A review of asymmetry measurements in vector meson photoproduction experiments

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Curtis A. Meyer
Carnegie Mellon University

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Abstract

In vector meson photoproduction, the spin observables of the vector mesons are represented by their density matrix. The elements, the spin-density matrix elements, or SDMEs are the physical observables. With polarized and unpolarized photon beams, it is possible to measure 11 of the 17 real values that describe the complex SDMEs. Historically, many experiments have reported various asymmetries, where it appears that many of these measurements were trying to measure a combinations of a small number of SDMEs. In this report, we survey the various asymmetries found in the literature and discuss. when possible, how they are related to the SDMEs. Finally, we provide a summary of existing data on ρ^0 meson photoproduction using linearly-polarized photons.

1 Introduction

In the photoproduction of vector mesons, it is possible for the vector meson to be polarized, even in the case of unpolarized photon beams. The spin-state of the vector meson is described by its density matrix, and spin-density matrix elements (SDMEs) provide information on both the underlying production mechanism, as well the decay distribution, $W(\cos\theta, \phi)$, of vector meson. We measure the SDMEs through the decay distribution. The decay distribution depends not only on the SDMEs, but also on the decay mode of the vector mesons. In particular, purely hadronic, radiative, and lepton decays may have different decay distributions. Throughout this note, we use $W_h(\cos\theta, \phi)$ to describe the purely hadronic decay, while $W_r(\cos\theta, \phi)$ describes the radiative decays.

From experiments utilizing linearly-polarized photon beams, a number of different asymmetries have been reported. As we will see, some of these are easily related to some SDMEs, while for others, the relation is not so clear. The nomenclature used for these asymmetries is also confusing. We will survey the history of various asymmetries reported for vector mesons, and to the extent possible, show how they relate to the SDMEs.

2 Spin Observables in Vector Meson Photoproduction

2.1 Production Amplitudes

We can characterize the photoproduction of vector mesons by an amplitude T, which connects the density matrix for the initial photon beam, $\rho(\gamma)$, the the density matrix of the produces vector meson, $\rho(V)$. Following Schilling from reference [1], we can write

$$\rho(V) \;\; = \;\; T\,\rho(\gamma)\,T^{\dagger}\,.$$

This can be written in the center-of-mass frame helicity representation [2] as

$$\rho_{\lambda_V\lambda'_V}(V) \;\; = \;\; \frac{1}{\mathcal{N}}\sum_{\lambda'_N\lambda_\gamma\lambda_N\lambda'_\gamma} T_{\lambda_V\lambda'_N,\lambda_\gamma\lambda_N}\,\rho_{\lambda_\gamma\lambda'_\gamma}\,T^{\dagger}_{\lambda'_V\lambda'_N,\lambda'_\gamma\lambda_N}\,,$$

where the λ_x represent the helicity of the incoming (N) and outgoing (N') nucleon, the photon spin-density matrix elements, (γ, γ'), and the vector meson spin-density matrix elements (V, V'). The term $\mathcal{N}$ is a normalization factor given as

$$\mathcal{N} \;\; = \;\; \frac{1}{2}\sum_{\lambda_V\lambda'_N\lambda_\gamma\lambda_N}\left|T_{\lambda_V\lambda'_N,\lambda_\gamma\lambda_N}\right|^2\,,$$

which for center-of-mass momentum of the incoming photon, k, can be related to the unpolarized differential cross section as

$$\frac{d\sigma}{d\Omega} \;\; = \;\; \frac{1}{2}\left(\frac{2\pi}{k}\right)^2\mathcal{N}\,.$$

Thus, measuring the differential cross section and the density matrix elements of the vector meson photoproduction allows us to study the production processes.

Experimentally, we can access elements of the vector meson density matrix through the decay angular distributions of the daughter products of the vector meson as discussed on Section 2.3.

2.2 Spin Density Matrix Elements

We can now include the photon polarization into our description of the vector meson density matrices. We start by writing down the photon polarization vectors, P_γ for linearly- and circularly polarized photons. In the case of linearly-polarized photons, the photon polarization is given as

$$\mathbf{P}_\gamma \;\; = \;\; P_\gamma\,(-\cos 2\Phi, -\sin 2\Phi, 0) \qquad (1)$$

where P_γ is the degree of linear polarization (between 0 and 1), and Φ is the angle between the polarization vector of the photon and the production plane of the vector meson. For circularly-polarized photons,

$$\mathbf{P}_\gamma \;\; = \;\; P_\gamma\,(0, 0, \pm 1) \qquad (2)$$

where P_γ is again the degree of polarization, and the ± 1 corresponds to the helicity of the photon, $\lambda_\gamma = +1$ or $\lambda_\gamma = -1$. If we now consider the three components of the photon polarization (1 and

2 for linearly polarized and 3 for circularly polarized), we can now write the vector meson density matrix as the sum

$$\rho(V) \;=\; \rho^0_V + \sum_{\alpha=1}^{3} P^\alpha_\gamma \, \rho^\alpha \,, \tag{3}$$

where the ρ^α form a decomposition of the total density matrix, based on photon polarization.

The ρ^α are related to the amplitudes, T, as

$$\rho^0_{\lambda_V \lambda'_V}(V) \;=\; \frac{1}{2\mathcal{N}} \sum_{\lambda'_N \lambda_\gamma \lambda_N} T_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N} \, T^*_{\lambda'_V \lambda'_N, \lambda_\gamma \lambda'_N} \tag{4}$$

$$\rho^1_{\lambda_V \lambda'_V}(V) \;=\; \frac{1}{2\mathcal{N}} \sum_{\lambda'_N \lambda_\gamma \lambda_N} T_{\lambda_V \lambda'_N, -\lambda_\gamma \lambda_N} \, T^*_{\lambda'_V \lambda'_N, \lambda_\gamma \lambda'_N} \tag{5}$$

$$\rho^2_{\lambda_V \lambda'_V}(V) \;=\; \frac{i}{2\mathcal{N}} \sum_{\lambda'_N \lambda_\gamma \lambda_N} \lambda_\gamma \, T_{\lambda_V \lambda'_N, -\lambda_\gamma \lambda_N} \, T^*_{\lambda'_V \lambda'_N, \lambda_\gamma \lambda'_N} \tag{6}$$

$$\rho^3_{\lambda_V \lambda'_V}(V) \;=\; \frac{i}{2\mathcal{N}} \sum_{\lambda'_N \lambda_\gamma \lambda_N} \lambda_\gamma \, T_{\lambda_V \lambda'_N, \lambda_\gamma \lambda_N} \, T^*_{\lambda'_V \lambda'_N, \lambda_\gamma \lambda'_N} \tag{7}$$

Utilizing symmetry properties, including the fact that the density matrices are Hermitian, it is possible to write the ρ^α in terms of their spin-density matrix elements (SDMEs) as described by Schilling [1]. The spin-density matrices can be expressed in terms of 17 real numbers, of which 11 are measurable. These 17 real numbers are given below.

$$\rho^0 \;=\; \begin{pmatrix} \frac{1}{2}\left(1-\rho^0_{00}\right) & \mathrm{Re}\rho^0_{10} + i\mathrm{Im}\rho^0_{10} & \mathrm{Re}\rho^0_{1-1} \\ & \rho^0_{00} & -\left(\mathrm{Re}\rho^0_{10} - i\mathrm{Im}\rho^0_{10}\right) \\ & & \frac{1}{2}\left(1-\rho^0_{00}\right) \end{pmatrix}$$

$$\rho^1 \;=\; \begin{pmatrix} \rho^1_{11} & \mathrm{Re}\rho^1_{10} + i\mathrm{Im}\rho^1_{10} & \mathrm{Re}\rho^1_{1-1} \\ & \rho^1_{00} & -\left(\mathrm{Re}\rho^1_{10} - i\mathrm{Im}\rho^1_{10}\right) \\ & & \rho^1_{11} \end{pmatrix}$$

$$\rho^2 \;=\; \begin{pmatrix} \rho^2_{11} & \mathrm{Re}\rho^2_{10} + i\mathrm{Im}\rho^2_{10} & i\mathrm{Im}\rho^2_{1-1} \\ & 0 & \mathrm{Re}\rho^2_{10} - i\mathrm{Im}\rho^2_{10} \\ & & -\rho^2_{11} \end{pmatrix}$$

$$\rho^3 \;=\; \begin{pmatrix} \rho^3_{11} & \mathrm{Re}\rho^3_{10} + i\mathrm{Im}\rho^3_{10} & i\mathrm{Im}\rho^3_{1-1} \\ & 0 & \mathrm{Re}\rho^3_{10} - i\mathrm{Im}\rho^3_{10} \\ & & -\rho^3_{11} \end{pmatrix}$$

Because the trace of ρ^0 is 1, we can define $\rho^0_{11} = \frac{1}{2}(1-\rho^0_{00})$. As we will see in Section 2.3, with unpolarized photon beams, we can measure three of the four real values describing the the ρ^0 SDMEs. With circularly-polarized photons, we can measure the same three ρ^0 SDMEs plus two of the four real values from the ρ^3 SDMEs.With linearly-polarized photons, the same ρ^0 elements are accessible. In addition, we can measure four of the five real elements of the ρ^1 SDMEs, and two of the four form ρ^2.

