Analysis: What are the key observables and how do we make measurements?
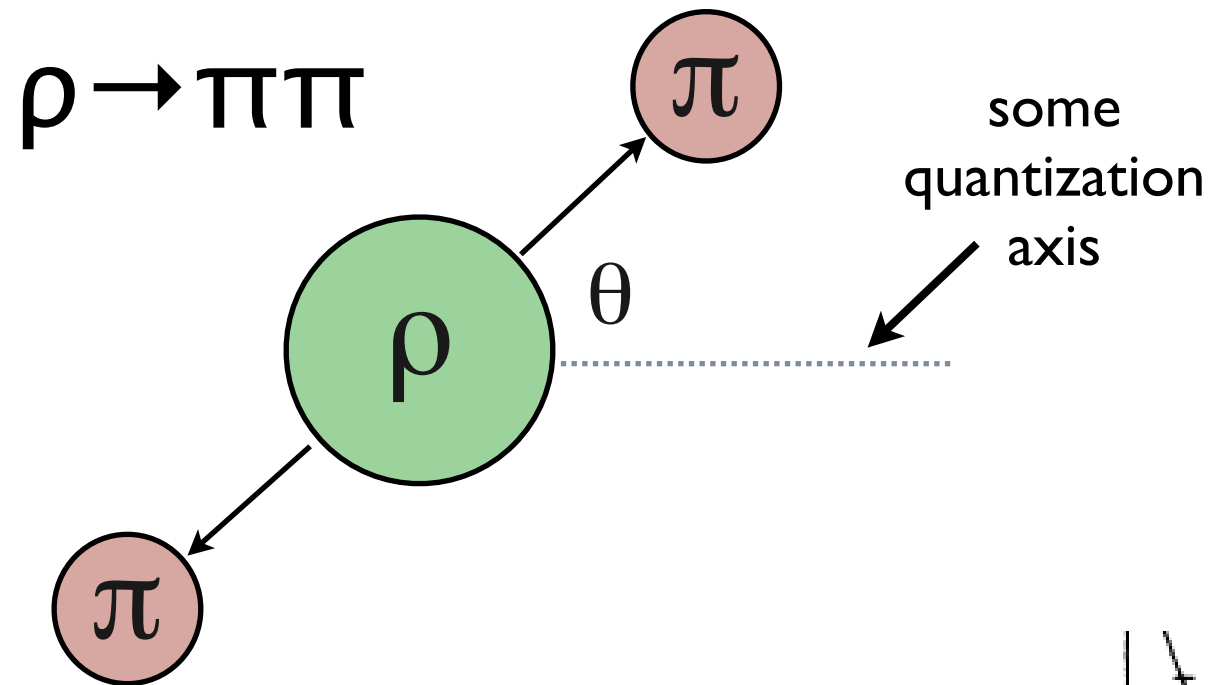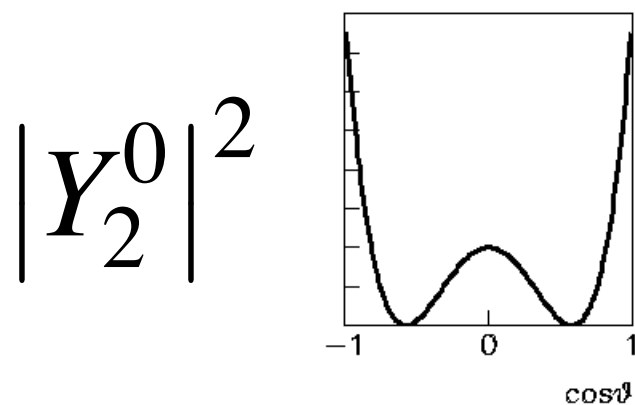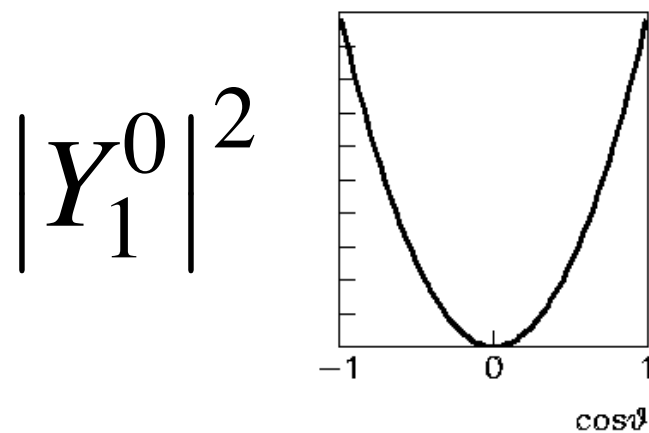
Matt Shepherd
Indiana University

Spectroscopy Tools

- Detector measures four-momenta of particles

- Many discoveries made by plotting invariant mass and looking for peaks ("bump hunting")

  - works best for narrow peaks (~10 MeV)

- GlueX needs to measure

  - mass and width of broad (~200 MeV) resonances

  - quantum numbers of resonances

- Need more than just magnitude of four-momentum

  - angular distributions also relevant!

PRL 91, 262001 (2003)

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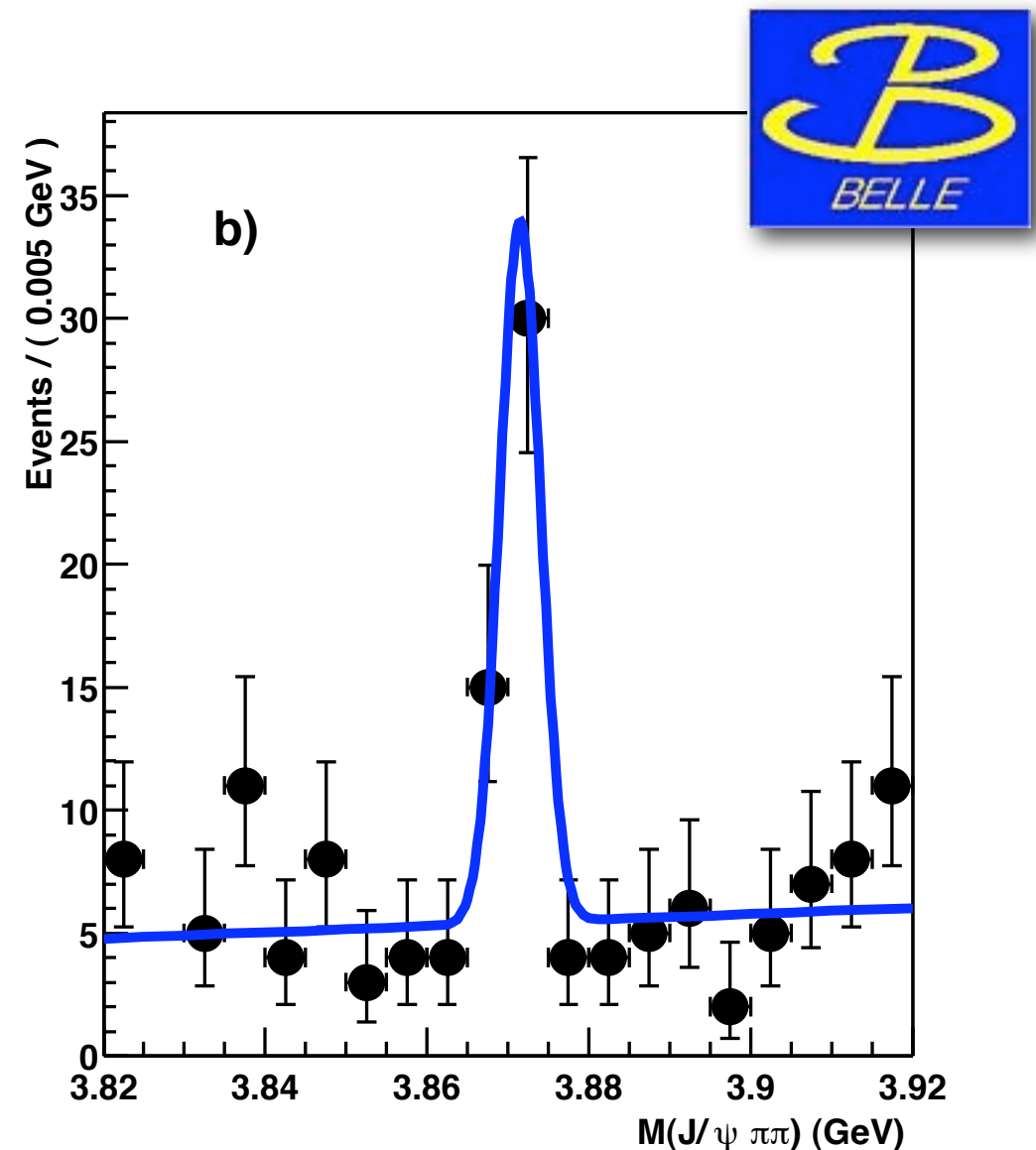
An Example: Measuring Spin
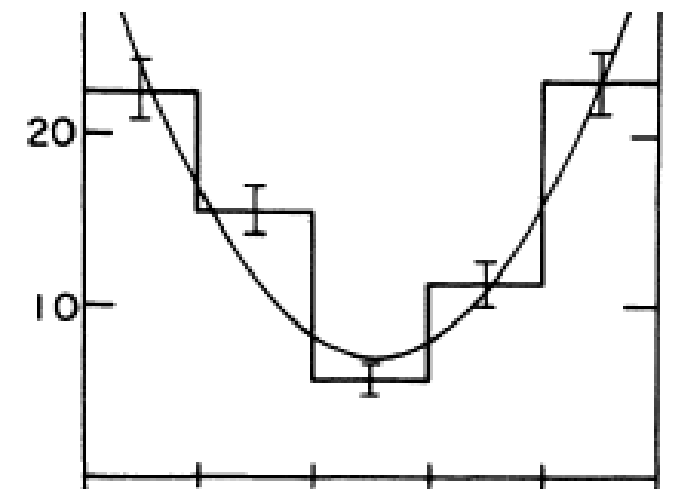
$$\left|Y_0^0\right|^2$$

