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MEMORANDUM

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To : GLUEX FDC group
Subject : FDC cathodes: tension, flatness and other issues

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1 Introduction

This memorandum presents the results of calculations on the following topics:

- Impact of the cathode flatness on the cathode readout resolution;
- The cathode foil tension and ways to measure it;
- The stresses in the foil with a round hole at the center;
- The impact of the hole in the cathode foils on the background rates.

2 Cathode Flatness

The FDC design is summarized elsewhere [1]. The cathode foil is made of Kapton 25 μm thick, with a deposited 5 μm or 2 μm layer of copper. The copper stripes are 4 mm wide and their pitch is 5 mm. The prototypes tested had a 5 μm layer, a 6 mm pitch and a 5 mm strip width. The cathode plane is composed of three pieces, glued together using 6 cm wide Kapton tape of the same (25 mum) thickness.

The flatness of two experimental cathodes has been measured by M. Veilleux. One is a tightly stretched cathode, another a loose one. Only the tight cathode is considered here, since the loose one had apparently too large deviation from flatness, on a few mm level. The tight cathode is flat within ± 0.5 mm, the deviation varying smoothly across the surface, but also demonstrates relatively sharp deviations of about 0.1 mm at the areas around the seams, across the gluing tape. The foil angle in these areas are about 0.005 rad.

Such a local tilt of the cathode plane may lead to a shift of the centroid of the signal across the stripes. The cathode signal is provided by the image of a charge located very close to the anode wire, and the order of magnitude of the shift is simple to evaluate: $\delta(x) \approx \alpha \cdot L$, where α is the cathode surface angle and $L = 5$ mm is the gap between the wire and cathode planes. An accurate calculations should take into account the string of charges reflected from both cathode planes. A numerical calculation was done using a 2-dimensional program POISSON [2]. The geometry and the field contour lines are shown in Fig. 1.

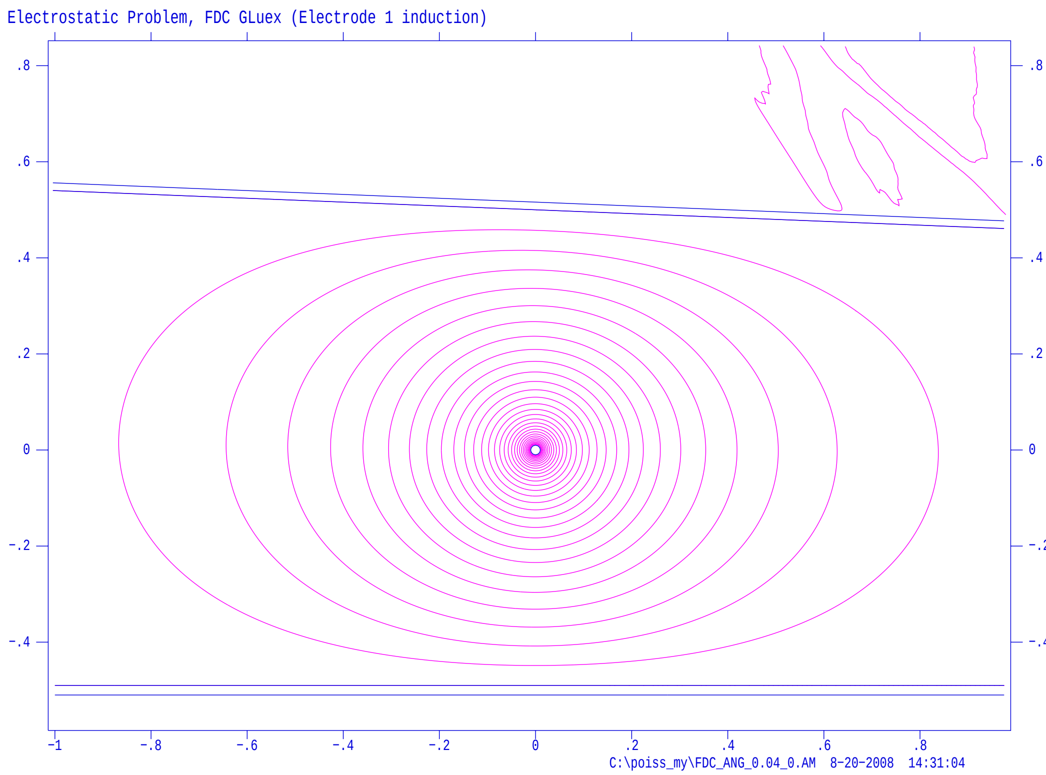

Figure 1: "POISSON" simulation of the charge image on the tilted cathode plane.

