Performance of the prototype module of the GlueX electromagnetic barrel calorimeter

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Abstract

A photon beam test of the 4 m long prototype lead/scintillating fibre module for the GlueX electromagnetic barrel calorimeter was carried out in Hall B at the Thomas Jefferson National Accelerator Facility with the objective of measuring the energy and timing resolutions of the module as well as the number of photoelectrons generated. Data were collected over an energy range of 150 to 650 MeV at multiple positions and angles along the module. Details of the analysis at the centre of and perpendicular to the module are shown herein; the results are $\sigma_E/E = 5.4\%/\sqrt{E(\text{GeV})} \oplus 2.3\%$, $\sigma_{\Delta T/2} = 70/\sqrt{E}$ ps, and 660 photoelectrons for 1 GeV at each end of the module.

Key words: electromagnetic calorimeter, scintillating fibres PACS: 29.40 Vj

1 **1** Introduction

The principal aim behind the GlueX experiment is to elucidate the phenomenon of confinement, by conducting advanced meson spectroscopy and searching for predicted exotic hybrid states with explicit gluonic degrees of freedom. Such states have a plethora of decays leading to photons in the final state, and require hermetic calorimetry for their detection and measurement of their four momentum. Test results from the cylindrical electromagnetic calorimeter for GlueX are reported herein.

A brief overview of the GlueX experiment is presented in Section 2. The photon
beam test – conducted in Hall B at the Thomas Jefferson National Accelerator
facility – and the setup of the experiment are covered in Section 3. The analysis
method and results for the energy resolution of the Hall B beam test are
described in Section 4. The timing resolution analysis and results are shown
in Section 5, while the photoelectron analysis is provided in Section 6. Lastly,
the results are summarized in Section 7.

¹⁶ 2 Overview of GlueX

To achieve the primary physics goal of GlueX, namely mapping the spectrum 17 of gluonic excitations of light mesons, it is essential to measure photons and 18 charged particles with sufficient acceptance and resolution to identify exclu-19 sive reactions, a requirement imposed by the amplitude analysis needed to 20 determine the J^{PC} quantum numbers of the produced mesons. The photons 21 of particular interest are those resulting from $\pi^0 \to \gamma \gamma$ and $\eta \to \gamma \gamma$ decays. 22 Photoproduction data at 9 GeV are sparse and mainly come from bubble 23 chamber experiments, in which reconstruction of final states with multiple 24 neutral particles is impossible. Such final states are expected to make up 60%25 of the photoproduction cross section, underscoring both the need and discov-26 ery potential for neutral particle reconstruction. GlueX will run in a dedicated 27 experimental hall (Hall D) at Jefferson Lab, to be constructed as part of the 28 12 GeV upgrade to the lab. 29

30 2.1 The GlueX Detector and Barrel Calorimeter

The GlueX detector design is ideally suited for a fixed-target photoproduction experiment. The 2.2 T solenoidal magnetic field traps low-energy electromag-

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netic background $(e^+e^-$ pairs) generated in the target inside a small diameter 33 beam hole that runs through the detector. The photon beam is incident on 34 a 30-cm LH_2 target. The target is surrounded by a start counter made of 35 plastic scintillator that provides event timing information, a cylindrical track-36 ing chamber (CDC) and a cylindrical electromagnetic calorimeter (BCAL). 37 Downstream of the target are circular planar tracking chambers (FDC) and a 38 circular planar electromagnetic calorimeter (FCAL). A schematic of the detec-30 tor is shown in Fig. 1; the two electromagnetic calorimeters are used to detect 40 and determine the four-momentum of the aforementioned decay photons. 41



Fig. 1. Schematic of the GlueX Detector. The detector has cylindrical symmetry about the beam direction. The detector subsystems and the dashed lines at angles (with respect to the beam direction) 10.8° through 126.4° are referenced in the text. The start counter is not shown for clarity.

The BCAL is shown schematically in Fig. 2. The dimensions of this calorime-42 ter are driven by the volume required for charged particle tracking and the 43 bore dimensions of the solenoidal magnet. The BCAL design is based on that 44 of the electromagnetic calorimeter used in the KLOE experiment at $DA\Phi NE$ -45 Frascati, which also operated in a solenoidal magnetic field [1–3]. The BCAL 46 and KLOE calorimeters both employ a lead/scintillating-fibre (Pb/SciFi) ma-47 trix of similar length with photosensors at either end to provide energy (ADC) 48 and time (TDC) measurements. The diameter of the KLOE calorimeter is 49 about three times that of the BCAL. 50

The relevant parameters that determine the π^0 and η mass resolutions are the photon energy (E) and the polar and azimuthal position resolutions (σ_{θ} and σ_{ϕ}). The energy resolution (σ_E) depends on the number of photoelectrons (N_{pe}) yielded by the photosensors, based on the collected light. The photo-



Fig. 2. The GlueX BCAL. (a) BCAL schematic; (b) a BCAL module side view; (c) end view of the BCAL showing all 48 modules and (d) an end view of a module showing readout segmentation. Details are given in the text.

electron statistics are strongly dependent on the stochastic fluctuations of the 55 energy deposited by the electromagnetic shower in the scintillating fibres of 56 the calorimeter modules. In addition, the number of photoelectrons collected 57 depends on the fraction of photon shower energy deposited in the fibres, the 58 efficiency with which the resulting scintillation light is captured in and trans-59 mitted down the fibre to the photosensor, and the photon detection efficiency 60 of the photosensor. The photon position is determined by the readout seg-61 mentation in the azimuthal direction and the difference in arrival time (ΔT) 62 of the scintillation light between the two ends of the barrel. The resolution 63 in the time difference $(\sigma_{\Delta T})$, and therefore the polar angle resolution, also 64 depend on the number of photoelectrons. The former is a critical input into 65 the momentum resolution for photons and for the particle identification for 66 charged particles, in conjunction with trajectories from the drift chambers. 67 As such, the time difference analysis is reported in this paper as being more 68 representative of the intrinsic BCAL resolution and independent of any ex-69 ternal timing reference. Other parameters of relevance for extracting physics 70 are adequate segmentation to avoid multiple occupancy, good linearity and a 71 sufficiently low-energy threshold for photon detection. 72