2.3 Decay Distributions of Vector Mesons

In photoproduction experiments, the decay angular distribution for vector mesons can be expressed in terms of the SDMEs, ρ^{α}_{ij}. We start by writing this as a sum

$$W(\cos\theta,\phi) \;=\; W^0(\cos\theta,\phi) + \sum_{\alpha=1}^{3} P^{\alpha}_{\gamma} W^{\alpha}(\cos\theta,\phi)\,, \tag{8}$$

where P^{α}_{γ} is the polarization vector of the photon given by equation 1 and 2. For hadronic decays of the vector mesons ($\rho^0 \to \pi^+\pi^-$, $\omega \to \pi^+\pi^-\pi^0$, $\phi \to K^+K^-$ and $\phi \to \rho\pi$) the W^{α}_h are given as

$$\begin{aligned} W^0_h(\cos\theta,\phi,\rho^0) &= \frac{3}{4\pi}\left[\frac{1}{2}\left(1-\rho^0_{00}\right)+\frac{1}{2}\left(3\rho^0_{00}-1\right)\cos^2\theta \right.\\ &\quad \left. -\sqrt{2}\,\mathrm{Re}\rho^0_{10}\sin 2\theta\cos\phi - \rho^0_{1-1}\sin^2\theta\cos 2\phi\right] \end{aligned} \tag{9}$$

$$W^1_h(\cos\theta,\phi,\rho^1) = \frac{3}{4\pi}\left[\rho^1_{11}\sin^2\theta + \rho^1_{00}\cos^2\theta - \sqrt{2}\,\mathrm{Re}\rho^1_{10}\sin 2\theta\cos\phi - \rho^1_{1-1}\sin^2\theta\cos 2\phi\right] \tag{10}$$

$$W^2_h(\cos\theta,\phi,\rho^2) = \frac{3}{4\pi}\left[\sqrt{2}\,\mathrm{Im}\rho^2_{10}\sin 2\theta\sin\phi + \mathrm{Im}\rho^2_{1-1}\sin^2\theta\sin 2\phi\right] \tag{11}$$

$$W^3_h(\cos\theta,\phi,\rho^3) = \frac{3}{4\pi}\left[\sqrt{2}\,\mathrm{Im}\rho^3_{10}\sin 2\theta\sin\phi + \mathrm{Im}\rho^3_{1-1}\sin^2\theta\sin 2\phi\right] \tag{12}$$

The angles θ and ϕ denote the direction of one of the daughter particles of the vector meson decay, nominally chosen in the helicity system, as described below. However the results are true for any system that can be reached by a rotation. For three-body decays, such as the $\omega \to \pi^+\pi^-\pi^0$, the angles describe the orientation of the normal to the decay plane of the vector meson. We note that the trace of ρ is invariant under these rotations.

The above formula account for the hadronic decays of vector mesons. There are also non-negligible radiative decays of vector mesons. The largest is $\omega \to \pi^0\gamma$, but others include $\rho \to \pi\gamma$, $\phi \to \eta\gamma$ and $K^* \to K\gamma$. We include here W^{α}_r for radiative decays, as taken from references [3, 4]. There is an overall sign difference between the two references for W^2_r and W^3_r. We present the sign from reference [4]. There is also a discrepancy in the ϕ dependence of W^2_r and W^3_r, where reference [4] have $\sin 2\theta\cos\phi$ and reference [3] have $\sin 2\theta\sin\phi$. We believe the latter is correct.

$$\begin{aligned} W^0_r(\cos\theta,\phi,\rho^0) &= \frac{3}{8\pi}\left[1-\rho^0_{11}\sin^2\theta - \rho^0_{00}\cos^2\theta \right.\\ &\quad \left. +\sqrt{2}\,\mathrm{Re}\rho^0_{10}\sin 2\theta\cos\phi + \rho^0_{1-1}\sin^2\theta\cos 2\phi\right] \end{aligned} \tag{13}$$

$$\begin{aligned} W^1_r(\cos\theta,\phi,\rho^1) &= \frac{3}{8\pi}\left[2\rho^1_{11} + \left(\rho^1_{00}-\rho^1_{11}\right)\sin^2\theta + \sqrt{2}\,\mathrm{Re}\rho^1_{10}\sin 2\theta\cos\phi \right.\\ &\quad \left. +\rho^1_{1-1}\sin^2\theta\cos 2\phi\right] \end{aligned} \tag{14}$$

$$W^2_r(\cos\theta,\phi,\rho^2) = -\frac{3}{8\pi}\left[\sqrt{2}\,\mathrm{Im}\rho^2_{10}\sin 2\theta\sin\phi + \mathrm{Im}\rho^2_{1-1}\sin^2\theta\sin 2\phi\right] \tag{15}$$

$$W^3_r(\cos\theta,\phi,\rho^3) = -\frac{3}{8\pi}\left[\sqrt{2}\,\mathrm{Im}\rho^3_{10}\sin 2\theta\sin\phi + \mathrm{Im}\rho^3_{1-1}\sin^2\theta\sin 2\phi\right] \tag{16}$$

2.4 Coordinate Systems

We now introduce the three coordinate systems in which SDMEs are reported in the literature.

1. The helicity system, where the z-axis is chosen opposite to the direction of the outgoing proton in the vector meson rest frame. This is equivalent to the direction of the vector meson in the overall center-of-mass frame.

2. The Adair system where the z-axis is chosen along the direction of the incident photon in the overall center-of-mass system.

3. The Gottfried-Jackson system where the z-axis is chosen along the direction of the incident photon in the vector meson rest frame.

For all three systems, the y-axis is taken to be normal to the production plane, and the direction of the x-axis is chosen to make the coordinate system right handed. For forward going particles, all three systems coincide. Transforming between these reference frames corresponds to appropriate rotations about the common $y - axis$.

$$\left(\rho^i_{MM'}\right)_B = \sum_{\mu\mu'} d^1_{M\mu}(-\alpha) \left(\rho^i_{\mu\mu'}\right)_A d^1_{\mu' M'}(\alpha) \tag{17}$$

where

$$\sin\alpha = \mid \hat{z}^A \times \hat{z}^B \mid .$$

2.5 Decay Distributions for Polarized and Unpolarized Photon Beams

We now examine equation 8 for various cases of photon polarization. For unpolarized photons,

$$W^{\text{unpol}}(\cos\theta, \phi) = W^0(\cos\theta, \phi)\,, \tag{18}$$

which from equation 9 means that with unpolarized photons, we can determine ρ^0_{00}, ρ^0_{1-1} and Reρ^0_{10}.

For linearly-polarized photons with polarization P_γ, we have

$$W^L(\cos\theta, \phi, \Phi) = W^0(\cos\theta, \phi) - P_\gamma \cos 2\Phi\, W^1(\cos\theta, \phi) - P_\gamma \sin 2\Phi\, W^2(\cos\theta, \phi)\,. \tag{19}$$

With a linearly-polarized photon beam, we can measure ρ^0_{00}, ρ^0_{1-1}, Reρ^0_{10},ρ^1_{00}, ρ^1_{11}, ρ^1_{1-1}, Reρ^1_{10}, Imρ^2_{1-1} and Imρ^2_{10}. Finally, for circularly-polarized photons of polarization P_γ, we have

$$W^{\pm}(\cos\theta, \phi) = W^0(\cos\theta, \phi) \pm P_\gamma\, W^3(\cos\theta, \phi)\,. \tag{20}$$

This can be expanded in terms of the SDMEs using equations 9 and 12. With circularly-polarized photons, it is possible to measure five SDMEs: ρ^0_{00}, ρ^0_{1-1}, Reρ^0_{10}, Imρ^3_{1-1} and Imρ^3_{10}.

For hadronic decays of vector mesons, we can expand equation 19 in terms of the SDMEs as

$$\begin{aligned} W^L_h(\cos\theta, \phi, \Phi) &= \frac{3}{4\pi}\Bigg[\frac{1}{2}\left(1-\rho^0_{00}\right) + \frac{1}{2}\left(3\rho^0_{00}-1\right)\cos^2\theta - \sqrt{2}\text{Re}\rho^0_{10}\sin 2\theta\cos\phi - \rho^0_{1-1}\sin^2\theta\cos 2\phi \\ &- P_\gamma\cos 2\Phi\left(\rho^1_{11}\sin^2\theta + \rho^1_{00}\cos^2\theta - \sqrt{2}\text{Re}\rho^1_{10}\sin 2\theta\cos\phi - \rho^1_{1-1}\sin^2\theta\cos 2\phi\right) \\ &- P_\gamma\sin 2\Phi\left(\sqrt{2}\text{Im}\rho^2_{10}\sin 2\theta\sin\phi + \text{Im}\rho^2_{1-1}\sin^2\theta\sin 2\phi\right)\Bigg]\,, \end{aligned} \tag{21}$$

where P_γ is the linear polarization of the incident photon and Φ is the angle between the polarization vector of the photon and the (x, z) production plane. With linearly-polarized photons.

We can carry out a similar exercise for the radiative decays of vector mesons produced using linearly-polarized photons. This decay distribution is equation 22.

$$\begin{aligned} W_r^L(\cos\theta,\phi,\Phi) &= \frac{3}{8\pi}\Big[1-\rho^0_{11}\sin^2\theta-\rho^0_{00}\cos^2\theta+\sqrt{2}\mathrm{Re}\rho^0_{10}\sin 2\theta\cos\phi+\rho^0_{1-1}\sin^2\theta\cos 2\phi \\ &- P_\gamma\cos 2\Phi\left(2\rho^1_{11}+\left(\rho^1_{00}-\rho^1_{11}\right)\sin^2\theta+\sqrt{2}\mathrm{Re}\rho^1_{10}\sin 2\theta\cos\phi+\rho^1_{1-1}\sin^2\theta\cos 2\phi\right) \\ &+ P_\gamma\sin 2\Phi\left(\sqrt{2}\mathrm{Im}\rho^2_{10}\sin 2\theta\sin\phi+\mathrm{Im}\rho^2_{1-1}\sin^2\theta\sin 2\phi\right)\Big] \end{aligned} \tag{22}$$

As a reminder, equation 21 and equation 22 differ because the decay amplitudes to hadronic final states are different from those for radiative decays.

