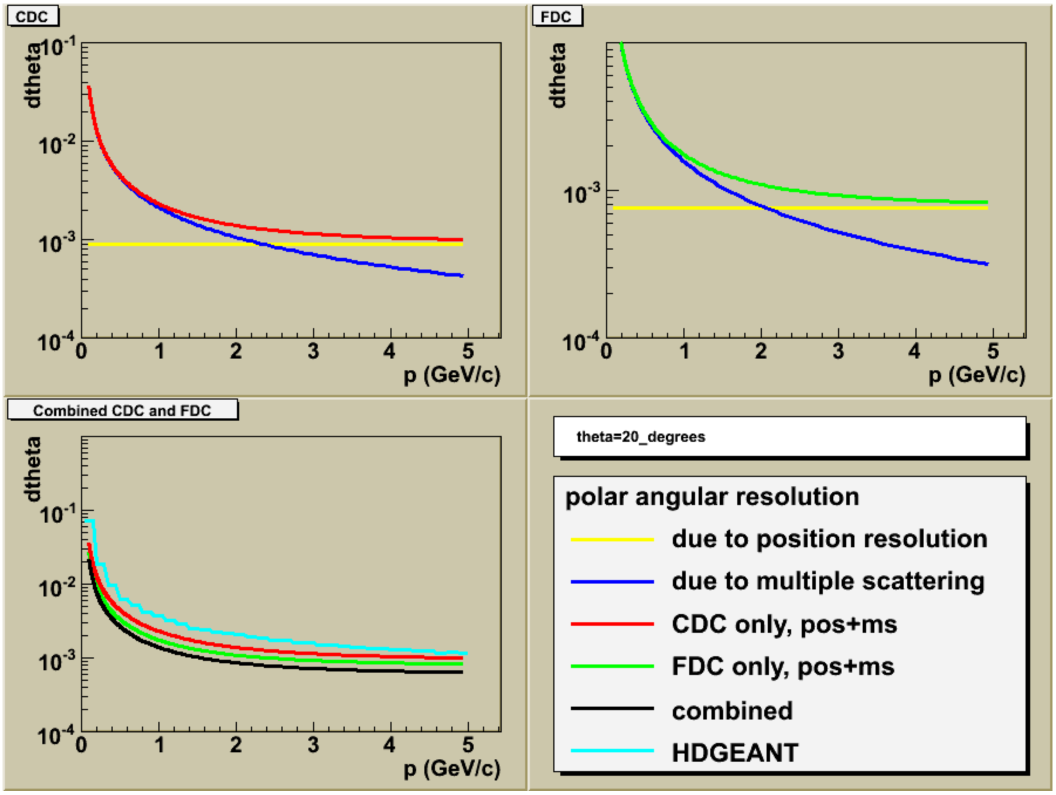


Figure 7: Estimates for resolution in polar angle as a function of total momentum at 20°.

