

# Speeding Up Tracking

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- Kalman Filter review
- Places to look for speed-up
- Vectorization
- Results

# Kalman Filter

- Track described by **5 parameter state vector** at each point along its path
  - Forward-going tracks:  $\{x, y, t_x, t_y, q/p\}$
  - Central tracks:  $\{q/p_T, \phi, \tan\lambda, D, z\}$
- **State vector** propagated step-by-step toward target
  - **Multiple scattering** and **energy loss** taken care of at each step
- State vector treated as a perturbation to an initial guess
  - Generate **reference trajectory** by swimming with initial parameters from the target
- Perform filter in multiple passes
  - Can iterate up to 10 times per pass, checking for  $\chi^2$  convergence
  - Regenerate reference trajectory based on results of previous pass

# Kalman Filter Algorithm

*Start with guess from track finding or wire-based stages*

*Propagate state  $S$  and covariance  $C$  through magnetic field*

*Use measurements to update state vector*

Seed  $S^0$

$$\begin{aligned} S_k^{k-1} &= S_k^{k-1,0} + F_k (S_{k-1}^{k-1} - S_{k-1}^{k-1,0}) \\ C_k^{k-1} &= F_{k-1} C_{k-1}^{k-1} F_{k-1}^T + Q_{k-1} \end{aligned}$$

Multiple scattering  
Energy loss

$$\begin{aligned} K_k &= C_k^{k-1} H_k^T [V_k + H_k C_k^{k-1} H_k^T]^{-1} \\ S_k^k &= S_k^{k-1} + K_k (m_k - h(S_k^{k-1})) \\ C_k^k &= C_k^{k-1} - K_k H_k C_k^{k-1} \end{aligned}$$

Use result (S,C) of iteration as new seed

# Where can we speed up the code?

- Code obtains magnetic field multiple times per step
  - Implemented *FineMesh* lookup table – no interpolation...

*Reconstruction rates with 4 threads on ifarm11*

	Particle	Interpolation(Hz)	FineMesh(Hz)	Note
$\pi$	p	24.6	34.9	two mass hypotheses
	-	44.1	62.3	one mass hypothesis

- Code performs copious amounts of matrix operations per iteration
  - Old code relied on TMatrix, a generic matrix package in ROOT
  - Largest matrix in Kalman Filter is  $5 \times 5$  → could benefit from custom matrix classes
    - Implemented new matrix classes

*Dmatrix2x1, Dmatrix3x1, DMatrix3x2, ..., DMatrix5x5*

# Vectorization

- Modern CPUs have special registers that support **Single Instruction, Multiple Data** operations (“vectorization”)

- Operations on 4 ints, 4 floats, or 2 doubles at a time can be done in parallel
- “Streaming SIMD Extensions”

- SSE2 instructions

- **ADDPD**       $\boxed{A1} \boxed{A0} + \boxed{B1} \boxed{B0} \rightarrow \boxed{A1+B1} \boxed{A0+B0}$
- **SUBPD**       $\boxed{A1} \boxed{A0} - \boxed{B1} \boxed{B0} \rightarrow \boxed{A1-B1} \boxed{A0-B0}$
- **MULPD**       $\boxed{A1} \boxed{A0} * \boxed{B1} \boxed{B0} \rightarrow \boxed{A1*B1} \boxed{A0*B0}$

- SSE3 instructions

- **HADDPD** = horizontal add       $\boxed{A1} \boxed{A0} \oplus \boxed{B1} \boxed{B0} \rightarrow \boxed{B1} \boxed{A0+A1}$

# Matrix Operation Benchmarks

- Compare Root-based matrices (TMatrixD) to custom SIMD-ized matrices (DMatrix)
- 10,000,000 “events” on ifarm16 (2.8 GHz Nehalem)

Operation	TMatrixD(s)	DMatrix(s)	T/D
$A_{2 \times 2}^{-1}$	1.493	0.090	16.59
$A_{5 \times 5}^{-1}$	6.274	2.274	2.76
$A_{5 \times 5} + B_{5 \times 5}$	1.470	0.609	2.41
$A_{5 \times 5} B_{5 \times 5}$	3.821	0.974	3.82
$A_{5 \times 5}^T$	1.178	0.393	3.00
$A_{5 \times 5}^T B_{5 \times 5} A_{5 \times 5}$	7.892	1.857	4.25
$C_{1 \times 5} A_{5 \times 5} D_{5 \times 1}$	2.466	0.520	4.74
$E_{2 \times 5} A_{5 \times 5} F_{5 \times 2}$	3.630	0.651	5.58