The charge profiles on the cathode planes for $\alpha = 0$ and $\alpha = 40$ mrad are shown in Fig. 2. The deviation is about $\delta(x) \sim 0.1$ mm. A 20 mrad tilt would provide a $\delta(x) \sim 0.05$ mm shift. This would make a tolerable impact on the cathode signal resolution, which is expected to be about 0.1 mm.

In summary, local tilts of the cathode plane should be limited to about 20 mrad, in order to keep the associated errors on the signal centroid coordinate below 0.05 mm.

3 Cathode Foil Tension

The foil properties are summarized in Appendix A.1. Most of the foil stiffness is provided by the copper layer, and the stiffness along the stripes is larger than one perpendicular to them.

The foil tension T should be optimized in order to provide the sufficient flatness, without an unnecessary stress on the frame. The production procedure must include relatively quick tension measurements. Several ways have been considered:

1. Measure the deflection caused by placing a disk or a ring of the weight W at the

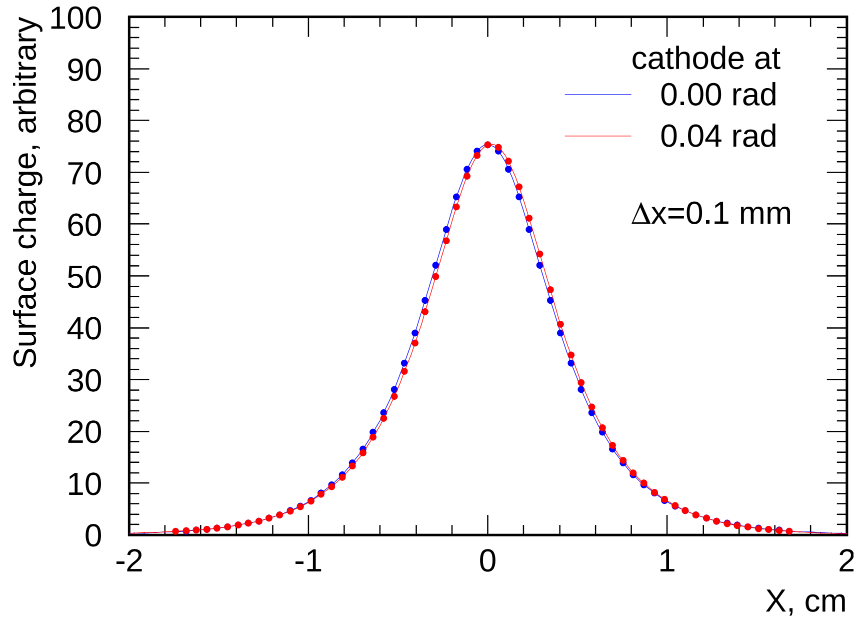

Figure 2: The charge profiles on planes tilted by 0 mrad and by 40 mrad. The shift of the mean values is about 0.1 mm.

center of the foil (see Appendix A.2.1). The deflection depends on the radius r:

$$\delta(r) = \frac{W}{2\pi T} \cdot \log \frac{r}{R}.$$

The weight should be selected to keep the deflection small enough, for example for $T = 600$ N/m, the maximum deflection should be below 1.7 mm.

2. Measure the deflection caused by an overpressure under the foil (see Appendix A.2.2). The deflection depends on the radius:

$$\delta(r) = \frac{P}{4T}(r^2 - R^2).$$

For $T = 600$ N/m, the maximum deflection should be below 2.7 mm.

3. Measure the frequency of foil vibrations (see Appendix A.2.3). The fundamental frequency is:

$$f_\circ = 2.40 \cdot \frac{1}{2\pi R}\sqrt{\frac{T}{\sigma}}.$$

For $R = 0.5$ m, $T = 600$ N/m:

- 5 μm copper : $f_\circ = 70$ Hz
- 2 μm copper : $f_\circ = 84$ Hz

It might be not easy to excite mechanically the fundamental vibration, since local perturbations of the foil couple better to higher modes. The fundamental frequency is not aliquot of the higher frequencies. It might be more promising to subject the foil to a sound of various frequencies and look for the lowest resonance.