The performance metrics for these quantities were set by simulating hadronic 73 74

photoproduction at GlueX energies using PYTHIA [4] and also by simulat-

ing several of the signature reactions expected to yield exotic mesons. These 75 studies included a GEANT-based simulation [5] of the entire GlueX detec-76 tor response, including detector material and cabling, photon reconstruction 77 and kinematic fitting. The PYTHIA simulations indicate that 70% of the pro-78 duced photons with energies up to about 2 GeV will be incident on the 79 BCAL. The photon population in the BCAL for one of the signature reac-80 tions, $\gamma p \to \eta \pi^0 p \to 4\gamma p$, where the distribution in $\eta \pi^0$ mass was uniform 81 from 1.0 to 2.0 GeV/c^2 and uniform in decay angles, is shown in Fig. 3. The 82 distribution of photons is plotted as a function of position from the upstream 83 end of the BCAL; the photons predominantly populate the downstream end 84 of the BCAL. The target occupies the region z = 33 - 63 cm. Also shown is 85 the average energy as a function of z with higher energy photons being more 86 forward. The integrated thickness of the BCAL matrix, in number of radiation 87 lengths, traversed by photons incident at various positions along the length of 88 the BCAL is also shown. Note that there is a narrow ($\sim 1^{\circ}$) angular range near 89 11° where the photon trajectory intercepts a small number of radiation lengths 90 of the Pb/SciFi matrix. Photons with angles less than 10°, with respect to the 91 beam direction, are detected in the FCAL. 92



Fig. 3. The distribution of photons, their energy and integrated path length through the Pb/SciFi matrix as a function of position along the length of the BCAL for one of the GlueX signature reactions, $\gamma p \rightarrow \eta \pi^0 p \rightarrow 4\gamma p$, is shown. The target position and angular range subtended by the BCAL are also presented.

⁹³ Moreover, the segmentation shown in Fig. 2d leads to double-occupancy in ⁹⁴ less than one-percent of events with two or more photons incident on the ⁹⁵ BCAL. This segmentation is also required for adequate determination of the ⁹⁶ azimuthal angle of tracks as well as for providing information on the energy ⁹⁷ deposition profile in depth, for good cluster identification. Additionally, studies ⁹⁸ of the lowest energy photons in high-multiplicity reactions that are expected ⁹⁹ to yield exotic hybrids such as $\gamma p \rightarrow b_1(1235)\pi n \rightarrow 2\pi^+\pi^-2\pi^0 n$ indicate that ¹⁰⁰ an energy threshold of 40 MeV suffices.

Finally, it is important to point out differences in the GlueX and KLOE appli-101 cations of barrel calorimetry. KLOE is a symmetric colliding beam experiment 102 with the intersection region at the centre of its barrel calorimeter. As a result, 103 that calorimeter is illuminated symmetrically and nearly uniformly by pho-104 tons having energies, on average, between 100 and 200 MeV and with very few 105 photons greater than 400 MeV. On the other hand, GlueX is a fixed target 106 experiment, resulting in a highly asymmetric photon distribution: 30% of the 107 photons in the BCAL will have energies considerably higher than 500 MeV. 108 Despite these differences, the KLOE experience provides valuable guidance in 109 the design and construction of the BCAL. The achieved KLOE resolutions [3] 110 of $\sigma_E/E = 5.4\%/\sqrt{E(\text{GeV})}$ and $56/\sqrt{E(\text{GeV})}$ ps are also adequate to achieve the GlueX physics requirements, as indicated by our simulation studies. The 111 112 extracted resolutions are a direct result of the internal Pb/SciFi matrix geom-113 etry such that similar resolutions should be expected for the BCAL [6]. 114

115 2.2 Module Geometry

Table 1 summarizes the salient features of the BCAL. These parameters are 116 based on the KLOE experience, detailed GEANT-based simulations and tests 117 of a full-scale prototype with charged particles, photon beam and cosmic rays. 118 Aside from the attenuation length, the SciFi parameters are not brand specific 119 but rather represent the generic parameters of double-clad fibres. The latter 120 have a higher capture ratio compared to single clad fibres, such as used in 121 KLOE. The nominal increase in capture ratio is over 50%, thus resulting in a 122 similar increase in the number of photoelectrons, which can be important for 123 low energy photons incident on the BCAL and the corresponding thresholds 124 of the detector. 125

The first prototype module (Module 1), used in the beam test described in this 126 paper, was constructed of alternating layers of 99.98% pure lead of 0.5 mm 127 thickness that were grooved ("swaged"), creating channels to accommodate 128 the fibres. This was accomplished by passing the lead sheets between the two 129 grooved rollers of a custom-designed machine thereby creating the channels 130 by plastic deformation of the lead. The fibres were obtained from PolHiTech¹ 131 and are type PHT-0044 double-clad scintillating fibres of 1 mm diameter. 132 These were bonded in the lead channels with Bicron- 600^{2} optical epoxy. The 133 thickness of the module is 23 cm, its length is 400 cm and the width is 12 cm 134 with the internal matrix geometry as indicated in Fig. 4. The matrix was built 135

¹ PolHiTech SRL, 67061 Carsoli (AQ), Italy (www.polhitech.it)

² Saint-Gobain Crystals & Detectors, USA (www.bicron.com)



Fig. 4. The BCAL fibre matrix showing the placement of 1 mm diameter fibres in the azimuthal and radial directions. The dimensions of the azimuthal and radial pitch, the glue box between the lead sheets and the glue ring around the fibres were determined from the prototype module using a measuring microscope. Particle tracks would appear to enter the matrix from the bottom. More details are given in Ref. [11].

