# Exotic Mesons: Measurement Techniques, Recent Evidence and Future Searches

Colloquium at CUA Department of Physics

Igor Senderovich<sup>1</sup>



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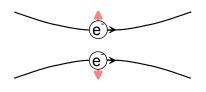
#### Outline

- Introduction
  - Fields, Quarks and Hadrons
  - Hadron Spectroscopy
- Partial Wave Analysis in Meson Spectroscopy
  - Essentials of PWA
  - PWA Example:  $\rightarrow X \rightarrow b_1 \pi \rightarrow 5\pi$
- Experimental Search for Exotic Mesons
  - Evidence from Past Experiments
  - Future Experiments

## Outline

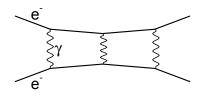
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- - Evidence from Past Experiments



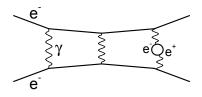
## Classical view of interactions

- smooth field permeates space
- continuous change in momentum from resulting force
- deterministic outcome



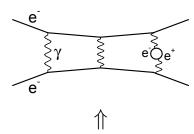
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- field quanta (e.g. photons) distributed through space
- discrete momentum kicks from exchange of these force carriers
- probabilistic outcome from superposition of possible scattering states



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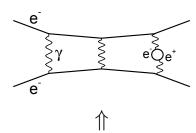
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- more than forces: particle creation/annihilation subtle but precisely calculated and confirmed effect in electrodynamics! (e.g. Lamb shift)



- $e^+/e^-$  pair pulled out of vacuum! What else is in there?
- no longer safe to assume interactions occur in empty space

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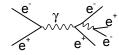
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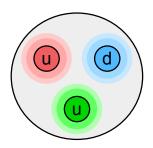
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## Quantum field view of interactions

- field quanta (e.g. photons) distributed through space
- discrete momentum kicks from exchange of these force carriers
- probabilistic outcome from superposition of possible scattering states
- more than forces: particle creation/annihilation subtle but precisely calculated and confirmed effect in electrodynamics! (e.g. Lamb shift)
- new particles can also be liberated from the vacuum:



# The Proton...



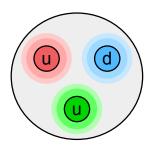
...turned out to have substructure!

Spin 1/2 "quarks": charge | I П Ш +2/3charm top up down strange bottom • A new theory, Quantum Chromodynamics (QCD), and 3 sets of charges proposed:

+	_
red	anti-red (cyan)
green	anti-green (magenta)
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- All quark composites (hadrons) must end up "white"
- New force carrier (gauge boson): gluon
  - carries a pair of color charges!
  - couples to quarks and other gluons

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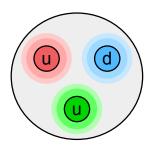
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Consider a simpler object:  $\Leftarrow$  quark-anti-quark pair, *i.e.* "meson"  $(\pi^+ \text{ in this case})$ 

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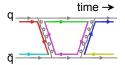
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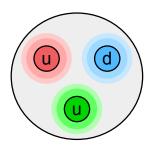
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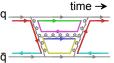
 $\begin{array}{c|cccc} \underline{Spin \ 1/2} \ ``quarks": \\ \hline charge & I & II & III \\ \hline +2/3 & up & charm & top \\ \hline -1/3 & down & strange & bottom \\ \end{array}$ 

• A new theory, *Quantum Chromodynamics* (QCD), and 3 sets of charges proposed:

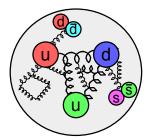
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# The Proton...



## More complex view:

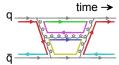
- valence quarks
- sea quarks
- proton mass, spin depend on the full dynamics!