2.6 Models Values of SDMEs

The production mechanism of the vector mechanism leads to specific values of the SDMEs. In this section, we summarize a few specific cases, where we look at the SDMEs in the helicity frame. We recall that the eleven measurable SDMEs are ρ^0_{00}, ρ^0_{1-1}, $\mathrm{Re}\rho^0_{10}$, ρ^1_{00}, ρ^1_{11}, ρ^1_{1-1}, $\mathrm{Re}\rho^1_{10}$, $\mathrm{Im}\rho^2_{1-1}$, $\mathrm{Im}\rho^2_{10}$, $\mathrm{Im}\rho^3_{1-1}$ and $\mathrm{Im}\rho^3_{10}$. From Schilling [1], for models in which the t-channel exchange is $J^P=0^\pm$, or for helicity-conserving models, the $\rho^{0,1,2,3}$ are independent of energy and production angle, $\rho^{0,3}$ are diagonal, and $\rho^{1,2}$ are anti-diagonal. Hence, from the eleven measurable SDMEs, six are zero.

$$\begin{aligned} 0 &= \rho^0_{1-1} = \mathrm{Re}\rho^0_{10} \\ 0 &= \rho^1_{00} = \rho^1_{11} \\ 0 &= \mathrm{Im}\rho^3_{1-1} = \mathrm{Im}\rho^3_{10} \end{aligned}$$

For helicity-conserving models, Schilling notes that the SDMEs can be expressed in terms of a single real parameter, a. For $J^P=0^+$ exchange, $a=\frac{1}{2}$, while for $J^P=0^-$ exchange, $a=-\frac{1}{2}$. For a spin-independent model, one has $a=\frac{1}{2}$. In a diffraction model, $a=\frac{1}{2}$ for natural-parity exchange in a helicity conserving model. He gives the following forms for the SDMEs in the helicity system.

$$\begin{aligned} \rho^0 &= \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \\ \rho^1 &= \begin{pmatrix} 0 & 0 & a \\ 0 & 0 & 0 \\ a & 0 & 0 \end{pmatrix} \\ \rho^2 &= \begin{pmatrix} 0 & 0 & -ia \\ 0 & 0 & 0 \\ ia & 0 & 0 \end{pmatrix} \\ \rho^3 &= \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix} \end{aligned}$$

In many models, it is assumed that $\mathrm{Im}\rho^2_{1-1}=-\rho^1_{1-1}$. This is explicit in the above models, but seems to be more general than this. However, as we can see in Section A.2, if they are equal in one frame, after a rotation they are almost certainly not equal in the new frame. Thus, we reiterate that this equality assumption is in the helicity frame.

3 Asymmetries

3.1 A Summary of Asymmetries from Literature

3.1.1 Asymmetries from Counter Experiments

There are several asymmetry variables defined and used in the literature. An early quantity was reported by counter experiments at DESY [5] and Cornell [6]. These used linearly-polarized photons to produce ρ^0 mesons, and had a very-limited acceptance device which detected the the $\pi^+\pi^-$ pair in a plane either parallel to or perpendicular to the direction of the photon's polarization. Over their limited acceptance, they measured a differential cross section in both cases as $\sigma_\parallel$ and $\sigma_\perp$, respectively. They then formed what the called the beam asymmetry, Σ, in terms of those cross sections as equation 23.

$$\Sigma = \frac{\sigma_\parallel - \sigma_\perp}{\sigma_\parallel + \sigma_\perp} \tag{23}$$

In Schilling [1], they show that this quantity can be expressed in terms of the SDMEs as

$$\frac{\sigma_\parallel - \sigma_\perp}{\sigma_\parallel + \sigma_\perp} = \frac{\rho^1_{11} + \rho^1_{1-1}}{\rho^0_{11} + \rho^0_{1-1}} . \tag{24}$$

Both the numerator and the denominator of the right-hand side of this expression are rotationally invariant, see Section A.3, so this asymmetry can be measured in any of the standard coordinate systems. It is important to note that the parallel and perpendicular cross sections have a meaning that may not be obvious. Paraphrasing from reference [7], the quantity $\sigma_\parallel$ represents the pion cross section of the vector meson decay with the pions submerged in the photon polarization plane, while the quantity $\sigma_\perp$ represent the pions perpendicular to it. In terms of equation 21 written as $W^L(\cos\theta,\phi,\Phi)$, these two cross sections are related to $\cos\theta = 0$, $\phi = \frac{\pi}{2}$ and $\Phi = \frac{\pi}{2}, 0$.

In reference [7], the beam asymmetry given by equations 23 and 24 is called $\check{\Sigma}_A$. We will use this notation to refer to this asymmetry in this more. Zhao also define a second related beam asymmetry, $\check{\Sigma}_B$, where $\cos\theta = 0$, $\phi = 0$ and $\Phi = \frac{\pi}{2}, 0$. The two beam asymmetries are given as

$$\check{\Sigma}_A = \frac{\rho^1_{11} + \rho^1_{1-1}}{\rho^0_{11} + \rho^0_{1-1}} \tag{25}$$

$$\check{\Sigma}_B = \frac{\rho^1_{11} - \rho^1_{1-1}}{\rho^0_{11} - \rho^0_{1-1}} . \tag{26}$$

We note here (as shown in Section A.3) that equation 25 is rotationally invariant, but that equation 26 is not. Hence, $\check{\Sigma}_B$ is only defined in the helicity system.

We can derive the above form for $\check{\Sigma}_B$ in equation 23 by evaluating $\sigma_\parallel$ and $\sigma_\perp$. These are defined through the decay distribution, $W^L(\cos\theta,\phi,\Phi)$, from equation 19. As noted above these cross sections are defined at very specific values of the angles.

$$\sigma_\parallel = (P_\gamma + 1)\, W^L(0, \frac{\pi}{2}, \frac{\pi}{2}) + (P_\gamma - 1)\, W^L(0, \frac{\pi}{2}, 0)$$

$$\sigma_\perp = (P_\gamma - 1)\, W^L(0, \frac{\pi}{2}, \frac{\pi}{2}) + (P_\gamma + 1)\, W^L(0, \frac{\pi}{2}, 0)$$

From equation 19 using $\Phi = 0, \frac{\pi}{2}$, we have

$$\begin{aligned} W^L(\cos\theta, \phi, \frac{\pi}{2}) &= W^0(\cos\theta, \phi) + P_\gamma W^1(\cos\theta, \phi) \\ W^L(\cos\theta, \phi, 0) &= W^0(\cos\theta, \phi) - P_\gamma W^1(\cos\theta, \phi) \end{aligned}$$

Now expanding in the form of equation 21 with $\cos\theta = 0$ and $\phi = \frac{\pi}{2}$, and using the definition of $\rho^0_{11} = \frac{1}{2}(1 - \rho^0_{00})$, we get

$$\begin{aligned} W^L(0, \frac{\pi}{2}, \frac{\pi}{2}) &= \frac{3}{4\pi}\left(\rho^0_{11} + \rho^0_{1-1}\right) + P_\gamma \frac{3}{4\pi}\left(\rho^1_{11} + \rho^1_{1-1}\right) \\ W^L(0, \frac{\pi}{2}, 0) &= \frac{3}{4\pi}\left(\rho^0_{11} + \rho^0_{1-1}\right) - P_\gamma \frac{3}{4\pi}\left(\rho^1_{11} + \rho^1_{1-1}\right) . \end{aligned}$$

From these, we can determine the two cross sections, $\sigma_\parallel$ and $\sigma_\perp$ as

$$\begin{aligned} \sigma_\parallel &= \frac{3}{2\pi} P_\gamma \left(\rho^0_{11} + \rho^0_{1-1} + \rho^1_{11} + \rho^1_{1-1}\right) \\ \sigma_\perp &= \frac{3}{2\pi} P_\gamma \left(\rho^0_{11} + \rho^0_{1-1} - \rho^1_{11} - \rho^1_{1-1}\right) . \end{aligned}$$

Now combining these according to equation 23, we get

$$\check{\Sigma}_A = \frac{\rho^1_{11} + \rho^1_{1-1}}{\rho^0_{11} + \rho^0_{1-1}} ,$$

which is what we presented in equation 24.

3.1.2 The Parity Asymmetry

Schilling also discuss the parity asymmetry, P_σ, which is shown to be related to the relative importance of natural and un-natural parity exchange in the photoproduction of vector mesons. The parity asymmetry, P_σ is defined as

$$P_\sigma = \frac{\sigma^N - \sigma^U}{\sigma^N + \sigma^U} , \tag{27}$$

where the σ^N is the cross section due to natural parity exchange, and σ^U is that due to unnatural parity exchange. It can be expressed in terms of the SDMEs as

$$P_\sigma = 2\,\rho^1_{1-1} - \rho^1_{00} , \tag{28}$$

and is rotationally invariant (see Section A.3. The easiest way to obtain P_σ is to measure the SDMEs.