$$\left|Y_1^0\right|^2$$

$$\left|Y_2^0\right|^2$$

ρ → ππ

some quantization axis

θ

From data conclude J = 1
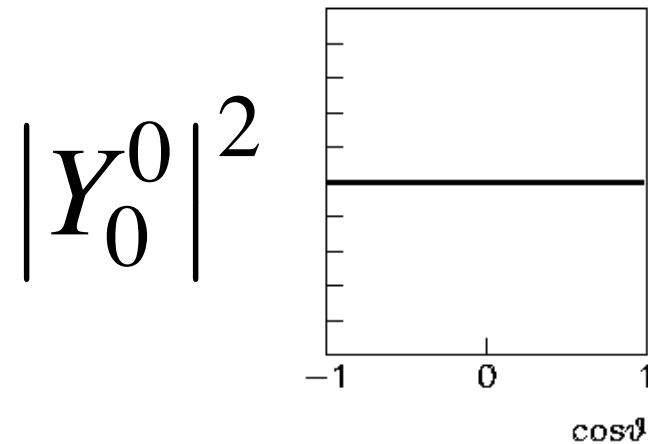
Pions are spinless so spin of ρ is carried in the orbital angular momentum of the two pions.

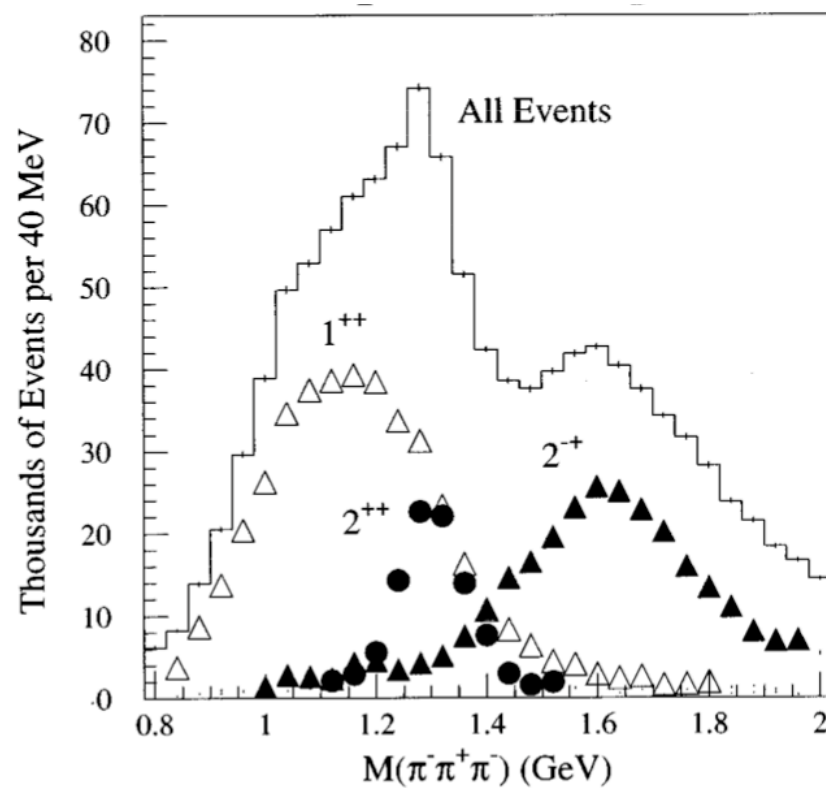VOLUME 8, NUMBER 2   PHYSICAL REVIEW LETTERS   JANUARY 15, 1962

DIFFERENTIAL π-π CROSS SECTIONS: EVIDENCE FOR THE SPIN OF THE ρ MESON*

D. Duane Carmony† and Remy T. Van de Walle‡
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received November 6, 1961; revised manuscript received December 27, 1961)

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A Modern Example

from pion production: π⁻p → π⁻π⁻π⁺p at 18 GeV/c

Partial Wave Analysis Method:

In every bin of 3π mass, fit for the (complex) sizes of different J^{PC} amplitudes using all available event kinematics.
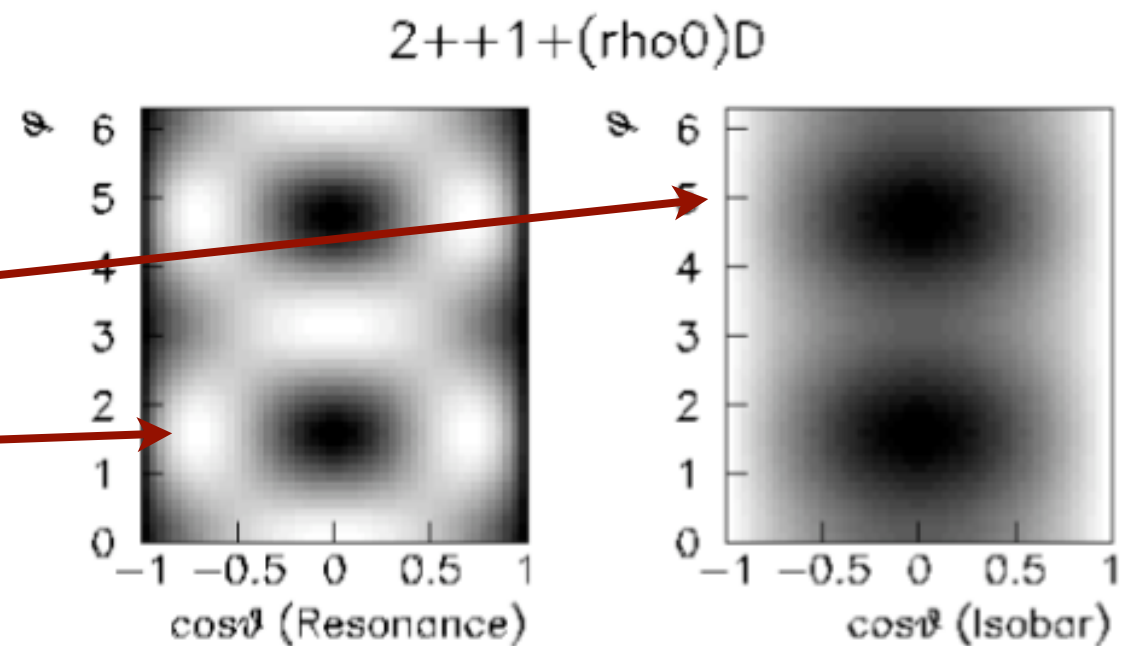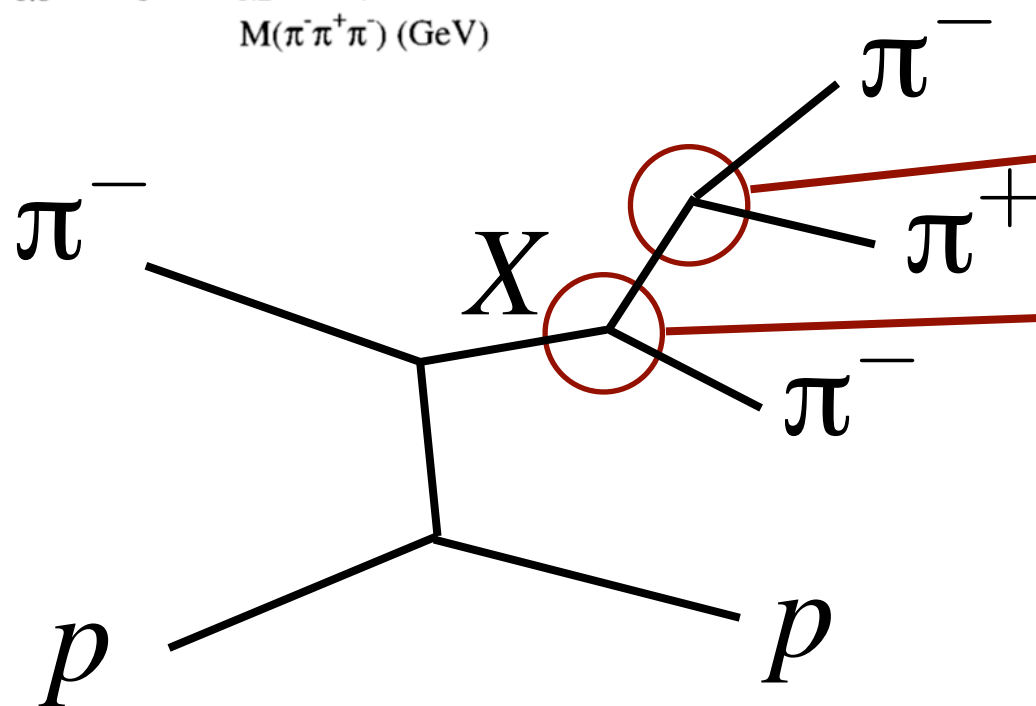
Amplitude Analysis