Using the method 1. M. Veilleux and I measured the tension of the tightly stretched cathode prototype. We used a disk $W = 18.6$ N (1.9 kg) and measured the deflection with the same laser device which is used for the flatness measurements. The results are:

r, mm	δ, mm	T, N/m
221±5	3.73±0.05	650±20
347±5	1.53±0.05	720±40

The average tension is about 660 N/m.

The foil strain (elongation) perpendicular to the stripes was measured to be 0.25%. Using the foil stiffness of $2.82 \cdot 10^5$ N/m (see Table 2) we expect a tension of 705 N/m, which is close to the measured value.

4 Electrostatic Simulation

Electrostatic calculations for the FDC had been done by D. Carman and S. Taylor analytically [3] and with the help of a program GARFIELD [4]. The former approach was used to calculate the electrostatic attraction between the anode and cathode planes, in order to estimate the cathode foil deflection. The usual simplifications have been done, namely it was assumed that the field close to the cathode plane is nearly uniform. The GARFIELD program calculates the field without such simplifications, however it does not allow to simulate the stripe structure of the cathode[1].

Therefore I tried to calculate the field numerically, using a 2-dimensional program POISSON [2] (see Fig. 3).

The calculated field on the cathode plane is far from uniform (see Fig. 4).

The electrostatic pressure is $0.5\langle E\rangle^2\varepsilon_\circ = 0.030$ Pa. From Eq. 7 we derive the maximum foil deflection for $T = 600$ N/m as $\delta = 3$ μm. This deflection is negligible. The result is not much different from the analytical calculations [3] $P = 0.022$ Pa.

[1]S. Taylor, private communication

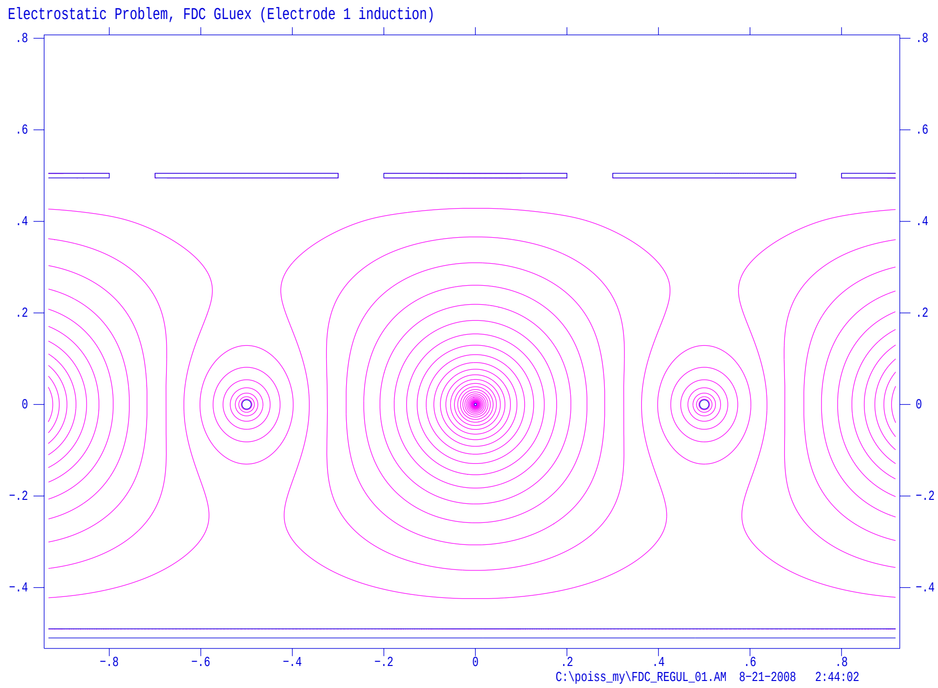

Figure 3: "POISSON" simulation of the FDC field with the regular potentials: V_s = 2420 V, $V_f = -500$ V, $V_c = 0$ V. The anode and field wires were 2.5 thicker than the real ones, in order to reduce the number of the grid elements.