The parity asymmetry, P_σ, allows us to disentangle the relative sizes of the natural and unnatural parity exchanges in the photoproduction of vector mesons. The parity asymmetry is determined by measuring the SDMEs using linearly polarized photons through equation 28. We recall from equation 27 this can be written as

$$P_\sigma = \frac{\sigma^N - \sigma^U}{\sigma^N + \sigma^U} ,$$

where the differential cross section is given as

$$\sigma^{tot} = \sigma^N + \sigma^U .$$

This allows us to write

$$\begin{aligned}\sigma^N &= \frac{1}{2}(1+P_\sigma)\,\sigma^{tot} \\ \sigma^U &= \frac{1}{2}(1-P_\sigma)\,\sigma^{tot} .\end{aligned}$$

3.1.3 The Beam Asymmetry

There is on additional asymmetry defined in the literature [8]. The polarized photon beam asymmetry, Σ^γ_x, defined in that reference as

$$\Sigma^\gamma_x = 2\,\rho^1_{11} + \rho^1_{00} . \tag{29}$$

This is the trace of the ρ^1 SDME, and therefore is rotationally invariant. Zhao [7] talks about the beam asymmetry (equation 29 in reference [8] and calls this $\check{\Sigma}$ (their equation 21)

$$\check{\Sigma} = \vec{\Sigma}\cdot\hat{x}\mathcal{T} .$$

From this they get

$$\Sigma^\gamma_x = \frac{2\,\rho^1_{11} + \rho^1_{00}}{2\,\rho^0_{11} + \rho^0_{00}} , \tag{30}$$

where we note that the denominator of equation 30 is the trace of ρ^0, which is 1. Thus, we get equation 29. It is noted that the derivation of Σ^γ_x in experiment requires integration over the angle ϕ in equation 21. For exclusive natural or unnatural exchange, both ρ^1_{11} and ρ^1_{00} vanish in the helicity frame, so Σ^γ_x also vanishes.

This quantity is reported by several groups for lower-energy ω photoproduction [9, 10, 11] as well as several unpublished CLAS analyses. In this sense is defined in terms of the averaged decay distributions for orthogonal photon beam orientations.

$$\Sigma^\gamma_x = \frac{\bar{W}_\perp(\Phi=90^\circ,\rho) - \bar{W}_\parallel(\Phi=0^\circ,\rho)}{\bar{W}_\perp(\Phi=90^\circ,\rho) + \bar{W}_\parallel(\Phi=0^\circ,\rho)}$$

Rather than actually measuring these averaged decay distributions, a moment analysis is often employed. From reference [7], equation 30 can be written as

$$\Sigma^\gamma_x = \frac{1}{2}\left\langle H \mid \Gamma^4\omega^A \mid H\right\rangle ,$$

which reference [3] expand in terms of moments as

$$\begin{aligned}\Sigma^\gamma_x = \frac{1}{2}\Big[&-\mathrm{Re}H_{1-1}\mathrm{Re}H_{41} - \mathrm{Im}H_{1-1}\mathrm{Im}H_{41} + \mathrm{Re}H_{10}\mathrm{Re}H_{40} + \mathrm{Im}H_{10}\mathrm{Re}H_{40} \\ &-\mathrm{Re}H_{11}\mathrm{Re}H_{4-1} - \mathrm{Im}H_{11}\mathrm{Im}H_{4-1} + \mathrm{Re}H_{2-1}\mathrm{Re}H_{31} + \mathrm{Im}H_{2-1}\mathrm{Re}H_{31} \\ &-\mathrm{Re}H_{20}\mathrm{Re}H_{20} - \mathrm{Im}H_{20}\mathrm{Im}H_{30} + \mathrm{Re}H_{21}\mathrm{Re}H_{3-1} + \mathrm{Im}H_{21}\mathrm{Re}H_{3-1}\Big] ,\end{aligned}$$

which can also be written as

$$\Sigma_x^\gamma \;=\; \mathrm{Re}\Big[H_{1-1}\,H_{41}^* - H_{10}\,H_{40}^* + H_{11}\,H_{4-1}^* - H_{2-1}\,H_{31}^* + H_{20}\,H_{30}^* - H_{21}\,H_{3-1}^*\Big]\,.$$

For GlueX energies, we expect that this observable is close to zero as we are dominated by helicity conserving production mechanisms. For lower energies, where a mix of s and t channel processes contribute, this will likely be quite different from zero.

3.2 Decay Angular Distributions and Asymmetries

3.2.1 Decay Angular Distribution in Models

Wolf in reference [12] looked at ρ^0 meson photoproduction in a model where s-channel helicity conservation was valid up to $|\,t\,|\approx 0.4\,GeV^2$. They then take from reference [13] the assumption that

$$\mathrm{Re}\rho^1_{1-1} \;=\; -\mathrm{Im}\rho^2_{1,-1} \;=\; \frac{1}{2}\,, \tag{31}$$

and that the remaining measurable SDMEs are all 0. Making these assumptions, we can write equation 21 as

$$\begin{aligned} W_h^L(\cos\theta,\phi,\Phi) \;=\; & \frac{3}{4\pi}\left[\frac{1}{2}\sin^2\theta \,+ P_\gamma\,\cos 2\Phi\left(\frac{1}{2}\sin^2\theta\cos 2\phi\right)\right.\\ & \left.+P_\gamma\,\sin 2\Phi\left(\frac{1}{2}\sin^2\theta\sin 2\phi\right)\right] \end{aligned} \tag{32}$$

which we can simplify to

$$W_h^L(\cos\theta,\phi,\Phi) \;=\; \frac{3}{8\pi}\,\sin^2\theta\,\left[1+P_\gamma\left(\cos 2\Phi\cos 2\phi + \sin 2\Phi\sin 2\phi\right)\right]\,.$$

Using standard trigonometric identities, this becomes

$$W_h^L(\cos\theta,\phi,\Phi) \;=\; \frac{3}{8\pi}\,\sin^2\theta\,\left[1+P_\gamma\,\cos 2\left(\Phi-\phi\right)\right]\,,$$

and if we rotate the polarization by 90°, then $\Phi \to \Phi + \frac{\pi}{2}$, and we get

$$W_h^L(\cos\theta,\phi,\Phi) \;=\; \frac{3}{8\pi}\,\sin^2\theta\,\left[1-P_\gamma\,\cos 2\left(\Phi-\phi\right)\right]\,.$$

These assumptions yield a common formula presented in literature that for vector meson photoproduction,

$$W(\cos\theta,\psi) \;\propto\; \sin^2\theta\,(1\pm P_\gamma\,\cos 2\psi)\,. \tag{33}$$

If we were to relax the assumption in equation 31 to have the two SDMEs equal to some asymmetry Σ_ψ multiplied by $\frac{1}{2}$, then equation 32 would become

$$W_h^L(\cos\theta,\phi,\Phi) \;=\; \frac{3}{4\pi}\,\left[\frac{1}{2}\sin^2\theta \,+ P_\gamma\,\cos 2\Phi\left(\Sigma_\psi\,\frac{1}{2}\sin^2\theta\cos 2\phi\right) + P_\gamma\,\sin 2\Phi\left(\Sigma_\psi\,\frac{1}{2}\sin^2\theta\sin 2\phi\right)\right]\,.$$

This simplifies to the form

$$W_h^L(\cos\theta,\phi,\Phi) = \frac{3}{8\pi}\sin^2\theta\,\left[1+\Sigma_\psi\,P_\gamma\,\cos 2\,(\Phi-\phi)\right]\,, \tag{34}$$

which is also seen in the literature, where the claim is that Σ_ψ is said to represent the fraction of the polarization transfered from the initial photon to the vector meson. If the assumption that all SDMEs are zero except ρ^1_{1-1} and $\mathrm{Im}\rho^2_{1,-1}$, is no longer true, and the values of ρ^1_{1-1} and $-\mathrm{Im}\rho^2_{1,-1}$ are not equal, then equation 34 will no longer be true.

We do note that under the assumption on the values of the SDMEs above, the calculated value of ρ^0_{11} would also be $\frac{1}{2}$, and using these, we find that the parity asymmetry, P_σ, from equation 28, the asymmetry, $\check{\Sigma}_B$, from equation 24, and the the polarized photon asymmetry, Σ^γ_x, from equation 29 are

$$\begin{aligned} P_\sigma &= 1 \\ \check{\Sigma}_B &= 1 \\ \Sigma^\gamma_x &= 0\,. \end{aligned}$$

We note that in the case of photoproduction of pseudoscalar mesons, the beam asymmetry, Σ_{ps}, is defined as

$$\Sigma^{ps} = \frac{I^\perp - I^\parallel}{I^\parallel + I^\perp}$$

where the I's represent the intensity for parallel and perpendicular linear polarization. From this, one gets

$$I = I_0\,(1 - P_\gamma \Sigma_{ps}\cos 2\phi)\,.$$

The apparent use of this formulation by experiments when evaluating the photoproduction of vector mesons seems quite suspect.