- historically called partial wave analysis (PWA), but we're not dealing really with "partial waves" anymore

- needs two very different but rather complicated ingredients

  - <u>Experimental/Technical</u>: multidimensional unbinned likelihood fit that correctly deals with detector acceptance

  - <u>Theoretical</u>: a physics model with free parameters that describes the experimental data

- What follows will be a brief introduction to this technique

  - *warning*: some technical details have been glossed over

  - hopefully there is enough detail to be useful

  - ask questions!

Maximum Likelihood

- Amplitude analysis is built around the (extended) maximum likelihood method

- Start with a model that contains free parameters (θ) and predicts the probability of having an event with a particular set of kinematic variables x (angles, invariant mass, etc.)

$$\mathcal{P}(\vec{x}; \vec{\theta})$$

- Vary the free parameters to maximize the probability for the entire data set

$$\mathcal{L} = \prod_{i=1}^{N_{\text{events}}} \mathcal{P}(\vec{x}_i; \vec{\theta})$$

Example: 1D in x

this choice maximizes L

M(J/ψ $\pi\pi$) (GeV)
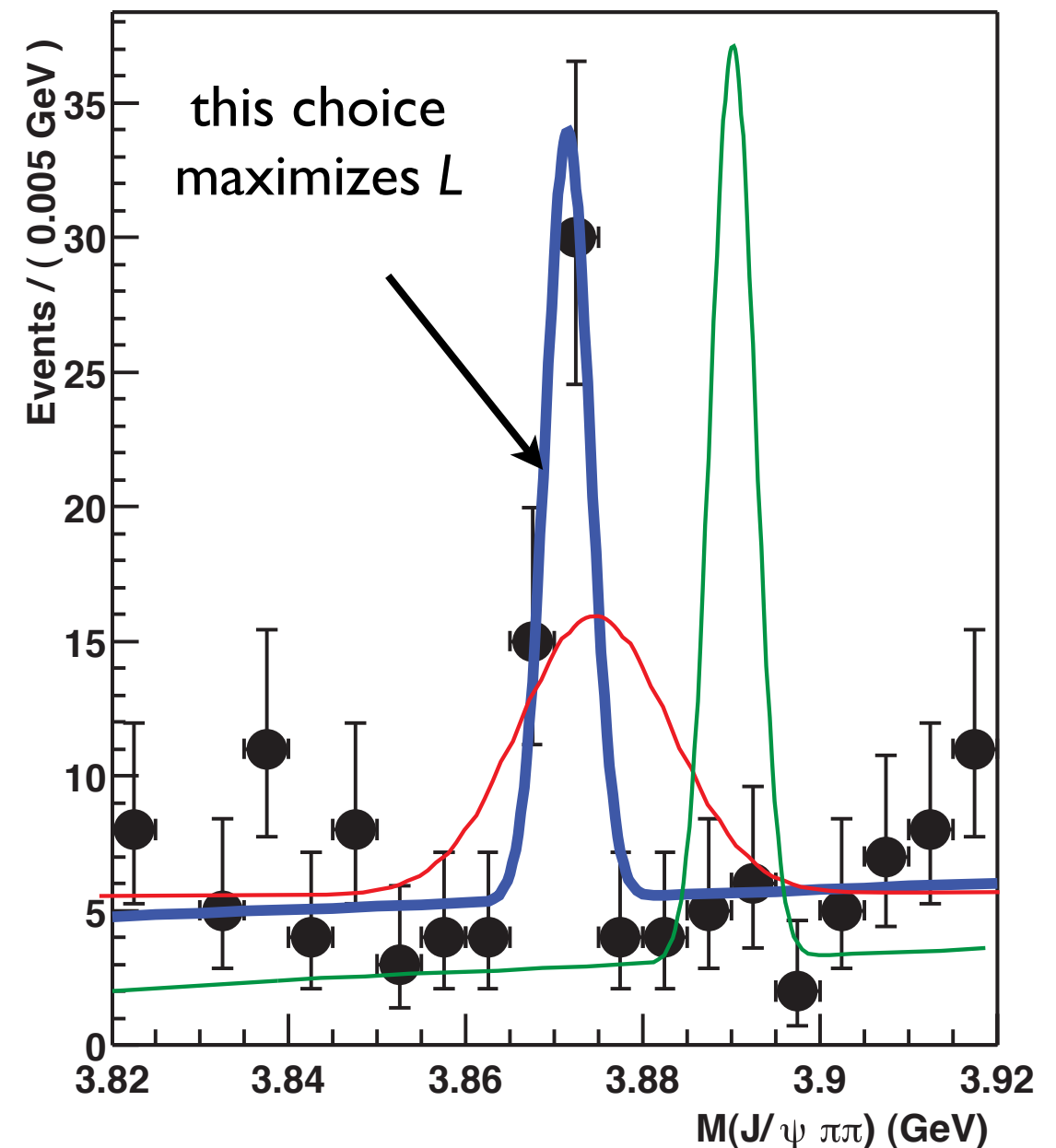
Experiment Application

Step 1: Shoot particles at slits

Probe
Beam of Particles
wavelength λ

Goal: determine the values of *d* and *D*

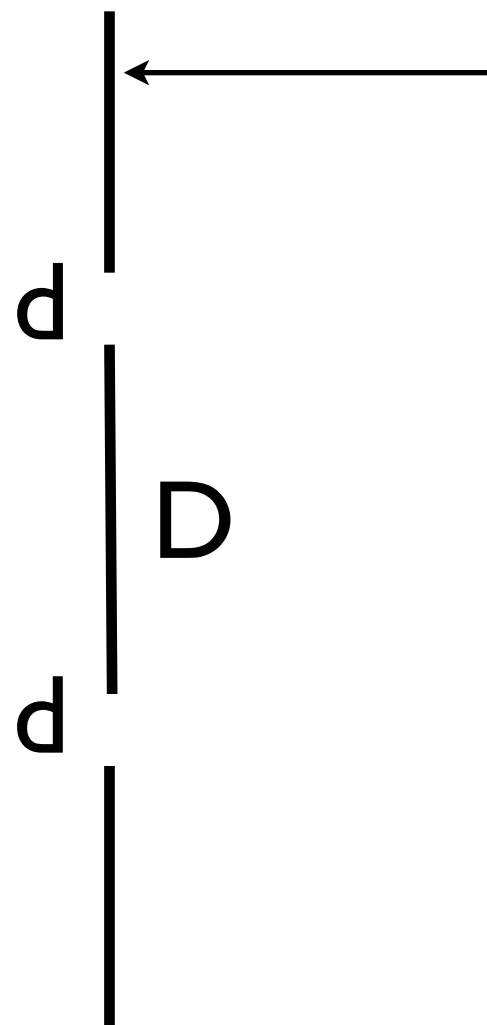Step 2: For each particle record location *x* where it was detected