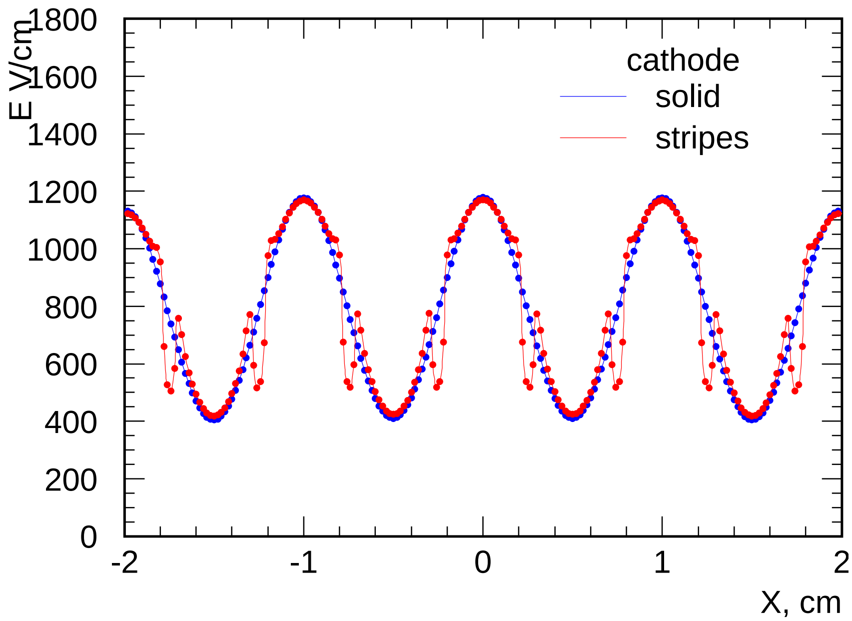

Figure 4: "POISSON" calculation of the electrical field on the cathode plane, for a solid cathode and for the striped structure.

5 Cathode Foil with a Hole

In order to reduce the beam and secondary electrons interactions in the chamber, it is being considered to make a round hole at the center of every cathode foil. Additionally, it may improve the gas distribution in the chambers. The hole diameter is a matter of optimization, a reasonable range is 2-10 cm.

The radial and azimuthal tensions in a stretched foil with a central hole are calculated in Appendix A.3:

$$T_r = \frac{T_\circ}{1+\frac{r_\circ^2}{R^2}}(1-\frac{r_\circ^2}{r^2})$$
$$T_t = \frac{T_\circ}{1+\frac{r_\circ^2}{R^2}}(1+\frac{r_\circ^2}{r^2}),$$

where T_r, T_t are the radial and azimuthal tensions, R is the radius of the foil, $r_\circ$ is the radius of the hole and $T_\circ$ is the initial tension, before the hole is made.

At the edge of the hole $T_r = 0$ and $T_t \approx 2T_\circ$. The tension dependence on the radius is shown in Fig. 5.

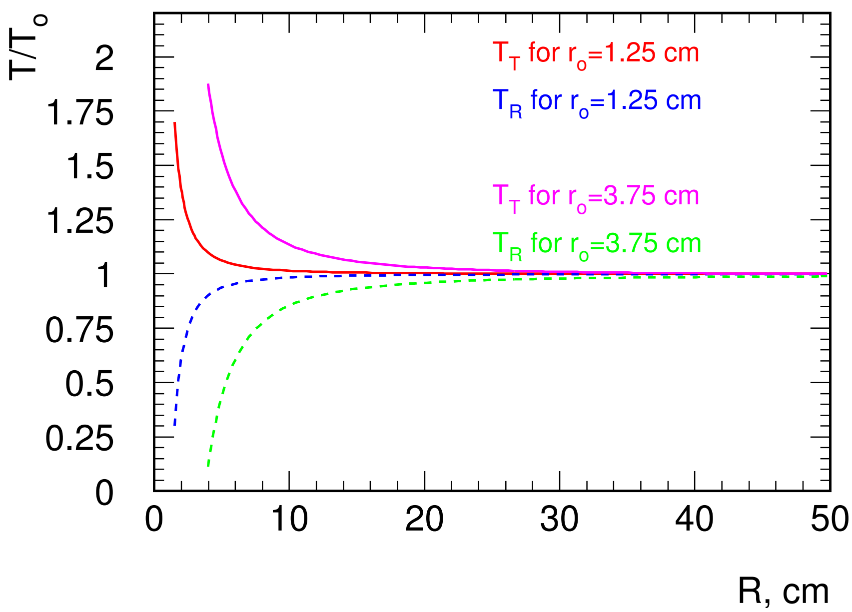

Figure 5: The radial and azimuthal tensions in the foil with a central hole of a radius $r_\circ$, for $r_\circ = 1.25$ and $r_\circ = 3.37$ cm.