3.2.2 Decay Distributions Averaged over Angles

Several papers discuss integrating equation 21 over the polar angles, $\cos\theta$ or ϕ [3, 4, 7]. In this section, we look at integrating over $\cos\theta$. To simplify things, we note can break the equation down into four classes of terms: independent of $\cos\theta$, follows $\cos^2\theta$, follows $\sin^2\theta$ and follows $\sin 2\theta$. From these, we have

$$\begin{aligned} 2 &= \int_{-1}^{1} 1\,d\cos\theta \\ 0 &= \int_{-1}^{1} \sin 2\theta\,d\cos\theta \\ \frac{2}{3} &= \int_{-1}^{1} \cos^2\theta\,d\cos\theta \\ \frac{4}{3} &= \int_{-1}^{1} \sin^2\theta\,d\cos\theta\,. \end{aligned}$$

We can use these to integrate equation 21, to obtain equation 35, where we have used the relation that $\rho^0_{11} = \frac{1}{2}(1 - \rho^0_{00})$.

$$\begin{aligned} W^L_h(\cos\theta, \phi, \Phi) &= \frac{3}{4\pi}\left\{(2)\ \rho^0_{00} + \left(\frac{2}{3}\right)\frac{1}{2}\left(3\rho^0_{00} - 1\right) - \left(\frac{4}{3}\right)\ \rho^0_{1-1}\cos 2\phi\right. \\ &- P_\gamma \cos 2\Phi \left[\left(\frac{4}{3}\right)\ \rho^1_{11} + \left(\frac{2}{3}\right)\ \rho^1_{00} - \left(\frac{4}{3}\right)\ \rho^1_{1-1}\cos 2\phi\right] \\ &- \left. P_\gamma \sin 2\Phi \left[\left(\frac{4}{3}\right)\ \mathrm{Im}\rho^2_{1-1}\sin 2\phi\right]\right\} \end{aligned} \tag{35}$$

We can now rewrite equation 35 in a form where we have collected terms with common factors.

$$\begin{aligned} W^L_h(\phi, \Phi) &= \frac{3}{4\pi}\left[\left(2\rho^0_{11} + \rho^0_{00} - \frac{1}{3}\right) - \frac{4}{3}\left(\rho^0_{1-1}\right)\cos 2\phi - \frac{2}{3}P_\gamma\left(2\,\rho^1_{11} + \rho^1_{00}\right)\cos 2\Phi\right. \\ &+ \left.\frac{4}{3}P_\gamma\left(\rho^1_{1-1}\cos 2\Phi\cos 2\phi - \mathrm{Im}\rho^2_{1-1}\sin 2\Phi\sin 2\phi\right)\right] \end{aligned}$$

The first term in line one contains the trace of ρ^0, which allows us to replace the left-most parenthetical term with $\frac{2}{3}$; a factor which can then be taken out to yield a slightly different form.

$$\begin{aligned} W^L_h(\phi, \Phi) &= \frac{1}{2\pi}\left[1 - 2\left(\rho^0_{1-1}\right)\cos 2\phi\ -\ P_\gamma\left(2\,\rho^1_{11} + \rho^1_{00}\right)\cos 2\Phi\right. \\ &+ \left. 2\,P_\gamma\left(\rho^1_{1-1}\cos 2\Phi\cos 2\phi - \mathrm{Im}\rho^2_{1-1}\sin 2\Phi\sin 2\phi\right)\right] \end{aligned}$$

We are now going to rewrite the second line using linear combinations of ρ^1_{1-1} and $\mathrm{Im}\rho^2_{1-1}$ as

$$\begin{aligned} W^L_h(\phi, \Phi) &= \frac{1}{2\pi}\left[1 - 2\left(\rho^0_{1-1}\right)\cos 2\phi\ -\ P_\gamma\left(2\,\rho^1_{11} + \rho^1_{00}\right)\cos 2\Phi\right. \\ &+ 2\,P_\gamma\left(\frac{\rho^1_{1-1} - \mathrm{Im}\rho^2_{1-1}}{2}\right)(\cos 2\Phi\cos 2\phi + \sin 2\Phi\sin 2\phi) \\ &+ \left. 2\,P_\gamma\left(\frac{\rho^1_{1-1} + \mathrm{Im}\rho^2_{1-1}}{2}\right)(\cos 2\Phi\cos 2\phi - \sin 2\Phi\sin 2\phi)\right], \end{aligned}$$

and we will now use trigonometric identities to simplify the last two terms,

$$\begin{aligned} W^L_h(\phi, \Phi) &= \frac{1}{2\pi}\left[1 - 2\left(\rho^0_{1-1}\right)\cos 2\phi\ -\ P_\gamma\left(2\,\rho^1_{11} + \rho^1_{00}\right)\cos 2\Phi\right. \\ &+ \left. P_\gamma\left(\rho^1_{1-1} - \mathrm{Im}\rho^2_{1-1}\right)\cos 2\left(\Phi - \phi\right) + P_\gamma\left(\rho^1_{1-1} + \mathrm{Im}\rho^2_{1-1}\right)\cos 2\left(\Phi + \phi\right)\right]. \end{aligned}$$

Following the procedure in reference [4], we can define the following asymmetries:

$$\Sigma^\phi_h = 2\rho^0_{1-1} \tag{36}$$

$$\Sigma^b_h = 2\rho^1_{11} + \rho^1_{00} \tag{37}$$

$$\Sigma^d_h = -\left(\rho^1_{1-1} - \mathrm{Im}\rho^2_{1-1}\right) \tag{38}$$

$$\Sigma^e_h = -\left(\rho^1_{1-1} + \mathrm{Im}\rho^2_{1-1}\right). \tag{39}$$

Using these, we can write our decay distribution as in equation 40.

$$W_h^L(\phi,\Phi) \;=\; \frac{1}{2\pi}\left[1-\Sigma_h^{\phi}\cos 2\phi - P_\gamma \Sigma_h^b \cos 2\Phi - P_\gamma \Sigma_h^d \cos 2\left(\Phi-\phi\right) - P_\gamma \Sigma_h^e \cos 2\left(\Phi+\phi\right)\right] \quad (40)$$

The term Σ_h^b is the same as Σ_x^γ in equation 29, defined in reference [8]. Because this is the trace of ρ^1, it is rotationally invariant. Now let us look at the Σ_h^b term in equation 40. If we average the equation over the angle ϕ, then the terms proportional to $\cos 2\phi$ and $\cos 2\left(\Phi-\phi\right)$ average to zero. Thus, we have

$$W_h^L(\Phi) \;=\; \frac{1}{2\pi}\left[1-P_\gamma\,\Sigma_h^b\,\cos 2\Phi\right] .$$

Thus, this beam asymmetry, Σ_x^γ or Σ_h^b, can be obtained by looking at the differential cross section for two orthogonal photon beam polarizations.

For the Σ_h^ϕ term, this is not rotationally invariant, so it is only defined in the helicity frame. The Σ^ϕ term would survive if we averaged over the photon polarization angle, Φ, so it can be determined using unpolarized photons, which is also clear from the fact that it is related to an element of ρ^0.

The term Σ_h^d may be thought of as a decay asymmetry, while the term Σ_h^e is an exchange term that is generally not present in literature. Rather, in many models, and most results reporting asymmetries with linearly-polarized photons, the is an assumption that $\mathrm{Im}\rho^2_{1-1} = -\rho^1_{1-1}$. Making this assumption, we get

$$\begin{aligned}\Sigma_h^d &= -2\,\rho^1_{1-1}\\ \Sigma_h^e &= 0\,,\end{aligned}$$

and the expression for W_h^L simplifies to

$$W_h^L(\phi,\Phi) \;=\; \frac{1}{2\pi}\left[1-\Sigma_h^{\phi}\cos 2\phi - P_\gamma \Sigma_h^b \cos 2\Phi - P_\gamma \Sigma_h^d \cos 2\left(\Phi-\phi\right)\right]$$

In this form, Σ_h^d can be associated with the asymmetry that we looked at in Section 3.2.1. However, to use this, we really need to have both Σ_h^ϕ and Σ_h^b zero. This yields

$$W_h^L(\phi,\Phi) \;=\; \frac{1}{2\pi}\left[1-P_\gamma\,\Sigma_h^d\,\cos 2\left(\Phi-\phi\right)\right] .$$

However, based on the more general work that we did above, we should really be fitting the decay distribution as

$$W_h^L(\phi,\Phi) \;=\; \frac{1}{2\pi}\left[\,1\;-\;P_\gamma\,\Sigma_h^d\,\cos 2\left(\Phi-\phi\right)-\;P_\gamma\,\Sigma_h^e\,\cos 2\left(\Phi+\phi\right)\right] . \quad (41)$$

There are really two angular terms to be fit, with one going by the normal $\cos 2\left(\Phi-\phi\right)$ and the other by the new term, $\cos 2\left(\Phi+\phi\right)$.

We can carry out a similar analysis for the radiative decays, where W_r^L is given by equation 22. Integrating over the angle θ as before, we get

$$\begin{aligned}W_r^L(\phi,\Phi) \;=\; \frac{3}{8\pi}\Bigg\{&(2)\,1-\left(\frac{4}{3}\right)\rho^0_{11}-\left(\frac{2}{3}\right)\rho^0_{00}+\left(\frac{4}{3}\right)\,\rho^0_{1-1}\cos 2\phi\\ -\;& P_\gamma\cos 2\Phi\left[(2)\;2\rho^1_{11}+\left(\frac{4}{3}\right)\,\left(\rho^1_{00}-\rho^1_{11}\right)+\left(\frac{4}{3}\right)\,\rho^1_{1-1}\cos 2\phi\right]\\ +\;& P_\gamma\sin 2\Phi\left[\left(\frac{4}{3}\right)\,\mathrm{Im}\rho^2_{1-1}\sin 2\phi\right]\Bigg\} .\end{aligned}$$

This can be simplified as before to yield

$$\begin{aligned}W_r^L(\phi,\Phi) &= \frac{1}{2\pi}\Big[1+\left(\rho^0_{1-1}\right)\cos 2\phi - P_\gamma\left(2\rho^1_{11}+\rho^1_{00}\right)\cos 2\Phi \\ &\quad -P_\gamma\left(\rho^1_{1-1}\cos 2\Phi\cos 2\phi - \mathrm{Im}\rho^2_{1-1}\sin 2\Phi\sin 2\phi\right)\Big]\,.\end{aligned}$$