L

d

D

d

X

Physical System Under Study
Two Slits: width d, separation D
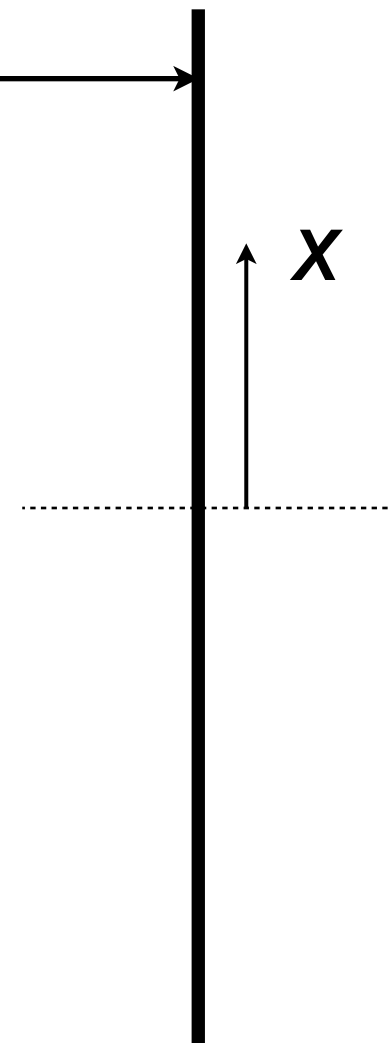
Detector
Measures location x_i
for each arriving particle

The Fit Procedure

- Our "theoretical model" that parametrizes the intensity of the particles in the detector is given by

$$I(x) = I_0 \left(\frac{\sin(d\pi x/\lambda L)}{d\pi x/\lambda L} \right)^2 \cos^2(2D\pi x/\lambda L)$$

- Start with a guess for values for *d* and *D*

- Convert *I(x)* into a properly normalized PDF -- multiple techniques are available for evaluating the integral

$$\mathcal{P}(x) = \frac{I(x)}{\int_{x_{\min}}^{x_{\max}} I(x)dx}$$

- Compute the likelihood by taking the product over all detected events

$$\mathcal{L} = \prod_{i=1}^{N} \mathcal{P}(x_i)$$

- Iterate with a new choice of *d* and *D* until the likelihood is maximized

I(x)

x

one guess for *I(x)*
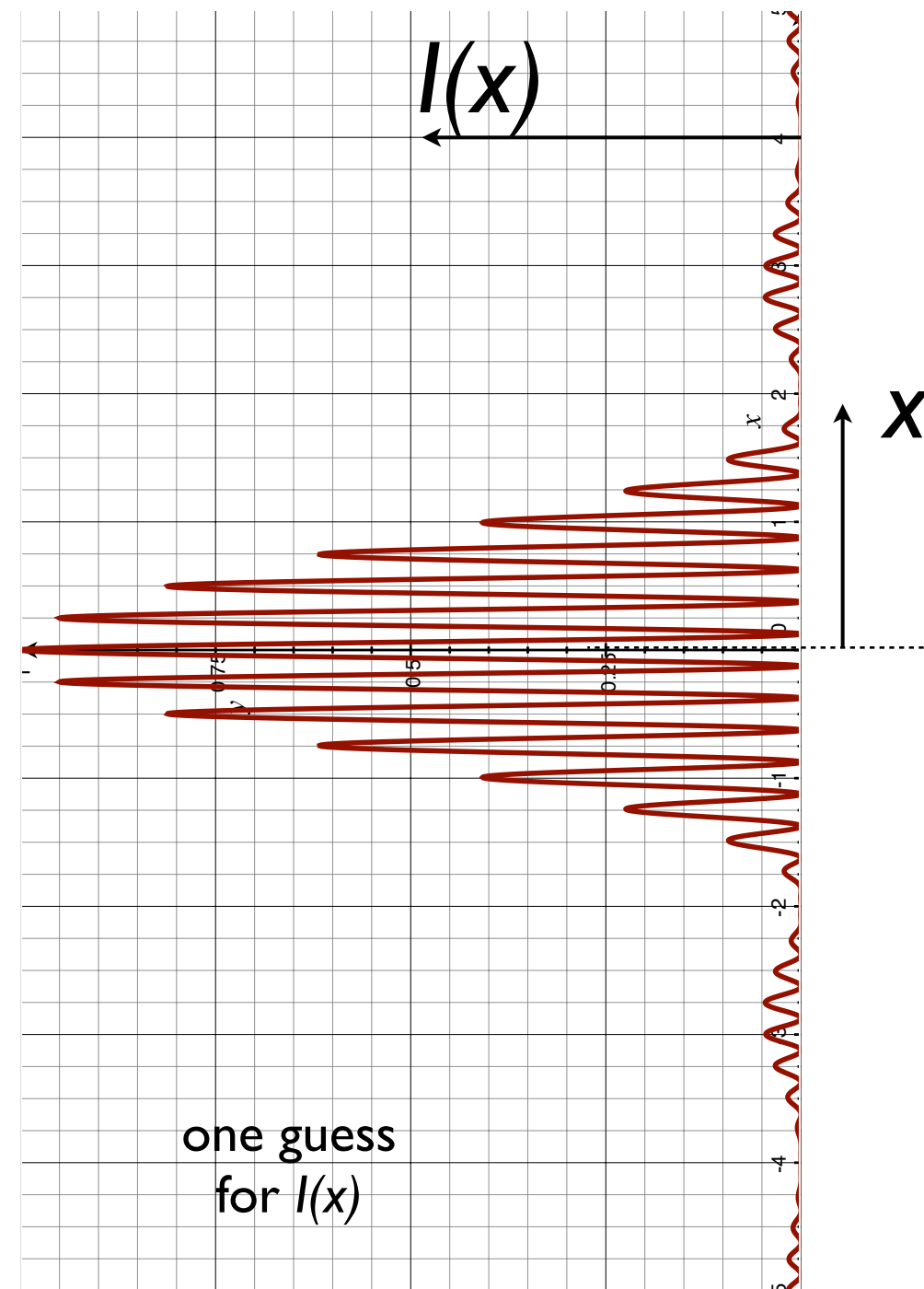
Connecting to GlueX

- Suppose we have $\gamma p \rightarrow \eta \pi^0 p$, we can draw two (of many) possible diagrams
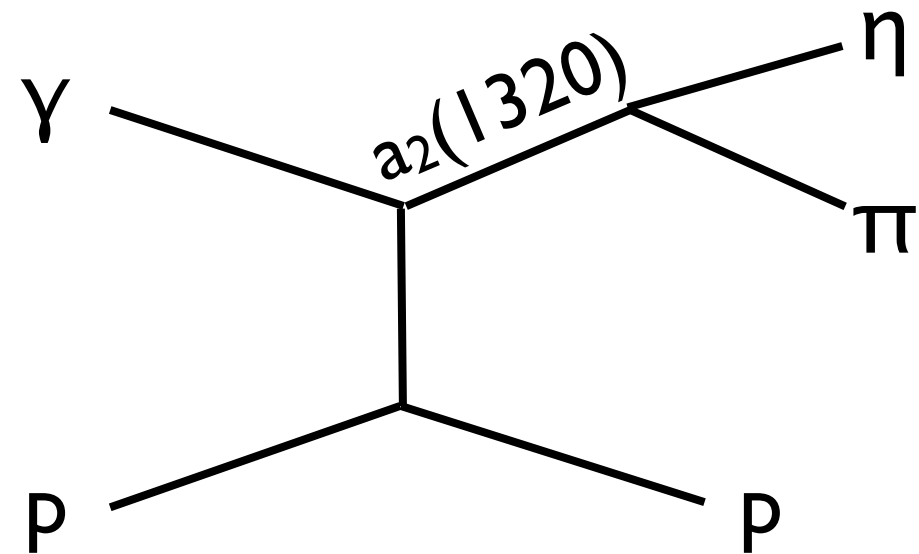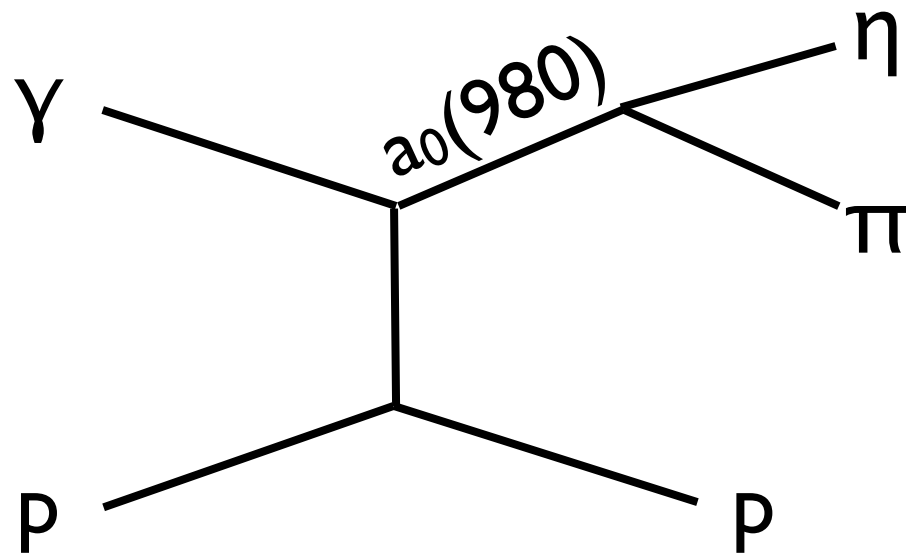
- Each of these can be related to an independent quantum mechanical *amplitude*