upon an aluminum base plate of 2.54 cm thickness that was further supported 136 by a steel I-beam for added stiffness and ease of handling. Module 1 was not 137 machined along its long sides at the 7.5° indicated in Fig. 2 and retained its 138 rectangular profile from production. In contrast, the two ends of the module, 139 where the read-out system was attached, were machined and polished. Visual 140 inspection revealed that only eight of the approximately 17 000 fibres had 141 been damaged in handling and construction. No optical defects affecting light 142 transmission were observed in the other fibres. 143

144 **3 Beam Test**

The goals of the beam test were to measure the energy, timing and position 145 resolutions of the prototype BCAL module as well as the response of the 146 module at different positions along its length and at various angles of the 147 incident beam. Results of this beam test will anchor further Monte Carlo 148 simulations of the GlueX detector and will aid in the development of the 48 149 modules for the full BCAL detector. The detailed analysis and results reported 150 in this paper are for Module 1 perpendicular to the beam ($\theta = 90^{\circ}$) with the 151 beam incident at its centre (z = 0 cm). 152

153 3.1 Experimental Facility

The beam test took place in the downstream alcove of Hall B at the Thomas 154 Jefferson National Accelerator Facility (Jefferson Lab). In order to accom-155 modate the module with its support frame, read-out system and cables, an 156 additional platform was installed in front of the alcove. This expanded space 157 allowed for the measurements with the photon beam perpendicular to the 158 module, as well as providing a greater range of lateral and rotational degrees 159 of freedom for the module when positioned inside the alcove. However, as il-160 lustrated in Fig. 5, the relative dimensions of the alcove and platform, with 161 respect to the length of the module, still allowed for only a limited range of 162 positions and incident angles that could be illuminated by the beam. Measure-163 ments, when the module was on the platform and oriented perpendicularly 164 to the beam, were possible for relative positions of the beam spot between 165 -100 cm to +25 cm with respect to the centre of the module. Within the 166 alcove, the angular range was limited to angles 40° and less, and a length scan 167 was carried out between -190 cm to -15 cm. The module was mounted on a 168 cart that could be remotely rotated with good precision to the required angle. 169 Lateral movements of the module with respect to the beam required a hall 170 access for manual positioning. 171



Fig. 5. Diagram of the Hall-B downstream alcove with schematic placements of the BCAL module. The drawing is not to scale.

The primary electron beam energy from the CEBAF accelerator at Jefferson 172 Lab was $E_0 = 675$ MeV and the current was 1 nA for most of the mea-173 surements. The electron beam was incident on a thin target (the "radiator") 174 located just upstream of the magnetic spectrometer (the "tagger"). The ener-175 gies of the electrons scattered from the radiator were measured, thus providing 176 timing and momentum information for the associated bremsstrahlung photons 177 with a spectrum of energies from 150 MeV up to 650 MeV, as described be-178 low. The photon beam was collimated with a 2.6 mm collimator reducing the 179

flux after collimation to 6.5% of its original value, resulting in a beam spot of virtually uniform density with a diameter of 1.9 cm on the BCAL module. The distance from the radiator to the collimator and the collimator to the BCAL were 5.8 m and ~39 m, respectively. See Ref. [15] for more details on the Hall B tagger.

The Hall B tagger system determines the electron momentum information 185 from 384 individual scintillator paddles, called E-counters, with a phototube 186 on one end. Each of these counters is arranged to cover constant momentum 187 intervals of $0.003E_0$ and to physically overlap with its adjacent neighbour by 188 1/3 of its width, thus creating 767 individual photon energy bins and providing 189 an energy resolution of $0.001E_0$. The timing information, on the other hand, is 190 provided by 61 individual scintillator counters, called T-counters, with photo-191 tubes attached to both ends. The T-counters are classified in two groups. The 192 first 19 (narrower) counters cover 75% to 90% of the incident electron energy 193 range and the remaining 42 counters cover the 20% to 75% range. 194

195 3.2 Readout and Electronics

The module was divided into 18 readout segments, each with dimensions $3.81 \times$ 196 3.81 cm^2 . This segmentation comprised six rows in depth and three columns 197 vertically with respect to the beam, as shown in Fig. 6. Acrylic light guides 198 having a square profile and with a 45° mirrored surface channelled the light 199 from the fibres to the PMTs that were placed perpendicularly to the fibre 200 direction on both the North and South ends of the module, as shown in Fig. 7. 201 The staggered, vertical placement of the PMTs was due to their diameter of 202 5 cm being larger than the 3.81 cm width of the readout segment size. Large, 203 rectangular silicone sheets, 2.5 mm thick, were used to interface the light guides 204 with the module and smaller, circular, 2.5 mm thick, silicone cookies coupled 205 the PMTs to the light guides. The readout ends and all their components were 206 enclosed in an aluminum box painted black with the top covered by Tedlar^{® 3} 207 PVF to maintain light-tightness. The shower profile was such that most of 208 the energy, nearly 90%, was deposited in the first 12 cm of the BCAL and 209 the largest number of photoelectrons originated in that part of the module. 210 For this reason, the three upstream columns of Fig. 6 were read out using 211 Philips⁴ XP2020 photomultiplier tubes. These tubes were selected for their 212 good timing characteristics. The last three rows were read out using Burle⁴ 213 8575 PMTs. 214

²¹⁵ The bases for the PMTs were designed with dual BNC outputs on the anode.

 $[\]overline{\ ^3 \ }$ Tedlar $^{\textcircled{R}}$ is a registered trademark of E. I. du Pont de Nemours and Company or its affiliates.

⁴ PHOTONIS SAS, Brive, France (www.photonis.com)



Fig. 6. The segmentation and readout for the BCAL module as viewed from its North end. The lead/scintillating fibre matrix would appear to be rotated by 90° with respect to Fig. 4. The electromagnetic shower that develops in the module approximately forms a cone shape and is illustrated with the shaded triangle in the figure. A very small percentage of the energy is deposited in the outer segments or leaks out the sides.



Fig. 7. (a) The box that encloses the 18 light guides and PMTs with cables attached for the South end of the BCAL module is shown. (b) The module is entirely wrapped in Tedlar[®] on the right and pressed against the light guides using a silicone sheet, as described in the text.