Following the procedure for the hadronic decays, this can be written as

$$\begin{aligned}W_r^L(\phi,\Phi) &= \frac{1}{2\pi}\Big[1+\left(\rho^0_{1-1}\right)\cos 2\phi - P_\gamma\left(2\rho^1_{11}+\rho^1_{00}\right)\cos 2\Phi \\ &- P_\gamma\left(\frac{\rho^1_{1-1}-\mathrm{Im}\rho^2_{1-1}}{2}\right)\cos 2\left(\Phi-\phi\right) - P_\gamma\left(\frac{\rho^1_{1-1}+\mathrm{Im}\rho^2_{1-1}}{2}\right)\cos 2\left(\Phi+\phi\right)\Big]\,.\end{aligned}$$

In order to put this in the form of equation 40, we define

$$\begin{aligned}\Sigma^\phi_r &= -\rho^0_{1-1}\\ \Sigma^b_r &= 2\rho^1_{11}+\rho^1_{00}\\ \Sigma^d_r &= \frac{1}{2}\left(\rho^1_{1-1}-\mathrm{Im}\rho^2_{1-1}\right)\\ \Sigma^e_r &= \frac{1}{2}\left(\rho^1_{1-1}+\mathrm{Im}\rho^2_{1-1}\right)\end{aligned}$$

This then gives us equation 42, which is the same as equation 40.

$$W_r^L(\phi,\Phi) = \frac{1}{2\pi}\Big[1-\Sigma^\phi_r\cos 2\phi - P_\gamma\Sigma^b_r\cos 2\Phi - P_\gamma\Sigma^d_r\cos 2\left(\Phi-\phi\right) - P_\gamma\Sigma^e_r\cos 2\left(\Phi+\phi\right)\Big] \quad (42)$$

We see that the radiative asymmetries can be related to those from the hadronic decays as

$$\begin{aligned}\Sigma^\phi_r &= -\frac{1}{2}\,\Sigma^\phi_h\\ \Sigma^b_r &= \Sigma^b_h\\ \Sigma^d_r &= -\frac{1}{2}\,\Sigma^d_h\\ \Sigma^e_r &= -\frac{1}{2}\,\Sigma^e_h\,.\end{aligned}$$

One interesting conclusion from this analysis is that for the radiative decay, the two oscillatory terms in

$$W_r^L(\phi,\Phi) = \frac{1}{2\pi}\Big[1-P_\gamma\,\Sigma^d_r\,\cos 2\left(\Phi-\phi\right) - P_\gamma\,\Sigma^e_r\,\cos 2\left(\Phi+\phi\right)\Big]$$

will have the opposite phase and $\frac{1}{2}$ the amplitude as the same terms from the hadronic decay.

4 Monitoring Beam Polarization in GlueX

In order to use vector meson photoproduction, and in particular ρ photoproduction, to monitor the beam polarization in GlueX, we need to first determine the SDMEs. The only existing measurement of these comes from reference [14], where measurements were with 9.3 GeV linearly polarized

$\lvert t\rvert$	0.02–0.05	0.05–0.08	0.08–0.12	0.12–0.18	0.18–0.25	0.25–0.40	0.40–0.80
ρ^0_{00}	$.02 \pm .01$	$-.06 \pm .02$	$-.01 \pm .02$	$.03 \pm .02$	$-.02 \pm .02$	$.00 \pm .03$	$.03 \pm .05$
ρ^0_{1-1}	$-.02 \pm .03$	$.06 \pm .03$	$-.06 \pm .03$	$.01 \pm .03$	$-.08 \pm .03$	$-.10 \pm .03$	$-.16 \pm .04$
$\mathrm{Re}\rho^0_{10}$	$.00 \pm .02$	$.04 \pm .02$	$.04 \pm .02$	$.03 \pm .02$	$.07 \pm .02$	$.06 \pm .02$	$.10 \pm .03$
ρ^1_{00}	$.03 \pm .02$	$-.05 \pm .03$	$.03 \pm .05$	$-.05 \pm .04$	$.01 \pm .04$	$-.05 \pm .05$	$-.05 \pm .08$
ρ^1_{11}	$-.06 \pm .04$	$.06 \pm .04$	$-.05 \pm .05$	$-.02 \pm .04$	$.02 \pm .05$	$.05 \pm .05$	$-.06 \pm .06$
ρ^1_{1-1}	$.48 \pm .04$	$.38 \pm .05$	$.48 \pm .05$	$.48 \pm .05$	$.49 \pm .05$	$.41 \pm .06$	$.57 \pm .07$
$\mathrm{Re}\rho^1_{10}$	$.03 \pm .03$	$.03 \pm .03$	$0.05 \pm .03$	$-.01 \pm .03$	$-.13 \pm .03$	$-.02 \pm .03$	$-.15 \pm .04$
$\mathrm{Im}\rho^2_{1-1}$	$-.50 \pm .04$	$-.42 \pm .05$	$-.57 \pm .04$	$-.48 \pm .04$	$-.49 \pm .05$	$-.42 \pm .06$	$-.64 \pm .10$
$\mathrm{Im}\rho^2_{10}$	$-.05 \pm .03$	$.02 \pm .03$	$.03 \pm .02$	$.03 \pm .03$	$.08 \pm .03$	$.03 \pm .03$	$.06 \pm .05$

Table 1: SDMEs for $9.3\,GeV$ photons from reference [14] for the reaction$\gamma p \rightarrow p\pi^+\pi^-$. The units of the momentum transfer, $|t|$ are $(\mathrm{GeV/c})^2$.

photons. The extracted SDMEs as measured in the helicity frame are given in Table 1. Within a couple of sigma, nearly all of these measured values are consistent with zero, except ρ^1_{1-1} and $\mathrm{Im}\rho^2_{1-1}$; both of which have magnitudes that are within errors of being $\frac{1}{2}$. Unfortunately, the errors are close to 10%, and there are a few anomalies as a function of $|t|$.

With this information, we can use equation 40 to monitor the degree of polarization. We see that ρ^0_{1-1} is probably consistent with zero for $|t| < 0.20$, and is small out to $|t| = 0.8$. Thus, Σ^ϕ_h is close to zero. Both ρ^1_{00} and ρ^1_{11} are consistent with zero over the measured range of $|t|$, but the errors are fairly large. This suggests that Σ^b_h is also zero. Finally, ρ^1_{1-1} is very close to $\frac{1}{2}$ and $\mathrm{Im}\rho^2_{1-1}$ is very close to $-\frac{1}{2}$ over the measured range of $|t|$. Thus, Σ^d_h is close to 1 and Σ^e_h is close to 0.

We also note that there are some physical constraints that are not satisfied by some of the measurements. In the helicity frame, the ρ^0_{00} term is related to the spin-flip amplitude and is constrained as $0 \leq \rho^0_{00} \leq 0$. Both ρ^1_{1-1} and $\mathrm{Im}\rho^2_{1-1}$ are constrained to be smaller than $\frac{1}{2}$ by

$$
\begin{aligned}
|\,\rho^0_{1-1}\,| &\leq \frac{1}{2}\left(1-\rho^0_{00}\right) \\
|\,\rho^1_{1-1}\,| &\leq \frac{1}{2}\left(1-\rho^0_{00}\right) \\
|\,\rho^2_{1-1}\,| &\leq \frac{1}{2}\left(1-\rho^0_{00}\right)\,.
\end{aligned}
$$

5 Summary

Photoproduction of the ρ meson will provide a good tool to monitor the linear polarization of the GlueX photon beam. However, in order to be able to utilize it, it will be necessary to accurately measure the SDMEs as a function of $|t|$, and from this, identify regions where they are close to optimal for s-channel helicity conservation.

A Rotating SDMEs

A.1 Rotation Angles

In many instances, we need to rotate some or all of the SDMEs between the three systems described in Section 2.4: helicity, Adair and Gottfried-Jackson. These rotations involve equation 17. The rotation angle, α, can be determined as follows for the rotations from helicity to the other two systems. In the following, θ_v is the angle of the vector meson in the center-of-mass frame and β is the relativistic velocity of the vector meson in the center-of-mass frame.

- For helicity to Adair, we have $\alpha = \theta_v$.
- For Adair to helicity, $\alpha = -\theta_v$.
- For helicity to Gottfried-Jackson, $\cos\alpha = (\beta - \cos\theta_v) / (\beta\cos\theta_v - 1)$.

In the center-of-mass frame, we can evaluate β using simle relativistic kinematics. Assuming a photon of energy E_γ interacts with a proton at rest to produce a meson of mass m_v recoiling against a proton. For this reaction,

$$s \;=\; m_p^2 + 2m_p E_\gamma\,.$$

In the center-of-mass frame, the meson and proton are emitted back-to-pack with equal and opposite momentum. The momentum, p, in the center-of-mass frame is

$$p \;=\; \frac{\sqrt{\left[s-(m_v+m_p)^2\right]\,\left[s-(m_v-m_p)^2\right]}}{2\sqrt{s}}\,.$$

We can evaluate β as this momentum, p, divided by the energy of the vector meson, $sqrtm_v^2 + p^2$.