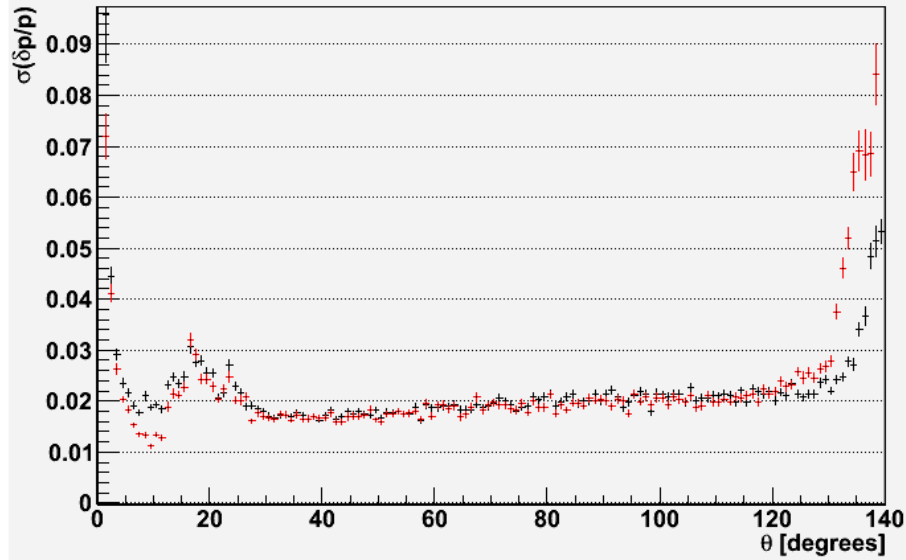
# Reconstruction Rates

- Generated 50000 events  $\rightarrow$  HDGeant  $\rightarrow$  MCSmear  $\rightarrow$  hd\_ana
- 4 threads on ifarm11(2.8 GHz Nehalem)

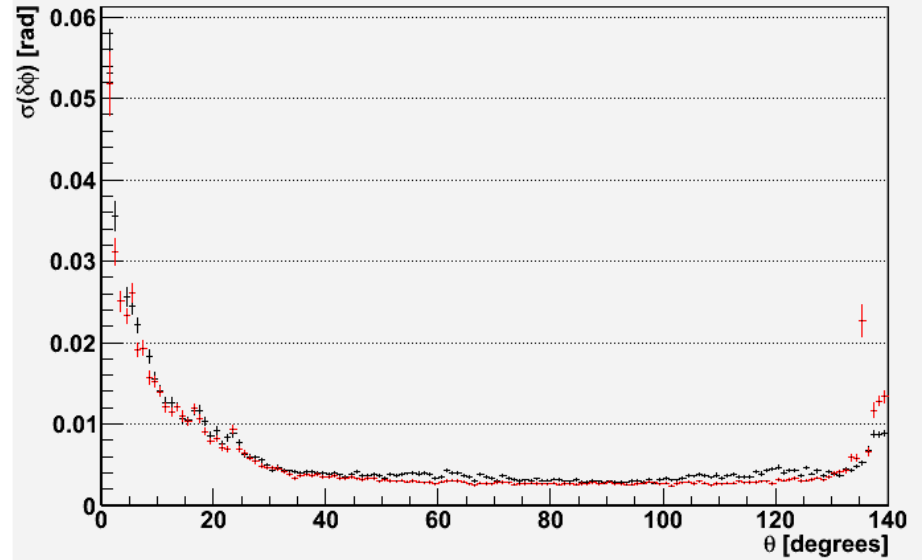
Topology	ALT1(Hz)	Kalman(Hz)	KalmanSIMD(Hz)
$\pi^+$ (particle gun)	56.0	32.8	141.7
$\pi^-$ (particle gun)	127.7	62.3	269.6
$\rho$ (particle gun)	85.9	34.9	156.8
$\rho\rho$	14.9	7.4	46.5
$n\pi^+\pi^+\pi^-$	7.5	6.4	31.8
$\rho b_1^+\pi^-$	3.8	4.7	18.2

# Resolution comparison

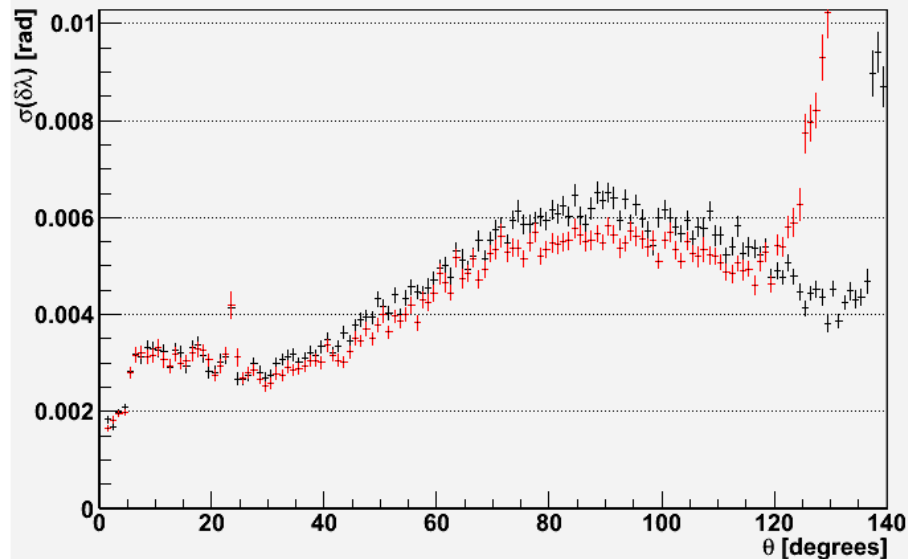
Momentum resolution, 0.3-3.1 GeV/c  $\pi^-$



Azimuthal angular resolution, 0.3-3.1 GeV/c  $\pi^-$



Dip angle resolution, 0.3-3.1 GeV/c  $\pi^-$



ALT1

KalmanSIMD

Particle gun



# Conclusion

- FineMesh option for magnetic field speeds up Kalman filter by ~40%
- SIMD-ized version of the Kalman Filter runs ~2-3× faster than ALT1
  - Except for back angles, no degradation in track parameter resolution

*Plan to switch to KalmanSIMD as default on 64bit machines  
in next software release*