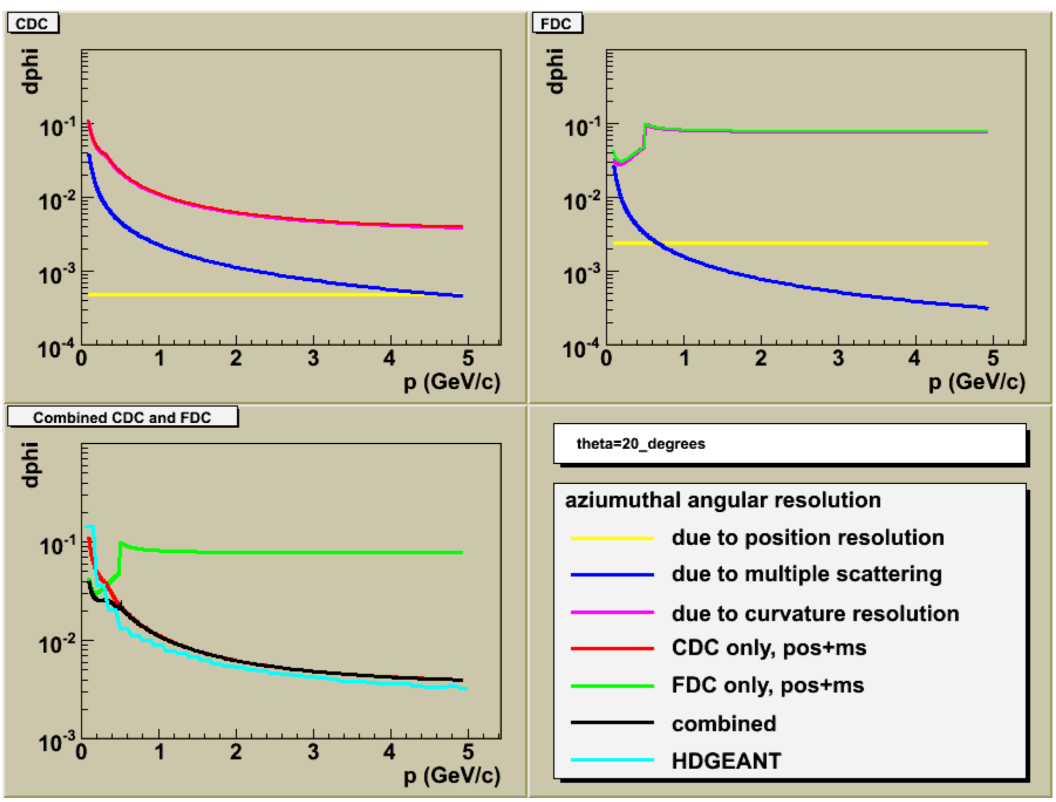


Figure 8: Estimates for resolution in azimuthal angle as a function of total momentum at 20°.

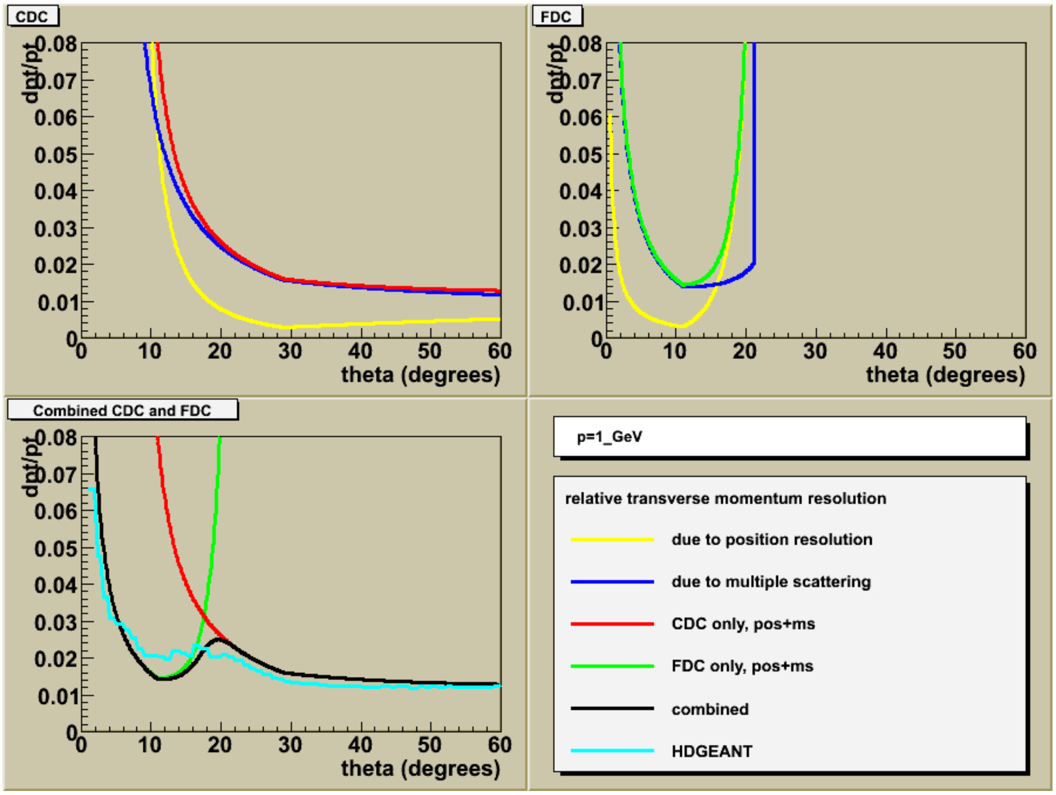


Figure 9: Estimates for resolution in relative transverse momentum as a function of polar angle at 20°.