- Given any single event we do not know which process occurred -- they are indistinguishable

Amplitude Structure

decay amplitude

$A_i(\vec{x})$
complex *function* of
the final state observables,
$\vec{x}$ is a location in multi-body
phase space

"known"

production amplitude

V_i
unknown complex
fit parameter

magnitude and phase
of amplitude

Produced State
J_i, m_i

Y

η

π

P

P

$$A_i(\theta_{GJ}, \phi_{GJ}) \propto D^{J_i}_{m_i,0}(\theta_{GJ}, \phi_{GJ}, 0)$$

I is intensity:
number of
events per unit
phase space

$$I(\vec{x}) = \frac{dN}{d\vec{x}} = \left| \sum_{\alpha}^{N_{\text{amps}}} V_\alpha A_\alpha(\vec{x}) \right|^2 = \sum_{\alpha,\beta}^{N_{\text{amps}}} V_\alpha V_\beta^* A_\alpha(\vec{x}) A_\beta(\vec{x})^*$$

Incorporating Mass Dependence

- For $\gamma p \to \eta \pi^0 p$ the spin of the $\eta\pi$ appears in just one variable related to the angle between the η (or π^0) and beam

- We can divide data into bins of $M(\eta\pi)$ and then do a fit in each bin of $M(\eta\pi)$ where we include 3 amplitudes corresponding to L=0, 2, and 4 in the $\eta\pi$ system

- We can then plot the fitted (complex) values of V for each of the three amplitudes as a function of $M(\eta\pi)$

- For a single resonance with mass M and width Γ, expect V to trace out the product of the production vertex and the Breit-Wigner propagator of the resonance
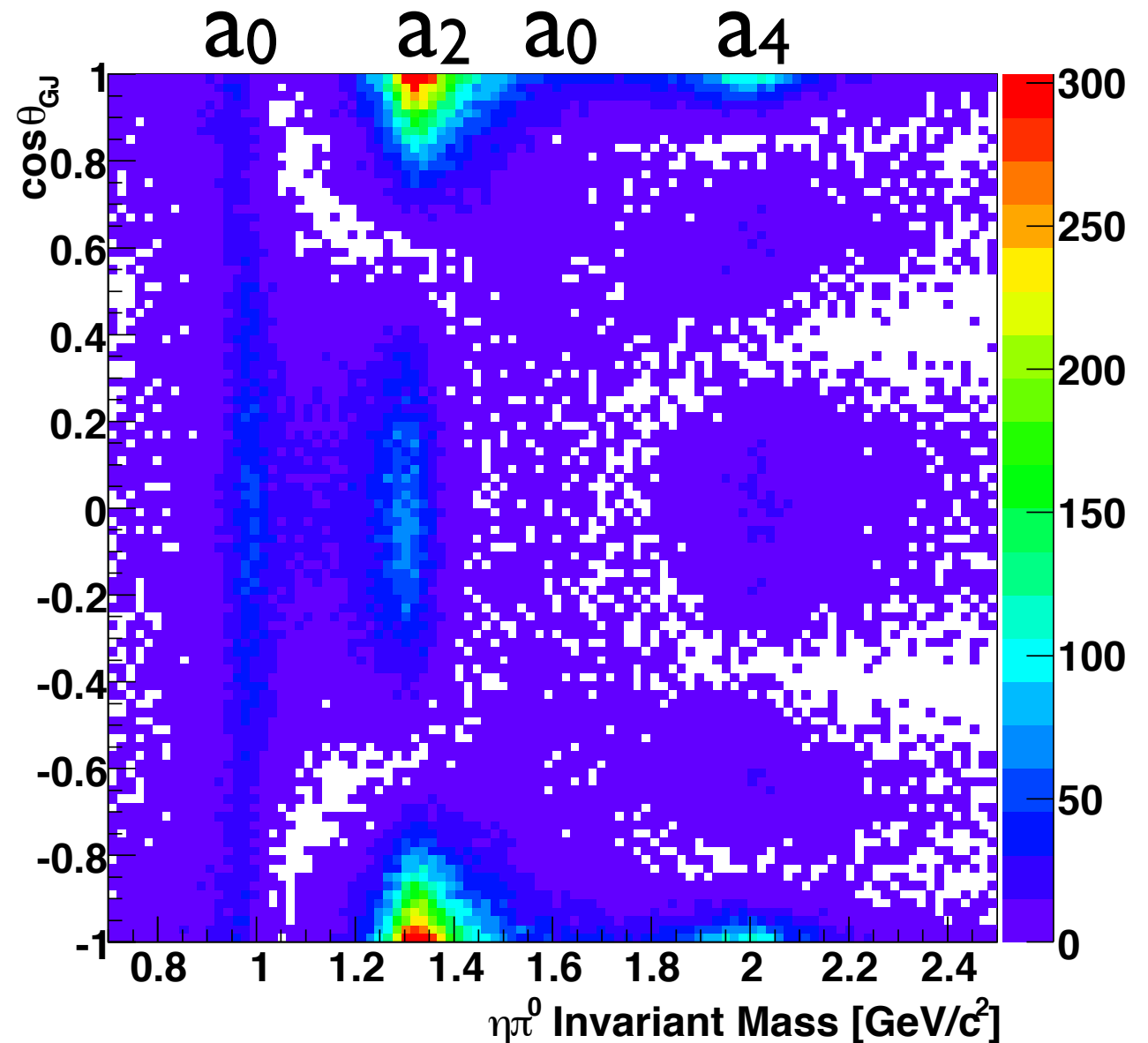
$$V(s) = \frac{V_0}{s - M^2 + iM\Gamma}$$