The rates for several conditions have been simulated with a standalone version of GEANT3. For events with a hit in the FCAL, the beam particle interaction point is shown in Fig. 6. The hole naturally reduces the interactions of the beam in the FDCs. The background rates in the FCAL are summarized in Table. 1.

Copper layer	hole $r_\circ$	rate in the FCAL
5 μm	no	125%
2 μm	no	100%
0 μm	no	90%
2 μm	3 cm	81%
2 μm	5 cm	80%

Table 1: Background rates in the FCAL for different FDC cathode configurations.

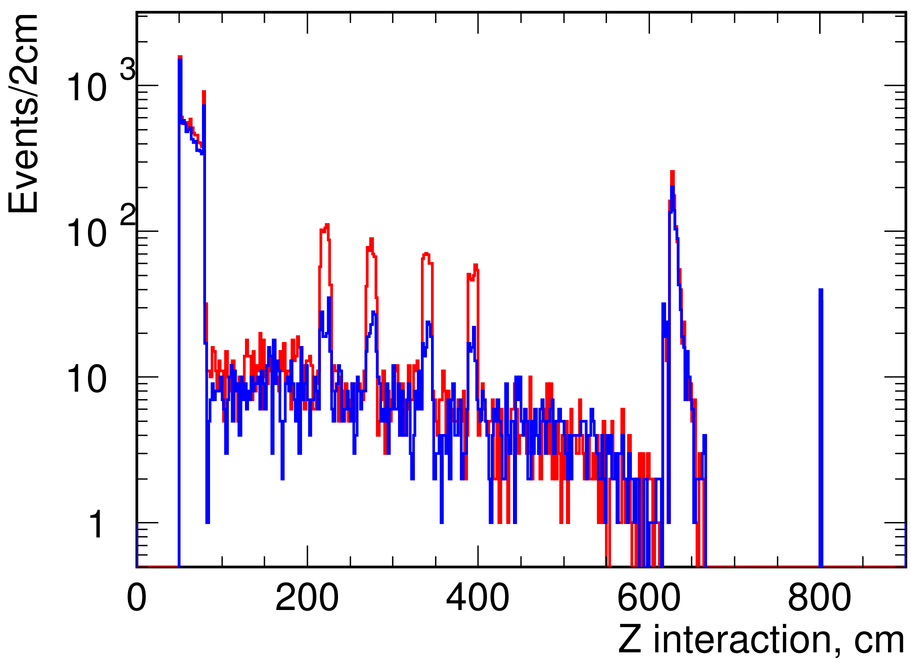

Figure 6: Electromagnetic background simulation. The beam particle interaction point is histogrammed, for events with a hit in FCAL. The red curve: no hole, 2 μm copper plating; blue curve: a hole $r_\circ = 3$ cm in all the cathodes.

A Auxiliary Calculations

A.1 Foil Properties

The properties of thin copper films on a Kapton foil have been studied [5]. It was shown that the yield strength of copper films of 2-5 μm are about 400-300 MPa, which is about 4 times higher than the yield strength of the bulk copper of 70 MPa. The mechanical properties of the cathode foil are summarized in Table. 2. The foil stiffness is $S = E \cdot t$, where E is the Young's modulus of the material, and t is the foil thickness. The stripe width is 4 mm, while the gap is 1 mm. The notations are:

- $w_1 = 0.2$ - the relative width of the gap
- $w_2 = 0.8$ - the relative width of the stripe
- $t_1 = 25$ μm - the Kapton thickness
- $t_2 = 5/2$ μm - the copper thickness
- E_1/E_2 - the Young's modulus of Kapton/copper

Along the stripes the foil stiffness is:

$$S = E_1 t_1 + E_2 t_2 \cdot w_2 \tag{1}$$

Perpendicular to the stripes, it can be calculated taking into account that the full relative elongation is $\delta = w_1\delta_1 + w_2\delta_2$, the tension $T = T_1 = T_2 \Rightarrow E_1 t_1 \delta_1 = (E_1 t_1 + E_2 t_2)\delta_2$:

$$S = \frac{T}{\delta} = \frac{E_1 t_1 + E_2 t_2}{1 + w_1 \frac{E_2 t_2}{E_1 t_1}}. \tag{2}$$

material	density	Young's modulus	Foil stiffness, N/m: || to stripes	Foil stiffness, N/m: $\perp$ to stripes	Yield strength
Kapton	1.42 g/cm^3	3.5 GPa			0.23 GPa
Copper	9.0 g/cm^3	110.0 GPa			0.30 GPa
25 μm Kapton	0.0355 kg/m^2		$0.88 \cdot 10^5$		
25/5 μm K/Cu	0.0715 kg/m^2		$5.28 \cdot 10^5$	$2.82 \cdot 10^5$	
25/2 μm K/Cu	0.0499 kg/m^2		$2.64 \cdot 10^5$	$2.05 \cdot 10^5$	

Table 2: Materials and foil properties, from [5].