A.2 Rotation of Spin-Density Matrix Elements

Using equation 17 and then using the symmetry properties of ρ^0 and ρ^1, it is easy to show that the result of a rotation from frame A to frame B results in the following two transformations. In the following, we have explicitly use ρ^0_{11}, where we use the trace of $\rho^0 = 1$ to yield equation 43

$$\rho^0_{11} \;=\; \frac{1}{2}\left(1-\rho^0_{00}\right)\,, \tag{43}$$

and from this we get the following rotation formulas.

$$\begin{aligned}\left(\rho^{0,1}_{11}\right)_B \;=&\; \left[\left(d^1_{11}\right)^2+\left(d^1_{1-1}\right)^2\right]\left(\rho^{0,1}_{11}\right)_A+\left[2\left(d^1_{11}\right)\left(d^1_{1-1}\right)\right]\left(\rho^{0,1}_{1-1}\right)_A\\ &+\left[\left(d^1_{10}\right)^2\right]\left(\rho^{0,1}_{00}\right)_A+2d^1_{10}\left[d^1_{1-1}-d^1_{11}\right]\,\mathrm{Re}\left(\rho^{0,1}_{10}\right)_A\end{aligned} \tag{44}$$

$$\begin{aligned}\left(\rho^{0,1}_{1-1}\right)_B \;=&\; \left[2\left(d^1_{11}\right)\left(d^1_{1-1}\right)\right]\left(\rho^{0,1}_{11}\right)_A+\left[\left(d^1_{11}\right)^2+\left(d^1_{1-1}\right)^2\right]\left(\rho^{0,1}_{1-1}\right)_A\\ &-\left[\left(d^1_{10}\right)^2\right]\left(\rho^{0,1}_{00}\right)_A-2d^1_{10}\left[d^1_{1-1}-d^1_{11}\right]\,\mathrm{Re}\left(\rho^{0,1}_{10}\right)_A\end{aligned} \tag{45}$$

We could in principle continue this for the remaining measurable SDMEs, but it is more useful at this point to utilize the trigonometric expansion of of the little d functions.

$$
\begin{aligned}
d^1_{11}(\alpha) &= d^1_{-1-1}(\alpha) = \frac{1+\cos\alpha}{2} \\
d^1_{1-1}(\alpha) &= d^1_{-11}(\alpha) = \frac{1-\cos\alpha}{2} \\
d^1_{10}(\alpha) &= d^1_{0-1}(\alpha) = -\frac{\sin\alpha}{\sqrt{2}} \\
d^1_{01}(\alpha) &= d^1_{-10}(\alpha) = \frac{\sin\alpha}{\sqrt{2}} \\
d^1_{00}(\alpha) &= \cos\alpha\,,
\end{aligned}
$$

For the three measurable ρ^0 SDMEs, we find the following relations when rotating by an angle α.

$$
\begin{aligned}
\left(\rho^0_{00}\right)_B &= \cos^2\alpha\,\left(\rho^0_{00}\right)_A + \left(1-\cos^2\alpha\right)\left[\left(\rho^0_{11}\right)_A - \left(\rho^0_{1-1}\right)_A\right] \\
&\quad -2\sqrt{2}\,\sin\alpha\cos\alpha\,\mathrm{Re}\left(\rho^0_{10}\right)_A \\
\left(\rho^0_{1-1}\right)_B &= \frac{1}{2}\left(1+\cos^2\alpha\right)\left(\rho^0_{1-1}\right)_A + \frac{1}{2}\left(1-\cos^2\alpha\right)\left[\left(\rho^0_{11}\right)_A - \left(\rho^0_{00}\right)_A\right] \\
&\quad -\sqrt{2}\sin\alpha\cos\alpha\,\mathrm{Re}\left(\rho^0_{10}\right)_A \\
\mathrm{Re}\left(\rho^0_{10}\right)_B &= \frac{\sin\alpha\cos\alpha}{\sqrt{2}}\left[\left(\rho^0_{00}\right)_A + \left(\rho^0_{1-1}\right)_A - \left(\rho^0_{11}\right)_A\right] + \left(\cos^2\alpha - \sin^2\alpha\right)\,\mathrm{Re}\left(\rho^0_{10}\right)_A
\end{aligned}
$$

If we use equation 43, we have the rotation for ρ^0_{00} written as

$$
\begin{aligned}
\left(\rho^0_{00}\right)_B &= \frac{1}{2}\sin^2\alpha + \frac{1}{2}\left(3\cos^2\alpha\;-1\right)\left(\rho^0_{00}\right)_A - \sin^2\alpha\,\left(\rho^0_{1-1}\right)_A \\
&\quad -2\sqrt{2}\,\sin\alpha\cos\alpha\,\mathrm{Re}\left(\rho^0_{10}\right)_A\;.
\end{aligned}
$$

The rotations of ρ^1 go as:

$$
\begin{aligned}
\left(\rho^1_{00}\right)_B &= \cos^2\alpha\,\left(\rho^1_{00}\right)_A + \left(1-\cos^2\alpha\right)\left[\left(\rho^1_{11}\right)_A - \left(\rho^1_{1-1}\right)_A\right] \\
&\quad -2\sqrt{2}\,\sin\alpha\cos\alpha\,\mathrm{Re}\left(\rho^1_{10}\right)_A \\
\left(\rho^1_{11}\right)_B &= \frac{1}{2}\left(1+\cos^2\alpha\right)\left(\rho^1_{11}\right)_A + \frac{1}{2}\left(1-\cos^2\alpha\right)\left[\left(\rho^1_{1-1}\right)_A + \left(\rho^1_{00}\right)_A\right] \\
&\quad +\sqrt{2}\sin\alpha\cos\alpha\,\mathrm{Re}\left(\rho^1_{10}\right)_A \\
\left(\rho^1_{1-1}\right)_B &= \frac{1}{2}\left(1+\cos^2\alpha\right)\left(\rho^1_{1-1}\right)_A + \frac{1}{2}\left(1-\cos^2\alpha\right)\left[\left(\rho^1_{11}\right)_A - \left(\rho^1_{00}\right)_A\right] \\
&\quad -\sqrt{2}\sin\alpha\cos\alpha\,\mathrm{Re}\left(\rho^1_{10}\right)_A \\
\mathrm{Re}\left(\rho^1_{10}\right)_B &= \frac{\sin\alpha\cos\alpha}{\sqrt{2}}\left[\left(\rho^1_{00}\right)_A + \left(\rho^1_{1-1}\right)_A - \left(\rho^1_{11}\right)_A\right] + \left(\cos^2\alpha - \sin^2\alpha\right)\,\mathrm{Re}\left(\rho^1_{10}\right)_A
\end{aligned}
$$

For completeness, we note that the imaginary (not observed) part of $\rho_{01}^{0,1}$ transforms as:

$$\mathrm{Im}\left(\rho_{01}^{0,1}\right)_B = \mathrm{Im}\left(\rho_{01}^{0,1}\right)_A$$

For the ρ^2 elements, we have

$$\begin{aligned}
\mathrm{Im}\left(\rho_{1-1}^{2}\right)_B &= \cos\alpha\,\mathrm{Im}\left(\rho_{1-1}^{2}\right)_A - \sqrt{2}\sin\alpha\,\mathrm{Im}\left(\rho_{10}^{2}\right)_A \\
\mathrm{Im}\left(\rho_{10}^{2}\right)_B &= \cos\alpha\,\mathrm{Im}\left(\rho_{10}^{2}\right)_A + \frac{\sin\alpha}{\sqrt{2}}\,\mathrm{Im}\left(\rho_{1-1}^{2}\right)_A
\end{aligned}$$

For the ρ^3 elements, we have

$$\begin{aligned}
\mathrm{Im}\left(\rho_{1-1}^{3}\right)_B &= \cos\alpha\,\mathrm{Im}\left(\rho_{1-1}^{3}\right)_A - \sqrt{2}\sin\alpha\,\mathrm{Im}\left(\rho_{10}^{3}\right)_A \\
\mathrm{Im}\left(\rho_{10}^{3}\right)_B &= \cos\alpha\,\mathrm{Im}\left(\rho_{10}^{3}\right)_A + \frac{\sin\alpha}{\sqrt{2}}\,\mathrm{Im}\left(\rho_{1-1}^{3}\right)_A
\end{aligned}$$

For completeness, we note that the real (not observed) part of $\rho_{01}^{2,3}$ transforms as:

$$\mathrm{Re}\left(\rho_{01}^{2,3}\right)_B = \cos\alpha\,\mathrm{Re}\left(\rho_{10}^{2,3}\right)_A - \frac{\sin\alpha}{\sqrt{2}}\left(\rho_{11}^{2,3}\right)_A$$

For all of these rotations, we note that in the case of forward-going vector mesons, $\alpha \approx 0^\circ$, that the rotation does not change the value of any of the SDMEs. The values should be the same in all three reference systems.

If we now apply these rotations to the specific examples in Section 2.6 which present the SDMEs in the helicity system, then rotating, we get the following forms for the SDMEs.

$$\rho^0 = \begin{pmatrix} \frac{1}{4}\left(1+\cos^2\alpha\right) & 0 & 0 \\ 0 & \frac{1}{2}\sin^2\alpha & 0 \\ 0 & 0 & \frac{1}{4}\left(1+\cos^2\alpha\right) \end{pmatrix}$$

$$\rho^1 = \begin{pmatrix} 0 & 0 & \frac{1}{2}\left(1+\cos^2\alpha\right)\,a \\ 0 & 0 & 0 \\ \frac{1}{2}\left(1+\cos^2\alpha\right)\,a & 0 & 0 \end{pmatrix}$$

$$\rho^2 = \begin{pmatrix} 0 & 0 & -i\cos\alpha\,a \\ 0 & 0 & 0 \\ i\cos\alpha\,a & 0 & 0 \end{pmatrix}$$

For forward-going vetor mesons, $\alpha \approx 0^\circ$, and the rotations do not do anything. The SDMEs should be the same in all three systems.