The signals were sent to a CAEN C207 equivalent leading edge discriminator 216 and from there they were sent directly to a JLab F1 TDC [16] that was used 217 to record the timing of the signals. The sum of the discriminator outputs was 218 sent to a second discriminator, the threshold of which was set to require signals 219 from at least four PMTs from each end of the module. The threshold logic 220 pulse from either end (North OR South) of the module and the Master OR 221 (MOR) signal from the T-counters of the tagger defined the trigger for the 222 experiment. On average, the event rate was between 1 to 4 kHz for the duration 223 of the beam test. A special electronics module was used to allow cosmic event 224 triggers from scintillator paddles placed above and below the module as well 225 as triggers from a pulser that were used to establish ADC pedestals, and 226 were recorded concurrently with beam data. Signal amplitudes from the the 227 second BNC output of the PMTs were digitized using CAEN V792 ADCs. 228



²²⁹ The complete logic diagram is shown in Fig. 8.

Fig. 8. The logic diagram for the BCAL Hall-B beam test electronics. It should be noted that segments 1 and 13 did not contribute to the trigger, and this explains the apparent discrepancy between the 18 outputs of the discriminator and the sum output (that is just 16).

230 4 Energy Resolution

231 4.1 Gain balancing and energy calibration

With the module divided into 18 segments on each of the North and South 232 sides, 36 PMTs were utilized in total. By adjusting the PMT supply voltage, 233 an initial, relative balancing of the PMT gains was performed using cosmic 234 data during the setup stage such that the means of the cosmic ADC spectra 235 were nominally within ten percent of a certain value; only a couple channels 236 deviated from this value by up to a factor of two. Further adjustments to the 237 gains were done in software during the analysis, using the spectra collected 238 during four dedicated cosmic runs. 239

²⁴⁰ By assuming that the energy deposited by cosmic rays is uniform in each ²⁴¹ segment of the BCAL, a gain balancing constant was found for each North and South segment by taking the ratio of each segment's spectra to that ofone particular segment,

$$C_{N,i} = \frac{N_{\text{ADC},i}}{N_{\text{ADC},7}},\tag{1}$$

where $C_{N,i}$ was the balancing constant for the *i*th segment on the North side, each balanced with respect to $N_{ADC,7}$. The procedure was identical for the South end, anchoring with respect to $S_{ADC,7}$. Keeping in mind the attenuation length of the BCAL, $N_{ADC,7}$ and $S_{ADC,7}$ were then balanced with respect to one another. An overall energy calibration constant for the BCAL was then found by plotting the balanced ADC values versus the tagged photon energy.

Once the BCAL was calibrated, the distribution of the difference between the reconstructed BCAL energy and the tagged photon beam energy was found. This ratio, *D*, is defined as

$$D = \frac{E_{\text{BCAL}} - E_{\text{BEAM}}}{E_{\text{BEAM}}},$$
(2)

where E_{BCAL} , the reconstructed energy in the BCAL module, is defined as

$$E_{\text{BCAL}} = K \cdot \sqrt{\left(\sum_{i=1}^{18} \frac{N_{\text{ADC},i}}{C_{\text{N},i}}\right) \left(\sum_{i=1}^{18} \frac{S_{\text{ADC},i}}{C_{\text{S},i}}\right)}$$
(3)

 E_{BEAM} is the photon energy measured in the tagger and K is the overall calibration constant. The reconstructed energy in the BCAL module is then the geometric mean of the balanced ADC values multiplied by K. The width of the distribution, σ_D , is the energy resolution, σ_E/E , for the module.

A plot of D vs. E_{BEAM} can be seen in Fig. 9. This shows how well the PMT gains are balanced and the energy in the BCAL is reconstructed. Although the deviations from zero are so small as to be inconsequential, typically less than 0.5%, there may be a number of physical reasons for these deviations such as non-linearities in the sampling fraction of the shower for each segment, albedo, background contributions to the ADC spectra which could not be removed at lower energies and leakage outside the module.

268 4.2 Energy resolution results

The calibrated spectra for D were fitted by a Gaussian function. A typical spectrum and its fit are shown in Fig. 10, this one for timing counter 40, corresponding to a beam energy of 273 MeV.



Fig. 9. $D = (E_{\text{BCAL}} - E_{\text{BEAM}})/E_{\text{BEAM}}$ is shown after gain balancing and calibration. Notice that the deviations from zero are typically less than 0.5%.



Fig. 10. The calibrated spectrum for D is shown for timing counter 40, corresponding to a beam energy of 273 MeV. The solid line is a Gaussian fit to the data.

Subsequently, the energy resolution was extracted for all timing counters and is shown in Fig. 11, plotted as a function of the tagged photon beam energy, for the data at $\theta = 90^{\circ}$ and z=0 cm. The fit to the data is also shown in Fig. 11, resulting in

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$$\frac{\sigma_E}{E} = \frac{5.4\%}{\sqrt{E(\text{GeV})}} \oplus 2.3\%,\tag{4}$$

where the \oplus indicates addition in quadrature. Small variations in the fits produced relatively large variations in the floor term (2.3 \pm 1%) but little variation in the stochastic term (5.4 \pm 0.1%).

²⁸⁰ In general, the energy resolution of an electromagnetic calorimeter is expressed



Fig. 11. Energy resolution vs. E_{BEAM} for photons for $\theta = 90^{\circ}$ and z = 0 cm. The fit gives $\sigma_E/E = 5.4\%/\sqrt{E(\text{GeV})} \oplus 2.3\%$. The fit of Fig. 10 corresponds to the 40^{th} datum from the right (19th from the left) in this figure.