s: invariant mass squared of $\eta\pi$
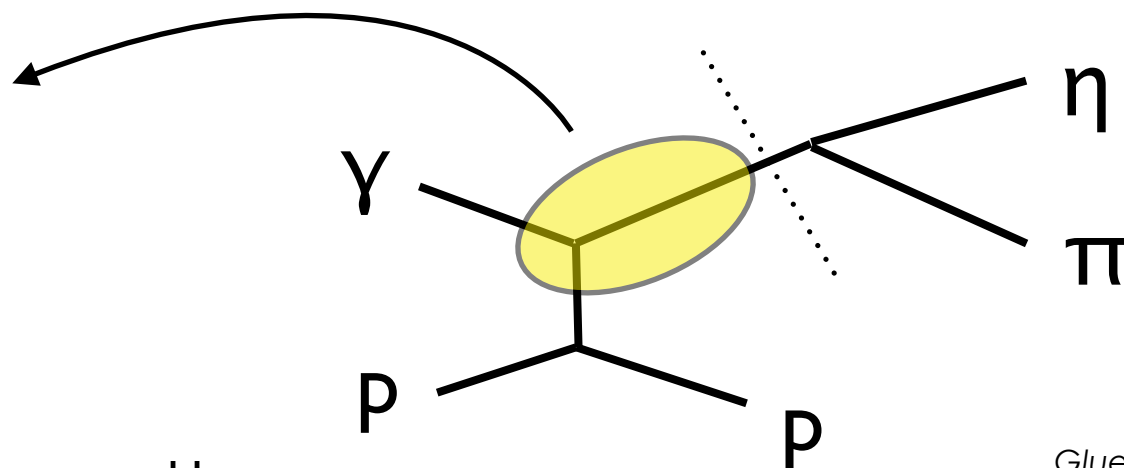
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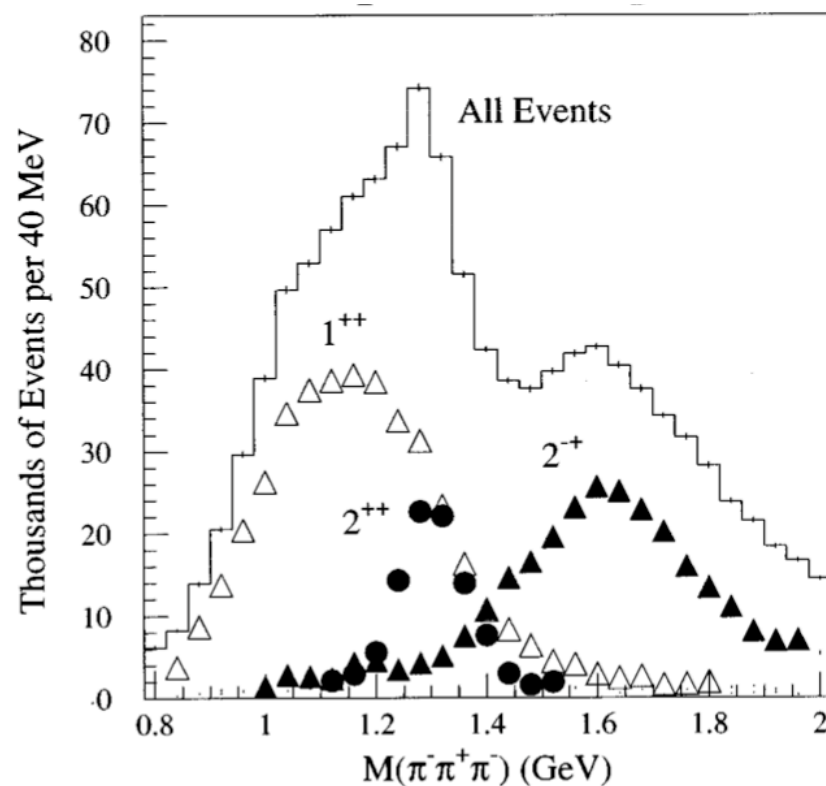
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2++1+(rho0)D
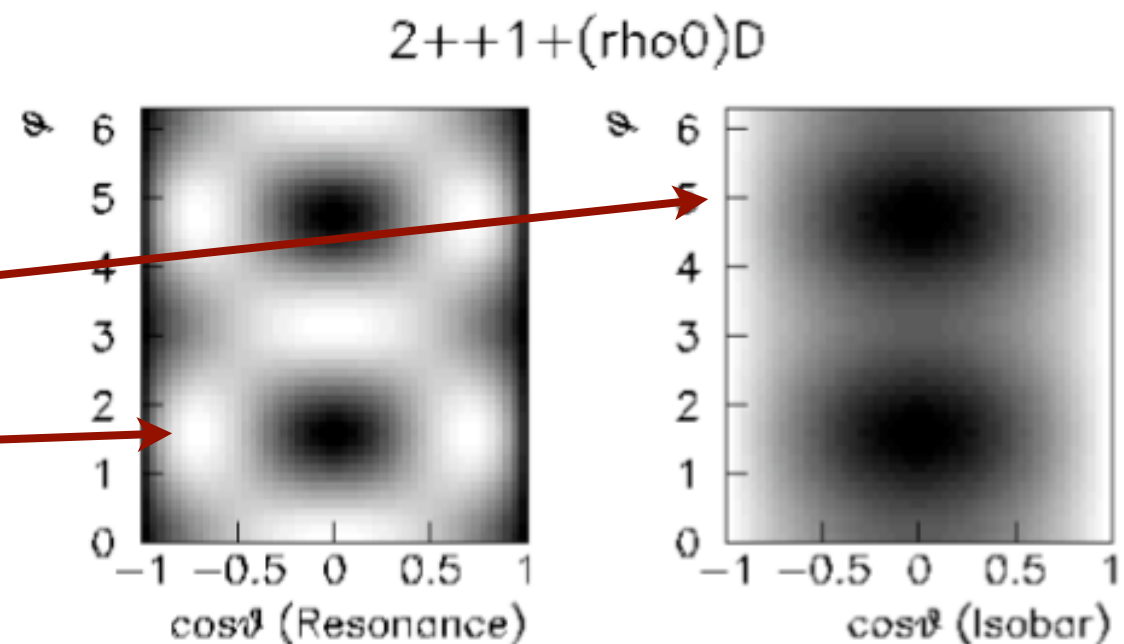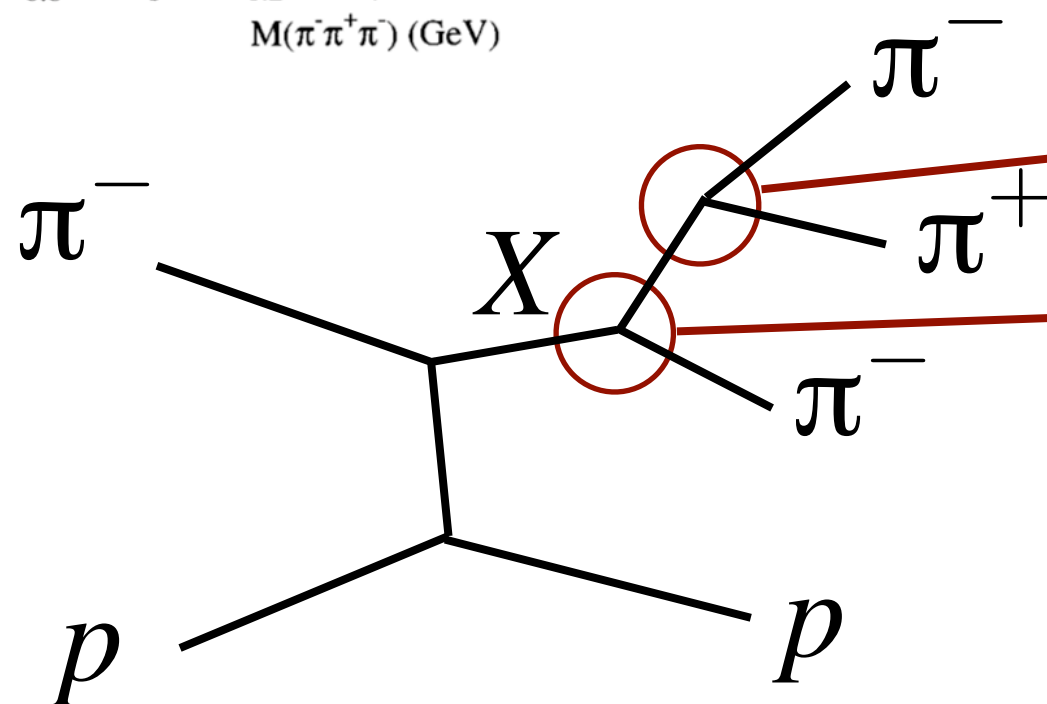
Implementation Details

- We fold in Poisson statistics and a detection efficiency η to arrive at the final likelihood expression for N observed events

$$\mathcal{L} = \frac{e^{-\mu}\mu^N}{N!} \prod_{i=1}^{N} \frac{\eta(\vec{x}_i)I(\vec{x}_i;\vec{V})}{\int \eta(\vec{x})I(\vec{x};\vec{V})d\vec{x}}$$

fit-predicted number of events:

$$\mu = \int \eta(\vec{x})I(\vec{x};\vec{V})d\vec{x}$$

- Removing constant terms, the log of the likelihood, which must be computed at every fit iteration, reduces to

$$\ln\mathcal{L} = \sum_{i=1}^{N} \ln \left(\sum_{\alpha,\beta}^{N_{\mathrm{amps}}} V_\alpha V_\beta^* \underline{A_\alpha(\vec{x}_i)A_\beta(\vec{x}_i)^*} \right) - \sum_{\alpha,\beta}^{N_{\mathrm{amps}}} V_\alpha V_\beta^* \int \eta(\vec{x})\underline{A_\alpha(\vec{x})A_\beta(\vec{x})^*}d\vec{x}$$

Sum of N logs each with N_{amps}^2 terms drives fit time

Model appears in A; if it does not contain parameters, A can be computed and cached for every event

Integrals are computed using Monte Carlo integration: needs η from high-statistics detector Monte Carlo sample; answer never changes if A are fixed

To fully explore the model space, we need to be able to parameterize A. This eliminates computational optimizations that have been historically used.