A.1.1 The Tension Variations

The calculations of the foil deflections are simple if the tension is isotropic and uniform across the foil. Otherwise the result depends on the foil stiffness. Let us estimate the applicability of this approximation. The foil ring, deflected at an angle $\frac{d\delta}{dr}$ becomes longer and produces an additional tension:

$$\Delta T = S \cdot (\sqrt{1 + (\frac{d\delta}{dr})^2} - 1) \approx S \cdot \frac{1}{2}(\frac{d\delta}{dr})^2, \tag{3}$$

where S is the foil stiffness (see Table 2). This additional tension should be kept considerably below the initial tension of the foil.

A.2 Foil Deflections

A.2.1 Weight on the foil

Let us calculate the deflection of a horizontal round foil of the radius R, at a uniform tension T, if a disk of weight W and radius $r_\circ$ is placed at the foil center. For small deflections we can consider the tension unchanged. The shape of the foil $\delta(r)$ is defined by the requirement that at any radius the vertical force is equal to the weight:

$$T \cdot \frac{d\delta}{dr} \cdot 2\pi r = W. \tag{4}$$

Assuming $\delta(R) = 0$:

$$\delta(r) = \frac{W}{2\pi T} \cdot \log \frac{r}{R}. \tag{5}$$

For $R = 0.5$ m, $T = 600$ N/m, $r = 0.2$ m and $W = 9.8$ N (equivalent to $m = 1$ kg) we obtain: $\delta \approx -2.4$ mm.

Requiring that the additional tension (see Eq. 3) is small, say $\Delta T < T \cdot 0.05$, we obtain: $|\delta(r_\circ)| < \sqrt{T \cdot 0.1/S} \cdot r \cdot \log{(R/r_\circ)}$. For $T = 600$ N/m and $S = 5 \cdot 10^5$ N/m, at $r_\circ = 0.1$ m $|\delta| < 1.7$ mm.

A.2.2 Overpressure on one side

In the same way one calculates the deflection cause by an overpressure P. The left side of Eq. 4 is unchanged, while the right side becomes $P \cdot \pi r^2$, leading to:

$$T \cdot \frac{d\delta}{dr} \cdot 2 = P \cdot r. \tag{6}$$

Assuming $\delta(R) = 0$:

$$\delta(r) = \frac{P}{4T} \cdot (r^2 - R^2). \tag{7}$$

For $R = 0.5$ m, $P = 10$ Pa (equivalent to 10^{-4} bar), $T = 600$ N/m and $r = 0$ m we obtain: $\delta \approx -1.0$ mm.

Requiring that $\Delta T < T \cdot 0.05$ (see Eq. 3), we obtain: $|\delta(0)| < 0.5R\sqrt{T \cdot 0.1/S}$. For $T = 600$ N/m and $S = 5 \cdot 10^5$ N/m, at $r = 0$ $|\delta| < 2.7$ mm.

A.2.3 Foil vibration

The equation of motion is:

$$T \cdot \nabla^2 \delta = \sigma \frac{\partial^2 \delta}{\partial t^2}, \tag{8}$$

where σ is the surface density. In the cylindrical frame the Laplacian is:

$$\nabla^2 \delta = \frac{\partial^2 \delta}{\partial r^2} + \frac{1}{r}\frac{\partial \delta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \delta}{\partial \phi^2} + \frac{\partial^2 \delta}{\partial z^2}. \tag{9}$$

For a thin membrane, looking for azimuthally symmetric modes only:

$$\frac{\partial^2 \delta}{\partial r^2} + \frac{1}{r}\frac{\partial \delta}{\partial r} = \frac{\sigma}{T}\frac{\partial^2 \delta}{\partial t^2}. \tag{10}$$