²⁸¹ in the form:

 $\frac{\sigma_E}{E} = \frac{a}{\sqrt{E(\text{GeV})}} \oplus b \oplus \frac{c}{E(\text{GeV})}.$ (5)

The a/\sqrt{E} term contains the combined effect of sampling fluctuations and 283 photoelectron statistics. It is commonly referred to as the stochastic term. 284 The $1/\sqrt{E}$ dependence is expected from the fact that the fluctuations are 285 proportional to the number of particle tracks, n, that cross the active mate-286 rial; n has a Poisson distribution with a variance \sqrt{n} . Since the energy of a 287 shower is proportional to n, the contributions to the resolution σ_E/E due to 288 the stochastic fluctuations is proportional to $1/\sqrt{E}$. The KLOE collaboration 289 concluded that the photon statistics from the light yield of their calorime-290 ter ranges from $1.6\%/\sqrt{E(\text{GeV})}$ [17] up to $2.7\%/\sqrt{E(\text{GeV})}$ [18] and, therefore, 291 contributes very little to the resolution since it is added in quadrature to the 292 sampling contribution. Indeed, the stochastic fluctuations in sampling domi-293 nate the resolution. 294

The constant term, *b*, in Eq. 5, originates from all other energy-independent sources that contribute to uncertainties in the energy reconstruction. These sources can be mechanical imperfections, material defects, segment-to-segment calibration variations, non-uniformity of response, instability with time and shower leakage. Much work has gone into removing any of these effects during the construction of the BCAL module, throughout the beam test, and in any subsequent analysis.

³⁰² If contributions from the noise term, c/E, existed, they would be from elec-³⁰³ tronics noise and pileup in high-rate environments. This term increases at lower energies but has not been observed to contribute in the beam test analysis as both the rates and noise were low. Fits to the beam test data including this term produce almost identical stochastic and constant terms with values for c consistent with zero.

The stochastic coefficient a = 5.4% in Eq. 4 compares well with the corre-308 sponding ones from KLOE determined from $e^+e^- \rightarrow e^+e^-\gamma$ reconstruction, 309 reported as a = 5.4% [3]. The KLOE beam tests [18] reported a value for 310 a = 5%. No value for b was reported in either case as it did not contribute 311 within KLOE's energy range due to its addition in quadrature to the resolu-312 tion. The KLOE calorimeter and BCAL Module 1 - as read out in the beam 313 test – have similar sampling fractions and photostatistics. Although the pro-314 duction readout for BCAL will be different, the beam test setup resulted in 315 benchmark metrics for any future upgrades to the BCAL matrix structure, 316 such as increasing the sampling fraction in the inner layers to improve low 317 energy photon detection for critical regions of exotic hybrid production phase 318 space and producing a better energy resolution. 319

As mentioned above, the stochastic fluctuations in sampling dominate the res-320 olution. This being the case, there should be little effect of the shower position 321 along the module (z) on the energy resolution, because attenuation only af-322 fects the number of photoelectrons at the read-out ends of the module and is 323 compensated for by the double ended read-out of the module. On the other 324 hand, increasing the photon beam energy results in more particle tracks over 325 a greater depth of the shower profile generated within the module, therefore 326 more fibre layers are intercepted by more particle tracks. The expectation, 327 then, is that the resolution will improve with increasing photon energy but 328 remain nearly independent of the position of the beam spot. This was veri-329 fied by examining the energy resolution for photon energies from 225 MeV to 330 575 MeV for three z-positions at normal incidence. 331

Finally, *b* is a reasonable indicator of the intrinsic constant term in the energy resolution of the BCAL. However, the maximum energy of the photon beam test was too low to determine it precisely, as the constant term contributes negligibly to the resolution at a few hundred MeV when added in quadrature to the stochastic term. Nevertheless, since approximately 30% of the photons in GlueX will have energies above 500 MeV, efforts to minimize the constant term and extract it more accurately in future beam tests will be fruitful.

339 4.3 Sampling fraction and energy

The sampling fraction – the fraction of energy deposited in the SciFi's – can be expressed as a ratio with respect to either the total energy deposited in the BCAL module (f) or the incident photon energy (f_{γ}) . These quantities, however, are difficult to measure in an experiment but fairly simple to simulate. A GEANT 3.21 simulation was developed to that end, based on modelling the BCAL as a standalone package and independent from the previously mentioned simulations. Individual fibre and epoxy volumes were programmed into the Monte Carlo with the appropriate Pb:SciFi:Glue ratios and material properties resulting in the geometry shown in Fig. 4.



Fig. 12. (a) The photon energy sampling fraction, f_{γ} , and deposited energy sampling fraction, f, are shown from simulation for $\theta = 90^{\circ}$. The data are fit to $a_0/E + a_1E + a_2$ functions, which were chosen among the simplest functions that described the data well. (b) The sampling fluctuations of the module, $\sigma_{f_{\gamma}}/f_{\gamma}$ and σ_f/f are plotted from simulation. The data are fit to $a/\sqrt{E} \oplus b$ functions with $a_{f_{\gamma}} = 4.6\%$, $b_{f_{\gamma}} = 1.6\%$, $a_f = 4.5\%$ and $b_f = 0.9\%$.

349 Simulations indicate that f_{γ} decreases as a function of photon energy due to

leakage and this is shown in Fig. 12a, with the loss being linear above 200 MeV. 350 It should be noted that the size of the module will primarily affect only f_{γ} in 351 the sense that a smaller module will result in a smaller f_{γ} due to energy from 352 the electromagnetic shower leaking outside the module. On the other hand, f353 depends only on the energy deposited in the matrix itself and is independent of 354 the incident photon energy or overall geometry of the module. The deviation 355 from linearity at low energy is probably due to the fact that more of the low 356 energy electrons and positrons from the electromagnetic shower stop in the 357 lead before being sampled by the scintillating fibres. One would expect this 358 reduction in both sampling fractions and this is what was observed. 359

The sampling fluctuations, σ_f/f , can be seen in Fig. 12b. These are the dominant contributor to the energy resolution, at about $4.5\%/\sqrt{E(\text{GeV})}$. Subtracting the simulated sampling fluctuation contributions from the measured energy resolution yields photoelectron statistics contribution to the energy resolution of about $3.1\%/\sqrt{E(\text{GeV})}$. This is similar to the estimated value of $\sim 2.7\%/\sqrt{E(\text{GeV})}$ from a KLOE beam test [18].