Parallel Computing

- This type of problem is perfect for parallel computing since all of the large sums over can be done in parts

$$\ln \mathcal{L} = \sum_{i=1}^{N} \ln \left(\sum_{\alpha,\beta}^{N_{\mathrm{amps}}} V_\alpha V_\beta^* A_\alpha(\vec{x}_i) A_\beta(\vec{x}_i)^* \right) - \sum_{\alpha,\beta}^{N_{\mathrm{amps}}} V_\alpha V_\beta^* \int \eta(\vec{x}) A_\alpha(\vec{x}) A_\beta(\vec{x})^* d\vec{x}$$

$$\int \eta(\vec{x}) A_\alpha(\vec{x}) A_\beta(\vec{x})^* d\vec{x} \rightarrow \frac{1}{N_{\mathrm{MC}}^{\mathrm{gen}}} \sum_{i=1}^{N_{\mathrm{MC}}^{\mathrm{acc}}} A_\alpha(\vec{x}_i) A_\beta(\vec{x}_i)$$

- Initially each node needs a sub-collection of data or MC and an algorithm for computing the A

- With each fit iteration the node just needs to know the new values of the fit parameters and it returns its contribution to the log likelihood

- Excellent application for GPU computing: compute amplitudes in parallel for all events and then collect sum

Amplitude Analysis Recap

- *The* analysis technique for extracting masses, widths, and quantum numbers of broad resonances at GlueX

- Maximizes precision by doing a multi-dimensional unbinned fit: binning data and or integrating over variables always results in a loss of knowledge

- Requires a theoretical model with parameters, which is typically a sum of amplitudes with production coefficients as parameters

- Very compute intensive but can be parallelized, implementations exist for multi-core, multi-machine, and graphics processing applications

How can we go wrong?

Poor Acceptance

- <u>Problem:</u> the predicted intensity in the active region doesn't vary much when a parameter, e.g. *d*, is varied which leads to insensitivity in measuring *d*
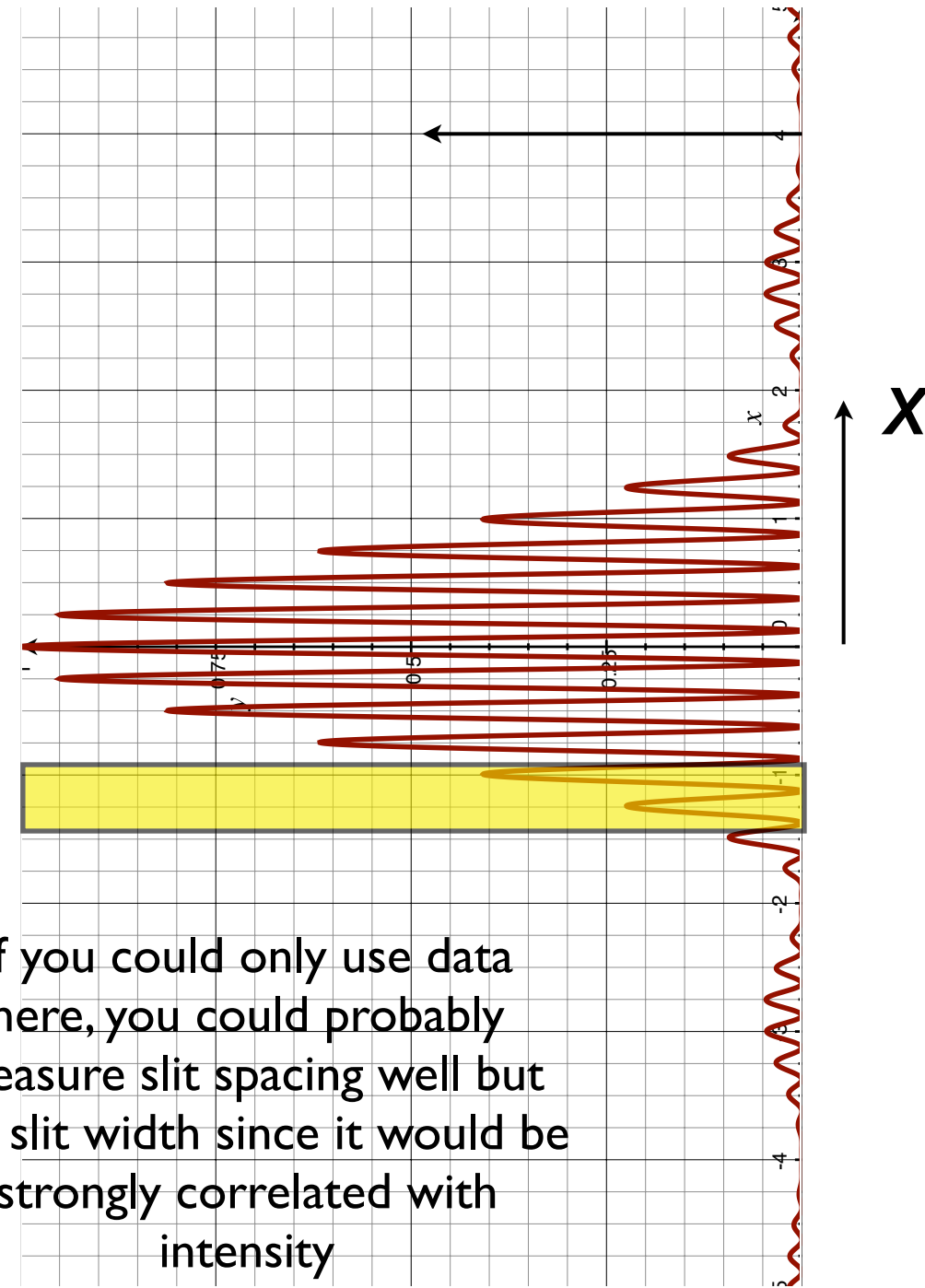
If you could only use data here, you could probably measure slit spacing well but not slit width since it would be strongly correlated with intensity

Poor Acceptance

- Problem: the predicted intensity in the active region doesn't vary much when a parameter, e.g. *d*, is varied which leads to insensitivity in measuring *d*

- Test:

  - generate a simulated set of four-vectors that includes some physics process

  - propagate the four-vectors through the detector and reconstruction to produce simulated data

  - fit these simulated data to extract parameters of the underlying physics process

GlueX needs to be a high acceptance detector!