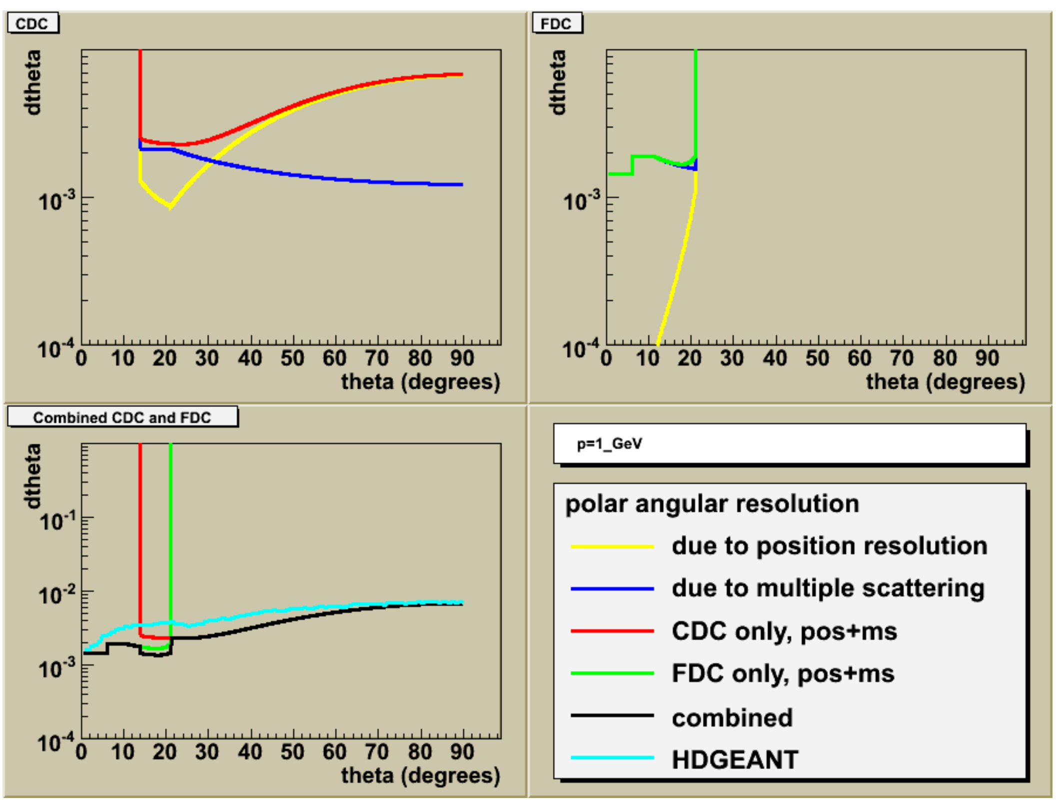


Figure 10: Estimates for resolution in polar angle as a function of polar angle at 20°.

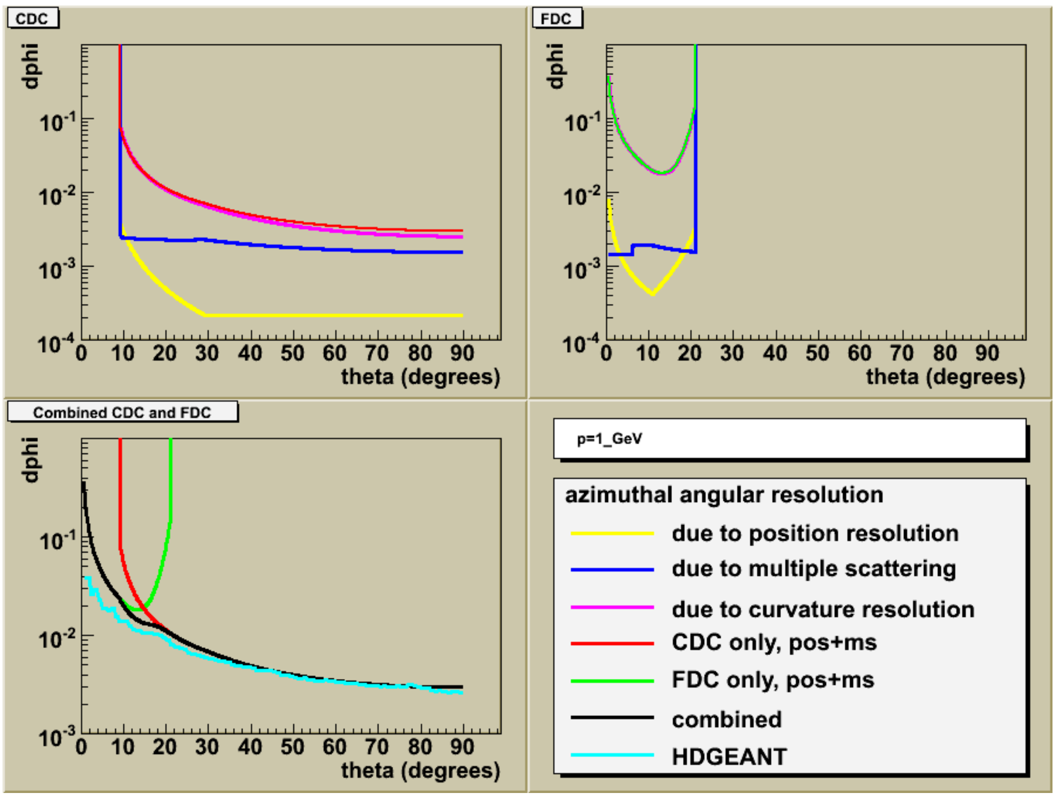


Figure 11: Estimates for resolution in azimuthal angle as a function of polar angle at 20°.