³⁶⁶ 5 Timing and Position Resolution

The time difference of the BCAL will provide position information for neutral particles, which is needed to reconstruct their four-momentum. The position resolution is related to the time difference resolution by the effective speed of light within the calorimeter. Thus, by using measurements of the effective speed of light ($c_{\text{eff}} = (16.2 \pm 0.4)$ cm/ns in Table 1) from a previous beam test at TRIUMF [10], the position resolution of the calorimeter can be easily extracted.

³⁷⁴ The time difference resolution will be of the form:

$$\sigma_{\Delta T/2} = \frac{c}{\sqrt{E(\text{GeV})}} \oplus d.$$
 (6)

In general, the constant term, d, in Eq. 6 is a result of residual calorimeter miscalibrations, but some fraction is also due to the finite width in z of the beam, which will contribute to the time difference resolution. With the beam width being $l \sim 1.9$ cm, the flat and square distribution of the beam contributes $(l/c_{\text{eff}})/\sqrt{12} = 30$ ps to the resolution.

The double-ended readout of the BCAL allowed for time difference measurements to be made, but because leading edge discriminators were used the timing had a dependence on pulse height which required a time-walk correc-



Fig. 13. ADC vs. TDC for segment South 8. The uncorrected time affected by the time walk due to the dependence on amplitude is seen in the top plot. The bottom plot shows the corrected time. The BCAL time was referenced with the tagger time. (colour online)

tion. A plot of ADC versus TDC for segment 8 can be seen in Fig. 13. Fits 384 with a function of the form $p_0/\sqrt{\text{ADC}} + p_1$ were performed, as the time delay 385 due to signal amplitude in leading edge discriminators follows this form. The 386 fit parameter p_1 is a constant term indicating the timing offset of the partic-387 ular readout segment from the tagger MOR timing signal. Parameter p_0 also 388 varies depending on the particular readout segment but has a nominal value 389 of ~ 35 ns \cdot GeV^{1/2}. The fit is poor for the downstream segments, specifically 390 segments 6 and 18 where the statistics are low, as there is very little energy 391 deposited there and the fluctuations are consequently large. For this reason, 392 most of the outer segments were not included in the timing analysis. Analysis 393 of the timing data focused mainly on segments 7, 8, 9 and 10 where nearly 394 90% of the energy was deposited. ADC values lower than channel 350 were 395 rejected, in the case of South 8, due to the resulting asymmetry from the walk 396 correction at low ADC values, which caused distortions in the time difference 397 resolution. This corresponds to 1 MeV of energy deposited in the segment or 398 0.125 MeV deposited in the fibres. Similar ADC cuts were made for the other 399 segments depending on the distortion at the lower end of the ADC spectra. 400 This results in a loss of efficiency at the lower energies but in a much improved 401 time difference resolution over the whole tagger spectrum. 402

⁴⁰³ The timing for an event was found by summing the TDC values of all the

segments in an event cluster, weighted by their energy; cuts on the ADC and timing determined whether a segment was included in the cluster. A cluster is defined by the energy weighted sum of the times of each segment such that the time difference, ΔT , is expressed as:

408



Fig. 14. The walk-corrected spectrum and Gaussian fit for timing counter 40. The solid line is a Gaussian fit to the data.

Subsequently, the walk-corrected spectra for each tagger timing counter were fitted by a Gaussian function. A typical spectrum and fit are shown in Fig. 14, this one for timing counter 40. All timing counter spectra were fitted in the same fashion, and the fit results are plotted on Fig. 15. From the subsequent fit in that figure, the time difference resolution including only the middle row segments 7, 8, 9 and 10 is found to be:

415
$$\sigma_{\Delta T/2,7-10} = \frac{75 \text{ ps}}{\sqrt{E(\text{GeV})}} \oplus 30 \text{ ps.}$$
 (8)

The floor term is equal to the finite width of the beam, as expected. This implies that the intrinsic time resolution of the BCAL is consistent with zero for the constant term. As the time difference resolution is dependent on the number of photoelectrons, the time difference resolution, $\sigma_{\Delta T/2,7-10}$, can be corrected to include the missing photoelectrons, after subtracting the beam width from the constant term, and is found to be

$$\sigma_{\Delta T/2} = \frac{70 \text{ ps}}{\sqrt{E(\text{GeV})}}.$$
(9)

⁴²³ The KLOE beam test result of 72 ps/ $\sqrt{E(\text{GeV})}$ [18] represents the timing



Fig. 15. The time difference resolution, in nanoseconds, for segments 7, 8, 9 and 10 as a function of energy. The fit gives $\sigma_{\Delta T/2} = 75 \text{ ps}/\sqrt{E(\text{GeV})} \oplus 30 \text{ ps}$. The fit of Fig. 14 corresponds to the 40^{th} datum from the right (19th from the left) in this figure.

resolution extracted from the signal average of both ends of each segment. 424 With better fibres and PMT's KLOE estimated they could achieve a resolution 425 of ~ $58 \text{ps}/\sqrt{E(\text{GeV})}$ and this was achieved[3]. The result shown here from 426 the BCAL beam test was extracted from the time difference of the signals. 427 It should be noted that old/degraded PMTs were used in this beam test, 428 especially the 18 Burle 8575's used in the three rear layers of the Module, 429 which had a timing resolution per pair averaging around 1.4 ns in contrast 430 to the forward XP2020's that averaged around 0.6 ns per pair. As such, it is 431 expected that the time difference resolution from Module 1 is actually better 432 than reported here and better fibres, light guides and light sensors will result 433 in an improved timing resolution. 434

Finally, the time difference resolution defines the position (z) resolution along the length of the module, since $\sigma_z = \sigma_{\Delta T/2} \cdot c_{\text{eff}}$. Therefore, the determined position resolution is calculated to be $\sigma_z = 1.1 \text{ cm}/\sqrt{E(\text{GeV})}$ for a 1 GeV photon. KLOE reported a similar position resolution from their beam test of $\sigma_z = 1.2 \text{ cm}/\sqrt{E(\text{GeV})}$ [18].