• A new theory, Quantum Chromodynamics (QCD), and 3 sets of charges proposed:

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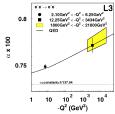


# Coupling Strength

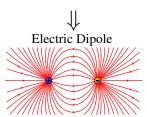
# **Electromagnetism (QED)**

coupling constant:  $\alpha$ 

(fine structure constant)

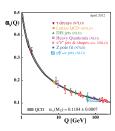


L3 Collab, Phys. Lett. B 623, 26 (2005)



# **Strong Interaction (QCD)**

coupling constant:  $\alpha_s$ 



Particle Data Group, 2012



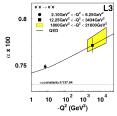


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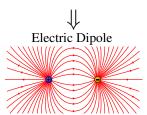
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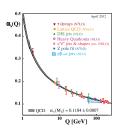


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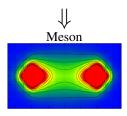


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## Introduction Overview

# Concepts covered so far:

- Nature of Fields
  - fields are quantized with discrete interactions
  - beyond forces, give rise to interactions that can create particles
- Quarks and Their Composites Hadrons
  - protons, neutrons, and later discovered pions, kaons, etc. are composite objects
  - so far understood as made of 2 or 3 valence quarks, with important dynamics of surrounding fields, other quarks, etc.
- Quantum Chromodynamics a special interaction holding quarks together
  - uses *gluons* as carriers which themselves have charge  $\Rightarrow$  self-interacting!
  - gets stronger at larger distances!
  - field condenses into dense regions
  - strong but confined the condensed field vacates the rest of space

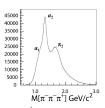
# Probing Hadrons: Spectroscopy

How did we explore quantum mechanics and atomic systems in the early days? Spectroscopy!

- work out the bound states, their excitations, line shapes from theory
- compare to experimentally-measured spectrum

Doing the same for hadrons is fruitful but challenging  $\rightarrow$ 

- strong coupling  $\Rightarrow$  no perturbative calculations possible How to cope?
  - build simpler models and try to calibrate to the full theory
  - numerical methods *lattice OCD*, but very computationally expensive
- short-lived ⇔ broad resonances hard to disentangle, identify states Solution: look at more than energy (mass) spectra
  - $\rightarrow$  Partial Wave Analysis (PWA)
    - decomposition of scattered intensity into states with different angular distributions  $\Rightarrow$  get angular momentum q.n. of original states from angular correlations
    - determination of wave (amplitude) components gives phase



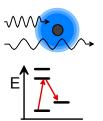
# Hadron Spectroscopy Example

How do we do spectroscopy on hadrons? How does it compare with atomic/optical spectroscopy?

# Atomic spectroscopy:

send in light, see at what frequencies the sample absorbs

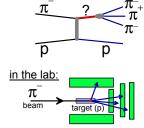
ionize sample, detect emitted light (from discrete transitions)



# Hadron spectroscopy:

create a particle/resonance in collision, analyze it from debris

- only descendent particles are detected
- use energy/momentum conservation to reconstruct original system properties
- map out its c.o.m. energy (mass) spectrum



Excitation modes of bound particles teach us about the internal field.

# But can we study the field more directly?

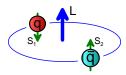


- Can a meson's condensed field (flux tube) vibrate?
- How would we detect/identify this?

Mesons are identified by  $J^{PC}$  quantum numbers:

• 
$$\vec{J} = \vec{L} + \vec{S}$$
,  $\vec{S} = \vec{S_1} + \vec{S_2} = 0, 1$ 

- Parity: symmetry under space inversion;  $P = (-1)^{L+1}$
- Charge conjugation: symmetry under matter/anti-matter inversion;  $C = (-1)^{L+S}$



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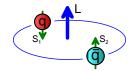


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 $\therefore q\overline{q}$  quantum numbers:

J		++	-+	+-
0		$0_{++}$	$0_{-+}$	
1 2	1	1++		1+-
2	2	$2^{++}$	$2^{-+}$	

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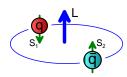
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J		++	-+	+-
0	0	$0_{++}$	-	$0_{+-}$
1	1 <sup></sup> 2 <sup></sup>	1++	$1^{-+}$	1+-
2	2	$2^{++}$	$2^{-+}$	$2^{+-}$



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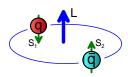
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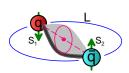
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0	0	$0_{++}$	$0_{-+}$	$0_{+-}$
1	1	1++		
2	1 <sup></sup> 2 <sup></sup>	2++	$2^{-+}$	$2^{+-}$





Exotic states  $\Longrightarrow$  unambiguous signature of new degrees of freedom

## Outline

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  - Essentials of PWA
  - PWA Example:  $\rightarrow X \rightarrow b_1 \pi \rightarrow 5\pi$
- - Evidence from Past Experiments

# Decay Amplitude Angular Distribution

Like a bound state, scattering/decay has some characteristic angular momentum.

Consider a P-wave (L = 1) decay  $\Rightarrow$ 

Each decay like this samples the probability distribution given by this  $\theta$ ,  $\phi$ -dependent amplitude.

 $\therefore$  We should be able to get the L quantum number from the angular distribution of particles in a detector!

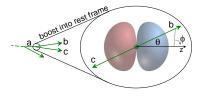


Figure: Illustration of a 2-body decay analysis.

# Finding Original Amplitude: Maximum Likelihood Method

Goal: Find the most likely probability distribution to have generated your data.

Construct a likelihood function with:

- probability of having seen all the detected events  $\vec{x}_1 \dots \vec{x}_N$ :
- Poisson probability of having seen the number of detected events

$$\mathcal{L} = \operatorname{Pois}(\mu; \mathbf{N}) \prod_{i}^{N} P(\vec{x}_{i}; u_{1}, u_{2} \dots u_{n}) = \frac{e^{-\mu} \mu^{N}}{N!} \prod_{i}^{N} \frac{1}{\mu} \left| \sum_{\alpha}^{n} c_{\alpha} \Psi_{\alpha}(\vec{x}_{i}) \right|^{2}$$

 $\mu$  is the total expected in the detector, calculated via Monte Carlo i.e. sampling of events  $\vec{y}_1 \dots \vec{y}_{N_{\text{gen}}}$  weighted by

- candidate probability distribution
- detector acceptance

$$\Rightarrow \qquad \mu = \frac{1}{N_{\rm gen}} \sum_{i}^{N_{\rm acc}} \left| \sum_{\alpha}^{n} c_{\alpha} \Psi_{\alpha}(\vec{x}_{i}) \right|^{2}$$

Other methods exist, but in all cases, Monte Carlo samples computed with detector simulation are needed.

# Example: simulated $\gamma p \to Xp \to b_1\pi p$

Let us simulate the following reaction:

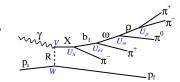
$$\gamma p \to X p \to b_1^\pm \pi^\mp \to \omega \pi^\pm \pi^\mp p \to \pi^+ \pi^- \pi^0 \pi^\pm \pi^\mp p$$

with two amplitude components for resonance *X*:

$$J^{PC} = 1^{--}, \{M, \Gamma\} = \{1.89, 0.16\} \text{ GeV}$$
  
 $J^{PC} = 2^{+-}, \{M, \Gamma\} = \{2.00, 0.25\} \text{ GeV}$ 

This means generating 5-pion final state events according to the probability distribution of this production/decay process.

A variant of **GEANT** software is used to simulate flight of particles through the detector, and data acquisition.