X

If you could only use data here, you could probably measure slit spacing well but not slit width since it would be strongly correlated with intensity
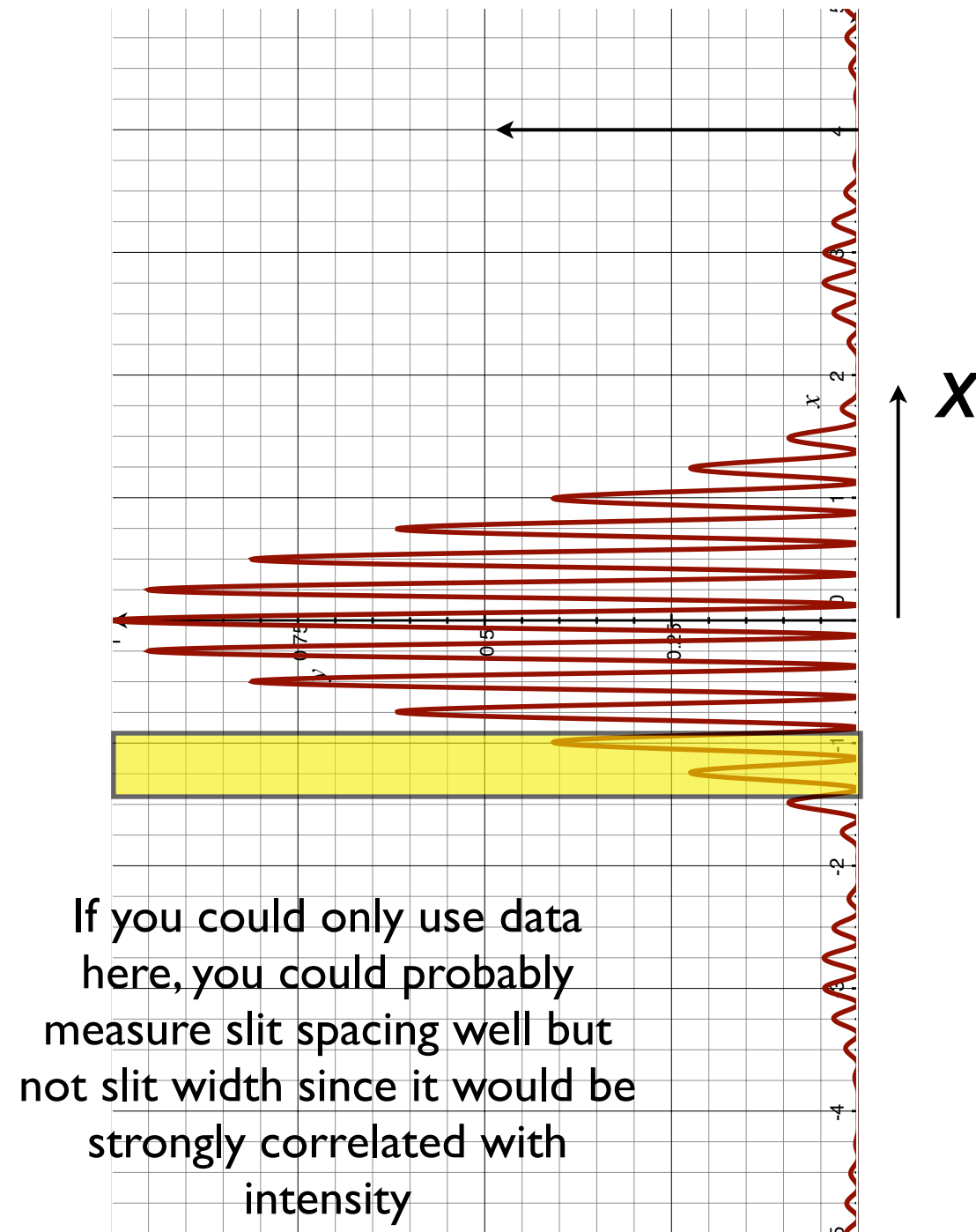
Poorly Understood Acceptance

- <u>Problem</u>: the acceptance function $\eta(\vec{x})$ used in the fit doesn't match the real detector acceptance, so the parameters of $I(\vec{x};\vec{V})$ come out incorrect when the product $\eta(\vec{x})I(\vec{x};\vec{V})$ is fit to data.

  - Acceptance appears in normalization integral, which is done numerically, by Monte Carlo techniques

$$\ln \mathcal{L} = \sum_{i=1}^{N} \ln \left(\sum_{\alpha,\beta}^{N_{\mathrm{amps}}} V_\alpha V_\beta^* A_\alpha(\vec{x}_i) A_\beta(\vec{x}_i)^* \right) - \sum_{\alpha,\beta}^{N_{\mathrm{amps}}} V_\alpha V_\beta^* \int \eta(\vec{x}) A_\alpha(\vec{x}) A_\beta(\vec{x})^* d\vec{x}$$

  - Monte Carlo integration example in one dimension: randomly generate for N different x_i that uniformly populate $[x_{\min}, x_{\max}]$

$$\int_{x_{\min}}^{x_{\max}} f(x)dx = (x_{\min} - x_{\max})\langle f(x) \rangle \qquad \text{and} \qquad \langle f(x) \rangle = \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$

Poorly Understood Acceptance

- The fit relies on integrals which can be calculated by using Monte Carlo physics events that are generated uniformly over the volume of multi-dimensional phase space in which the fit is done

$$\int \eta(\vec{x}) A_\alpha(\vec{x}) A_\beta(\vec{x})^* d\vec{x} = \mathcal{V} \frac{1}{N_{\text{gen}}} \sum_{i=1}^{N_{\text{gen}}} \eta(\vec{x}_i) A_\alpha(\vec{x}_i) A_\beta(\vec{x}_i)$$

need this integral for all combinations of α and β (amplitude indices)

volume of phase space, adds constant term in ln L so it can be ignored

simulated acceptance enters here: η=1 if Monte Carlo event is detected by detector and selected for analysis, η=0 otherwise

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- <u>Test:</u> Generate a simulated data set with some known physics process. Artificially "break" the detector simulation (introduce holes, degrade efficiencies, etc.) in the sample of phase space Monte Carlo used to compute the integrals above. Are the results of the fit robust?

Incorrect Physics Model

- Ideally the parameters of the fit correspond to quantities that have some fundamental physical interpretation, e.g., the masses, widths, and production rates of resonances

- <u>Remember the fitting procedure only returns the most likely values of the parameters given some model</u> -- it is up to us to supply the correct model for the data. (Goodness of fit is not reflected in the statistical errors on parameters!)

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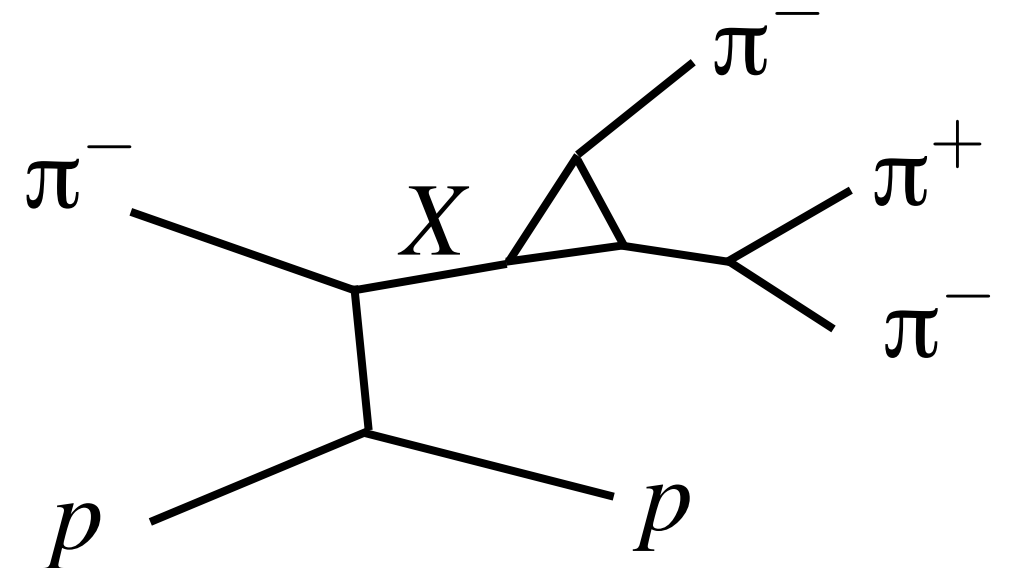
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- Theoretical problem: more complicated physics processes exist in reality, but not in the model.

  - Development of complex physics models was previously limited by lack of data to test them and computational resources to use them in the fit -- *no longer true with GlueX (we hope!)*

$$\pi^-$$
$$\pi^-$$
$$X$$
$$\pi^+$$
$$\pi^-$$
$$p$$
$$p$$

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Summary and Final Thoughts

- Amplitude analysis is a key tool for extracting physics from GlueX data.

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Summary and Final Thoughts

- Amplitude analysis is a key tool for extracting physics from GlueX data.

- The technique is "physics rich" and not as trivial as curve fitting, but its practicality has been demonstrated by past experiments.

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- Because of its complexity it is tempting to treat amplitude analysis as a magic black box where four-vectors and Monte Carlo go in and exotic meson discoveries come out **--** don't do it!

  - Understand the algorithms **--** are they correct and optimal?

  - Understand the experimental assumptions **--** are they valid?

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  - Understand the algorithms -- are they correct and optimal?

  - Understand the experimental assumptions -- are they valid?

  - Understand the theoretical assumptions -- are they valid?

- Because of its incredible statistical precision, many GlueX analyses will likely become systematics limited -- we will really need analysis experts (you!) to overcome these limitations.

  - *There are interesting problems in need of creative solutions that can be tackled right now using simulated data.*