The harmonic solutions have a form: $\delta = A \cdot \sin(\omega t + \phi_\circ) \cdot f(r)$. The Eq. 10 is reduced to:

$$\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr} + \frac{\sigma}{T}\omega^2 \cdot f = 0 \tag{11}$$

In order to convert it to the canonical Bessel equation we make a substitution: $\eta = r \cdot \omega\sqrt{\frac{\sigma}{T}}$, $f_1(\eta) \equiv f(r)$:

$$\frac{d^2 f_1}{d\eta^2} + \frac{1}{\eta}\frac{df_1}{d\eta} + f_1 = 0 \tag{12}$$

The solution is the Bessel function $f_1(\eta) = J_\circ(\eta)$, which crosses zero $J_\circ(\eta) = 0$ at $\eta_\circ, \eta_1, \eta_2, ... = 2.40, 5.52, 8.65, ...$ The boundary condition $f(R) = 0$ selects a set of frequencies $\omega_k = \eta_k \frac{1}{R}\sqrt{\frac{T}{\sigma}}$. The fundamental frequency is:

$$f_\circ = 2.40 \cdot \frac{1}{2\pi R}\sqrt{\frac{T}{\sigma}} \tag{13}$$

For $R = 0.5$ m, $T = 600$ N/m:

- 5 μm copper : $f_\circ = 70$ Hz
- 2 μm copper : $f_\circ = 84$ Hz

A.3 Mechanical impact of a hole in the foil

Let us consider a foil of a radius R under a uniform tension $T_\circ$, in which a central hole of a radius $r_\circ$ is made. The radial and azimuthal tensions are denoted as T_r and T_t.

The balance of forces applied to a sector $dr \cdot d\varphi$ is: $(T_r + \frac{dT_r}{dr}dr)(r+dr)d\varphi - T_r \cdot r \cdot d\varphi - 2T_t dr \cdot d\varphi = 0$. It follows:

$$T_r + \frac{dT_r}{dr} \cdot r = T_t \tag{14}$$

When a hole appears, the points on the foil move radially: $r \longrightarrow r + \rho(r)$. The foil element $dr \cdot d\varphi$ is contracted in the radial and stretched in the azimuthal directions, which can be described as follows:

$$T_r = T_\circ + \frac{d\rho}{dr}S \tag{15}$$

$$T_t = T_\circ + \frac{\rho}{r}S \tag{16}$$

where S is the foil stiffness. From Eq. 16 and Eq. 14 we get:

$$\rho S = \frac{dT_r}{dr}r^2 + T_r r - T_\circ r. \tag{17}$$

Therefore:

$$\frac{d\rho}{dr}S = \frac{d^2T_r}{dr^2}r^2 + 3 \cdot \frac{dT_r}{dr}r + T_r - T_\circ \tag{18}$$

Substituting Eq. 18 into Eq. 15:

$$\frac{d^2T_r}{dr^2}r = -3\frac{dT_r}{dr} \tag{19}$$

The solution is

$$\frac{dT_r}{dr} = \frac{A}{r^3}, \qquad T_r = -\frac{A}{2r^2} + B.$$

One constant is found using the boundary condition $T_r(r_\circ) = 0$:

$$T_r = \frac{A}{2r_\circ^2}(1 - \frac{r_\circ^2}{r^2}).$$

The second constant is found using the boundary condition $\rho(R) = 0$ (the foil is attached to a solid frame):

$$\rho \cdot S = \frac{A}{2}(\frac{1}{r} + \frac{r}{r_\circ^2}) - T_\circ r.$$

This makes:

$$A = \frac{2T_\circ r_\circ^2}{1 + \frac{r_\circ^2}{R^2}}.$$

The final result is:

$$T_r = \frac{T_\circ}{1 + \frac{r_\circ^2}{R^2}}(1 - \frac{r_\circ^2}{r^2})$$
$$T_t = \frac{T_\circ}{1 + \frac{r_\circ^2}{R^2}}(1 + \frac{r_\circ^2}{r^2}) \quad (20)$$

For $r_\circ \ll R$ the azimuthal tension around the hole is $T_t \approx 2T_\circ$.

References

[1] D. Carman and S. Taylor. Forward Drift Chamber Technical Design Report, 2008. GLUEX doc 754-v10.

[2] POISSON / SUPERFISH Program. URL http://laacg1.lanl.gov/laacg/services/download_sf.phtml. LANL.

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