⁴⁴⁰ 6 Determination of the number of photoelectrons

The number of photoelectrons per end of the prototype BCAL module, N_{pe} , was estimated at z = 0 cm and $\theta = 90^{\circ}$. The distribution in the ratio, R, of the North to the South readout sums, for each of ten bins in beam energy, E_i , $_{444}$ from 150 MeV to 650 MeV, was expressed as

445
$$R(E_j) = \frac{\sum_{i=1}^{18} E_{\mathrm{N},i;j}}{\sum_{i=1}^{18} E_{\mathrm{S},i;j}}$$
(10)

where $E_{N,i}$ and $E_{S,i}$ are the calibrated energies corresponding to the i^{th} segment on the North and South side, respectively. Using this ratio results in the suppression of shower fluctuations that dominate the statistical variance of the individual sums for each readout end. Under the assumption that each of the amplitude spectra has a Poisson-type shape, the ratio spectra were fitted to the function:

$$_{452} \qquad f(r) \sim \int P(x, N_{pe} \cdot \sqrt{R}) \cdot \frac{1}{r} P\left(\frac{x}{r}, \frac{N_{pe}}{\sqrt{R}}\right) \left[\frac{x}{r} dx\right] \tag{11}$$

where r is a North/South amplitude ratio, R is an average North/South amplitude ratio, N_{pe} is the average number of photoelectrons, and P is a Poissontype probability:

456
$$P(x,N) = \frac{e^{-N}N^x}{\Gamma(x+1)}.$$
 (12)

The (1/r) and (x/r) factors are needed to perform the integration over the uniform r-bins. The χ^2/ndf was nearly one for all the fits. The resulting photoelectron yield per GeV per end is plotted in Fig. 16 as a function of beam energy.



Fig. 16. The number of photoelectrons per GeV per end of the BCAL module is shown as a function of energy. A one parameter fit is plotted (dashed line). For more details see the text.

The one parameter fit in Fig.16 yields a mean value of \sim 660 photoelectrons per GeV for photons over the energy range of the beam test. A non-linearity of

 $\sim 5\%$ is apparent but is not worriesome due to the prelimanary nature of the 463 beam test. A similar effect can be seen in results from KLOE beam tests for 464 photons and positrons [17]. The non-lenearity in the number of photoelectrons 465 observed may be due to the non-linearity of the detector when sampling the 466 soft photons of an electromagnetic shower, variations in the light guides and 467 their couplings and shower leakage. Nevertheless, this is an adequate estima-468 tion of the number of photoelectrons from this work and future beam tests 469 over a wider range of energies with a more sophisticated readout system simi-470 lar to the final experiment will solidify this value and more thoroughly reveal 471 any non-linearities in the detector response. 472

In comparison, KLOE reported $N_{pe} \sim 700$ per end at 1 GeV. The BCAL 473 module used double-clad scintillating fibres, potentially giving rise to approx-474 imately 50% more photoelectrons than KLOE. However, the KLOE calorime-475 ter had light guides combined with Winston Cone collectors that resulted in a 476 much higher transport efficiency, typically $\sim 90\%$, than the light guides used in 477 the beam tests described in this work, estimated to have a transport efficiency 478 of $\sim 50\%$. This feature could easily compensate for the increased capture ratio 479 of the fibres in the BCAL but lower number of measured photoelectrons. 480

481 7 Summary and Conclusions

The first full-scale prototype module for the BCAL tested the construction 482 techniques and the performance of the matrix under beam conditions. An 483 energy resolution of $\sigma_E/E = 5.4\%/\sqrt{E({\rm GeV})} \oplus 2.3\%$ and a time difference 484 resolution of $\sigma_{\Delta T/2} = 70 \text{ ps}/\sqrt{E(\text{GeV})}$ ps were found from the Jefferson Lab 485 beam test data. The number of photoelectrons per GeV is about 660. The en-486 ergy and timing resolutions meet the original design goals and the performance 487 of the module closely matches that of KLOE, a proven sampling calorimeter. 488 The analysis for the more demanding regions of module and beam geometries, 489 near the end of the module and at small incident angles can now proceed 490 having established the performance under more benign conditions and having 491 the Monte Carlo simulations tested and anchored to the data. 492

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Property	Value	Ref.
Number of modules ^{a}	48	
Module length ^{a}	$390 \mathrm{~cm}$	
Module inner cord^a	$8.51 \mathrm{~cm}$	
Module outer cord^a	$11.77~\mathrm{cm}$	
Module thickness ^{a}	$22.5~\mathrm{cm}$	
Module azimuthal bite ^{a}	7.5°	
Radial fibre pitch ^{b}	1.22 mm	
Azimuthal fibre pitch ^{b}	$1.35 \mathrm{~mm}$	
Lead sheet thickness ^{c}	$0.5 \mathrm{~mm}$	
Fibre diameter ^{c}	$1.0 \mathrm{mm}$	[7]
First cladding thickness c	$0.03 \mathrm{~mm}$	[7]
Second cladding thickness ^{c}	0.01 mm	[7]
Core fibre refractive index c	1.60	[7]
First cladding refractive index c	1.49	[7]
Second cladding refractive index c	1.42	[7]
Trapping efficiency c,d,e	$5.3\% \ (\min) \ 10.6\% \ (\max)$	[7-9]
Attenuation $length^b$	$(307 \pm 12) \text{ cm}$	[10]
Effective speed of light ^b , c_{eff}	$(16.2\pm0.4) \text{ cm/ns}$	[10]
Volume ratios ^{b}	37:49:14 (Pb:SF:Glue)	[11]
Effective mass number ^{e}	179.9	[11]
Effective atomic number e^{e}	71.4	[11]
Effective density ^{e}	4.88 g/cm^3	[11]
Sampling fraction ^{f}	0.125	[12]
Radiation length ^{e}	$7.06~{\rm g/cm^2}$ or $1.45~{\rm cm}$	[11]
Number of radiation $lengths^e$	$15.5X_0$ (total thickness)	[11]
Critical energy ^{e}	11.02 MeV (8.36 MeV)	[13, 14]
Location of shower \max^{e}	$5.0X_0$ ($5.3X_0$) at 1 GeV	[13, 14]
Thickness for 95% containment e	$20.3X_0$ ($20.6X_0$) at 1 GeV	[13, 14]
Molière radius ^{e}	$17.7~{\rm g/cm^2}$ or $3.63~{\rm cm}$	[14]
Energy resolution ^b , σ_E/E	$5.4\%/\sqrt{E}\oplus 2.3\%$	
Time difference res. ^b , $\sigma_{\Delta T/2}$	$70 \text{ ps}/\sqrt{E}$	
z-position resolution ^b , σ_z	$1.1~{\rm cm}/\sqrt{E}$ (weighted)	
Azimuthal angle resolution f	\sim 8.5 mrad	
Polar angle resolution f	$\sim 8 \text{ mrad}$	