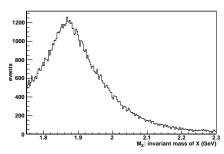


Figure : Resulting invariant mass distribution of the *X* resonance

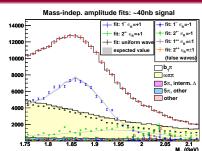
# Amplitude Fit Results: $40 \text{ nb } 2^{+-} \& \text{ Pythia (GlueX)}$

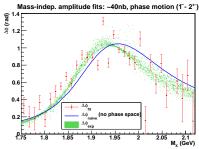
## Cross-section scaling:

- Pythia, a particle event generator approximating the dominant known reactions is used to create background
   13.9 G evts @ 9 GeV photon beam energy ~ 260 h run time
- $b_1\pi$ : 18 M evts  $\sim 25\% 2^{+-}$

#### Conclusions:

- one can pull out the individual waves
- some leakage seen between waves





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The tentative observations of exotic states thus far: (masses and widths from PDG)

$\pi_1$ state	Mass (GeV)	Width (GeV)	Prod.	Decays	Experiments
1400	$1.351 \pm 0.03$	$0.313 \pm 0.040$	$\pi^- p, \bar{p}n$	$\pi^-\eta$ , $\pi^0\eta$	E852, CBAR
1600	$1.662 \pm 0.015$	$0.234 \pm 0.050$	$\pi^- p, \bar{p}p$	$\eta'\pi$ , $b_1\pi$ ,	E852, CBAR, VES,
				$f_1\pi$ , $\rho\pi$	COMPASS
2015	$2.01 \pm 0.03$	$0.28 \pm 0.05$	$\pi^- p$	$b_1\pi f_1\pi$	E852

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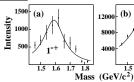
# Background: Experiment E852 at Brookhaven's AGS.

 $18 \,\mathrm{GeV/c}~\pi^-$  beam on a liquid  $H_2$  target

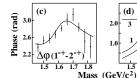
Reaction: 
$$\pi^- p \to \pi^- \pi^+ \pi^- p$$

# Figure:

- a)  $\pi_1(1600) 1^{-+}$  wave
- b) the dominant  $\pi_2(1670) 2^{-+}$  wave
- c) the relative phase
- d) individual phases:
  - 1.  $1^{-+}[\rho(770)]P1^{+}$
  - 2.  $2^{-+}[f_2(1270)]S0^+$
  - 3. relative production phase









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**Reanalysis:**  $\times 10$  more data, more waves to describe  $\pi_2(1670)$ :

- $\pi_1(1600)$  enhancement vanishes
- phase motion persists!

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# Background: CEBAF Large Acceptance Spectrometer (CLAS) at JLab

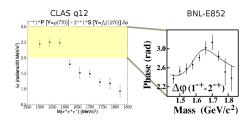
5.7 GeV photon beam on a liquid  $H_2$  target

Reaction: 
$$\gamma p \to (X)n \to \pi^+\pi^-\pi^+$$

Figure: No phase motion consistent with  $\pi_1(1600) 1^{-+}$  resonance observed

## Possible explanations:

- $\pi_1(1600)$  production just suppressed in charge exchange
- small cross-section in photoproduction (contrary to models)



The tentative observations of exotic states thus far: (masses and widths from PDG)

$\pi_1$ state	Mass (GeV)	Width (GeV)	Prod.	Decays	Experiments
1400	$1.351 \pm 0.03$	$0.313 \pm 0.040$	$\pi^- p, \bar{p}n$	$\pi^-\eta$ , $\pi^0\eta$	E852, CBAR
1600	$1.662 \pm 0.015$	$0.234 \pm 0.050$	$\pi^- p, \bar{p}p$	$\eta'\pi$ , $b_1\pi$ ,	E852, CBAR, VES,
				$f_1\pi$ , $\rho\pi$	COMPASS
2015	$2.01 \pm 0.03$	$0.28 \pm 0.05$	$\pi^- p$	$b_1\pi f_1\pi$	E852