 $\mathrm{Tabl}\overline{\mathrm{e}~1}$

BCAL properties. Superscript: a - design parameters of the BCAL specified for the final detector; b - quantities that have been measured; c - specifications from the manufacturer; d - from literature; e - parameter calculated from known quantities; f = parameter estimated from simulations. The number of radiation lengths as well as the resolutions in the table are all at $\theta = 90^{\circ}$ incidence.

509 Figure Captions

Fig. 1. Schematic of the GlueX Detector. The detector has cylindrical symmetry about the beam direction. The detector subsystems and the dashed lines at angles (with respect to the beam direction) 10.8° through 126.4° are referenced in the text. The start counter is not shown for clarity.

Fig. 2. The GlueX BCAL. (a) BCAL schematic; (b) a BCAL module side view; (c) end view of the BCAL showing all 48 modules and (d) an end view of a module showing readout segmentation. Details are given in the text.

Fig. 3. The distribution of photons, their energy and integrated path length through the Pb/SciFi matrix as a function of position along the length of the BCAL for one of the GlueX signature reactions, $\gamma p \rightarrow \eta \pi^0 p \rightarrow 4\gamma p$, is shown. The target position and angular range subtended by the BCAL are also presented.

Fig. 4. The BCAL fibre matrix showing the placement of 1 mm diameter fibres in the azimuthal and radial directions. The dimensions of the azimuthal and radial pitch, the glue box between the lead sheets and the glue ring around the fibres were determined from the prototype module using a measuring microscope. Particle tracks would appear to enter the matrix from the bottom. More details are given in Ref. [11].

Fig. 5. Diagram of the Hall-B downstream alcove with schematic placements of the BCAL module. The drawing is not to scale.

Fig. 6. The segmentation and readout for the BCAL module as viewed from its North end. The lead/scintillating fibre matrix would appear to be rotated by 90° with respect to Fig. 4. The electromagnetic shower that develops in the module approximately forms a cone shape and is illustrated with the shaded triangle in the figure. A very small percentage of the energy is deposited in the outer segments or leaks out the sides.

Fig. 7. (a) The box that encloses the 18 light guides and PMTs with cables attached for the South end of the BCAL module is shown. (b) The module is entirely wrapped in Tedlar[®] on the right and pressed against the light guides using a silicone sheet, as described in the text.

Fig. 8. The logic diagram for the BCAL Hall-B beam test electronics. It
should be noted that segments 1 and 13 did not contribute to the trigger,
and this explains the apparent discrepancy between the 18 outputs of the
discriminator and the sum output (which is just 16).

Fig. 9. $D = (E_{\text{BCAL}} - E_{\text{BEAM}})/E_{\text{BEAM}}$ is shown after gain balancing and calibration. Notice that the deviations from zero are typically less than 0.5%.

Fig. 10. The calibrated spectrum for D is shown for timing counter 40, corresponding to a beam energy of 273 MeV. The solid line is a Gaussian fit to the data.

Fig. 11. Energy resolution vs. E_{BEAM} for photons for $\theta = 90^{\circ}$ and z = 0 cm. The fit gives $\sigma_E/E = 5.4\%/\sqrt{E(\text{GeV})} \oplus 2.3\%$. The fit of Fig. 10 corresponds to the 40^{th} datum from the right (19th from the left) in this figure.

Fig. 12. (a) The photon energy sampling fraction, f_{γ} , and deposited energy sampling fraction, f, are shown from simulation for $\theta = 90^{\circ}$. The data are fit to $a_0/E + a_1E + a_2$ functions, which were chosen among the simplest functions that described the data well. (b) The sampling fluctuations of the module, $\sigma_{f_{\gamma}}/f_{\gamma}$ and σ_f/f are plotted from simulation. The data are fit to $a/\sqrt{E} \oplus b$ functions with $a_{f_{\gamma}} = 4.56\%$, $b_{f_{\gamma}} = 1.55\%$, $a_f = 4.45\%$ and $b_f = 0.93\%$.

Fig. 13. ADC vs. TDC for segment South 8. The uncorrected time affected
by the time walk due to the dependence on amplitude is seen in the top plot.
The bottom plot shows the corrected time. The BCAL time was referenced
with the tagger time. (colour online)

Fig. 14. The walk-corrected spectrum and Gaussian fit for timing counter 40.
The solid line is a Gaussian fit to the data.

Fig. 15. The time difference resolution, in nanoseconds, for segments 7, 8, 9 and 10 as a function of energy. The fit gives $\sigma_{\Delta T/2} = 75 \text{ ps}/\sqrt{E(\text{GeV})} \oplus 30 \text{ ps}$. The fit of Fig. 14 corresponds to the 40th datum from the right (19th from the left) in this figure.

Fig. 16. The number of photoelectrons per GeV per end of the BCAL module
is shown as a function of energy. A one parameter fit is plotted (dashed line).
For more details see the text.

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