# Background: spectrometer at CERN's SPS with $\pi$ , p, K (?) beams

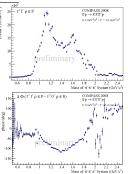
190 GeV  $\pi^-$  beam on a liquid  $H_2$  target

Reaction: 
$$\pi^- p \to \pi^- \pi^+ \pi^- p$$

Figure:

Top: exotic  $1^{-+}\rho\pi$  P-wave invariant mass

Bottom: phase difference with  $a_1$  1<sup>++</sup>



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2015	$2.01 \pm 0.03$	$0.28 \pm 0.05$	$\pi^- p$	$b_1\pi f_1\pi$	E852

# Background: CLEO-c @ Cornell $e^+/e^-$ storage ring

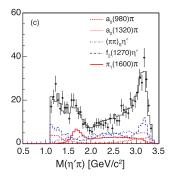
colliding  $e^+/e^-$  beams

Reaction: 
$$e^+/e^- \rightarrow \psi(2S) \rightarrow \gamma \chi_{c1} \rightarrow \eta' \pi^+ \pi^-$$

# Figure:

Amplitude invariant mass projections for  $\chi_{c1} \rightarrow \eta' \pi^+ \pi^- \text{ decay}$ 

G. S. Adams et al. [CLEO Collaboration], Phys. Rev. D 84, 112009 (2011)



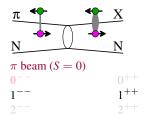
# **Using Photon Beams**

Data so far: production (mostly) with  $\pi$  beams Is the exotic hybrid production suppressed?

A possible argument: the spin flip needed for exotic q.n. is suppressed.

Proposal: use S = 1 beam  $\rightarrow$  photons!

Meson  $J^{PC}$  q.n. production scenarios without quark spin flip (flux-tube model):



γ <b>*</b>	<u>X</u>
N O	N
$\gamma$ beam ( $S = 1$ )	
$0^{-+}$	$0^{+-}$
1-+	1+-
2-+	2+-

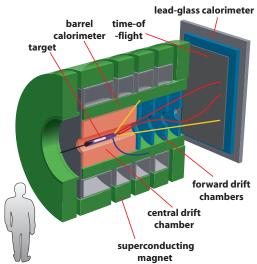
#### Criticism:

- Spin flip suppression may not be an issue for light quarks
- Search by CLAS (g12) did not see a signal for  $\pi_1(1600)$

# Gluonic Excitations Experiment (GlueX)

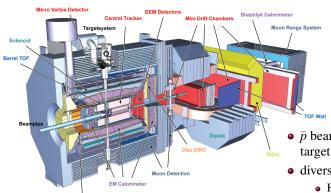
GlueX detector is being assembled in the new Hall D in Jefferson Lab, Newport News, VA

- 9 GeV (tagged) polarized photon beam on liquid H<sub>2</sub> target
- hermetic, low dead time detector for high statistics angular analysis



## Anti-Proton **An**nihilation at **Da**rmstadt (PANDA)

PANDA: a multi-purpose detector is being designed for the Facility for Antiproton and Ion Research (FAIR) project GSI, Darmstadt, Germany



- $\bar{p}$  beam on a flexible jet/fixed target system
- diverse program:
  - Hadron Spectroscopy
  - Nucleon Structure
  - Hypernuclei

Igor Senderovich

Hadrons in-medium

Barrel DIRC

# Summary

- Strong Interaction described by Quantum Chromodynamics: creates bound states that are difficult to study
- Overlapping resonances disentangled with Partial Wave Analysis, which yields states' quantum numbers and relative phases
- Hybrid Mesons (with excited flux tube identified by exotic quantum numbers) offer a new way to probe the dynamics of the gluon field
- Tantalizing hints of these from past experiments, particularly for  $\pi_1(1600) 1^{-+}$  state
- A new generation of experiments coming online to study these with good coverage and high statistics:
  - GlueX with photo-production
  - PANDA through  $p\bar{p}$  collisions