A.3 Rotation Invariance

While from equation 44 and 45, neither $\rho_{11}^{0,1}$ nor $\rho_{1-1}^{0,1}$ are rotationally invariant, their sum (equation 23) is. Performing this sum yields

$$\left(\rho_{11}^{0,1}\right)_B + \left(\rho_{1-1}^{0,1}\right)_B = \left(\rho_{11}^{0,1}\right)_A + \left(\rho_{1-1}^{0,1}\right)_A , \tag{46}$$

hence the asymmetry Σ given by equation 23 and 24 is rotationally invariant. Thus, it does not matter which of the three systems (helicity, Gottfried-Jackson or Adair) in which we evaluate Σ. Using the fact that the trace is also rotationally invariant, we have

$$2\left(\rho_{11}^{0,1}\right)_B + \left(\rho_{00}^{0,1}\right)_B = 2\left(\rho_{11}^{0,1}\right)_A + \left(\rho_{00}^{0,1}\right)_A . \tag{47}$$

This means that the quantity Σ_x^γ, given by equation 29 is also rotationally invariant. Taking two times equation 46 and subtracting equation 47, we get

$$2\left(\rho_{1-1}^{0,1}\right)_B - \left(\rho_{00}^{0,1}\right)_B = 2\left(\rho_{1-1}^{0,1}\right)_A - \left(\rho_{00}^{0,1}\right)_A , \tag{48}$$

from which we see that P_σ, as defined in equation 28 is also rotationally invariant.

For the quantity $\check{\Sigma}_B$ defined in equation 26, the difference of equations 44 and 45 yields

$$\left(\rho_{11}^{0,1}\right)_B - \left(\rho_{1-1}^{0,1}\right)_B = \cos^2\alpha\left[\left(\rho_{11}^{0,1}\right)_A - \left(\rho_{1-1}^{0,1}\right)_A\right] - \sin^2\alpha\left(\rho_{00}^{0,1}\right)_A , \tag{49}$$

which is not rotationally invariant. Equation 26 is only valid in the helicity frame.

B Existing Photoproduction Data

In this note, we have looked at existing data on the photoproduction of ρ mesons using linearly polarized photons. For measurements utilizing coherent bremsstrahlung on a diamond radiator, the polarization of the photon was calculated following the procedure in reference [5]. Similarly, the laser back-scattering experiments also computed the linear polarization of the incident photon based on its energy. The older experiments included counter measurements at Cornell (Section B.1) from reference [6] and DESY (Section B.2) from reference [15]. These experiments detected the three particles in the final state of

$$\gamma p \quad \rightarrow \quad p\pi^+\pi^- \tag{50}$$

using stacks of detectors where the two pions were detected in a plane on either side of the beam direction. Bubble chamber experiments using linearly polarized photons from laser back-scattering were carried out at SLAC (Section B.3). These are described in references [13, 14, 16]. The formalism for extracting the asymmetry is described in reference [1].

The results from these experiments are summarized in Table 2, where the asymmetry, $\check{\Sigma}_A$, is for $|t| < 0.4\,(GeV/c)^2$. A visual representation of these results is given in Figure 1 which has been taken from reference [15]. In Table 3 are summarized the beam asymmetry results from references [14, 16]. The reported values of $\check{\Sigma}_B$ are computed from the spin-density matrix elements, ρ^0 and ρ^1, reported in the references and presented in the helicity system. The numbers from reference [17] in Table 2 are computed from published values of the SDMEs. The number from reference [18] is computed from the ratio of the calculated to the expected linear polarization of the photon.

E_γ	$\check{\Sigma}_A$	Reference
1.6 GeV	0.96 ± 0.07	[15]
1.95 GeV	0.90 ± 0.07	[15]
2.25 GeV	0.89 ± 0.07	[15]
2.8 GeV	0.787	[16]
3.05 GeV	0.75 ± 0.13	[15]
3.5 GeV	0.87 ± 0.06	[6]
4.7 GeV	1.032	[16]
9.3 GeV	0.979	[14]
20.0 GeV	0.68 ± 0.28	[17]
20.0 GeV	0.94 ± 0.05	[18]

Table 2: A summary of measurements of beam asymmetry, $\check{\Sigma}_A$, for the photoproduction of the ρ mesons on hydrogen. These data are typically for $|t|$ smaller than 0.4 GeV/c. The values at 2.8, 4.7 and 9.3 are taken as averages of the values in Table 3. The values at 20 GeV are computed from other numbers given in the papers.

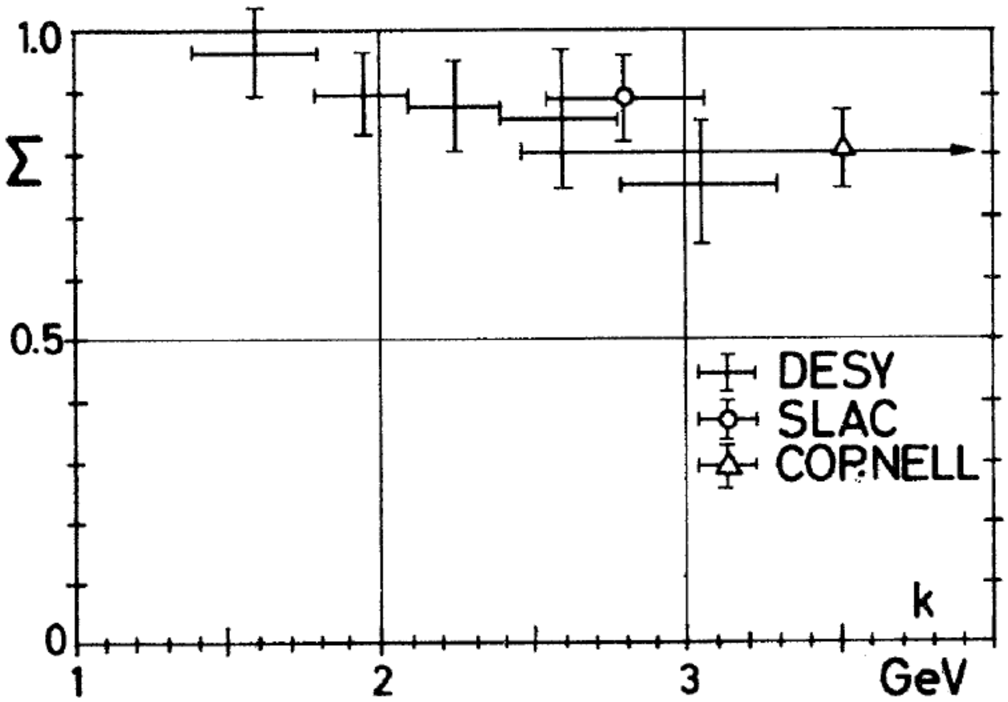

Figure 1: A summary of measurements of $\check{\Sigma}_A$ versus photon beam energy for $|t| < 0.4\,\mathrm{GeV/c}$. The figure is taken from reference [15]. The DESY reference refers to reference [15], the Cornell reference refers to reference [6] and the SLAC reference refers to [13].

$\lvert t\rvert\,(GeV/c)^2$	$\check{\Sigma}_A$		
	$E_\gamma = 2.8\,GeV$	$E_\gamma = 4.5\,GeV$	$E_\gamma = 9.3\,GeV$
0.02-0.05	0.968 ± 0.124	1.124 ± 0.132	0.894 ± 0.134
0.05-0.08	0.835 ± 0.148	0.847 ± 0.169	0.746 ± 0.116
0.08-0.12	0.967 ± 0.158	1.129 ± 0.142	0.966 ± 0.173
0.12-0.18	0.855 ± 0.181	1.012 ± 0.143	0.929 ± 0.142
0.18-0.25	1.075 ± 0.229	0.925 ± 0.164	1.186 ± 0.186
0.25-0.40	0.571 ± 0.172	1.156 ± 0.208	1.150 ± 0.218
0.40-0.80	1.148 ± 0.418	1.236 ± 0.330	1.569 ± 0.364

Table 3: The beam asymmetry, $\check{\Sigma}_A$, computed from the SDME's reported in reference [16] and [14]. The asymmetry is computed using equation 24.

B.1 Cornell Experiment

Measured photoproduction of ρ mesons at $0°$ using 3.5 GeV photons on hydrogen and carbon targets [6]. Photons produced using coherent bremsstrahlung on a diamond crystal using 9.7 GeV electrons. Photon beam is collimated and energy spectrum is measured using e^+e^- pairs. The plane of polarization being vertical, and thus perpendicular to the horizontal plane of detection of the $\pi^+\pi^-$ pair. The parallel configuration has the polarization in the horizontal plane. The define the beam asymmetry as

$$\check{\Sigma}_A = \frac{d^2\sigma_{\|} - d^2\sigma_{\perp}}{d^2\sigma_{\|} + d^2\sigma_{\perp}}$$

where the $\sigma_{\|}$ has the decay plane of the ρ parallel to the polarization and $\sigma_{\perp}$ has the decay plane perpendicular to the polarization of the photon.

This experiment has a single measurement of the asymmetry on hydrogen of $\check{\Sigma}_A = 0.87 \pm 0.06$. The polarization appears to come from a calculation, and is used to extract the beam asymmetry for ρ mesons produced on a carbon target. Figure 2(a) shows both the beam asymmetry, Σ, from the carbon target. Figure 2b shows the counting rate asymmetry, $\bar{P}_H\Sigma$ for carbon. Assuming that $\check{\Sigma}_A$ is one, as from the upper figure, this plot is a measure of the polarization. The solid line on the polarization is from the calculation for polarization.

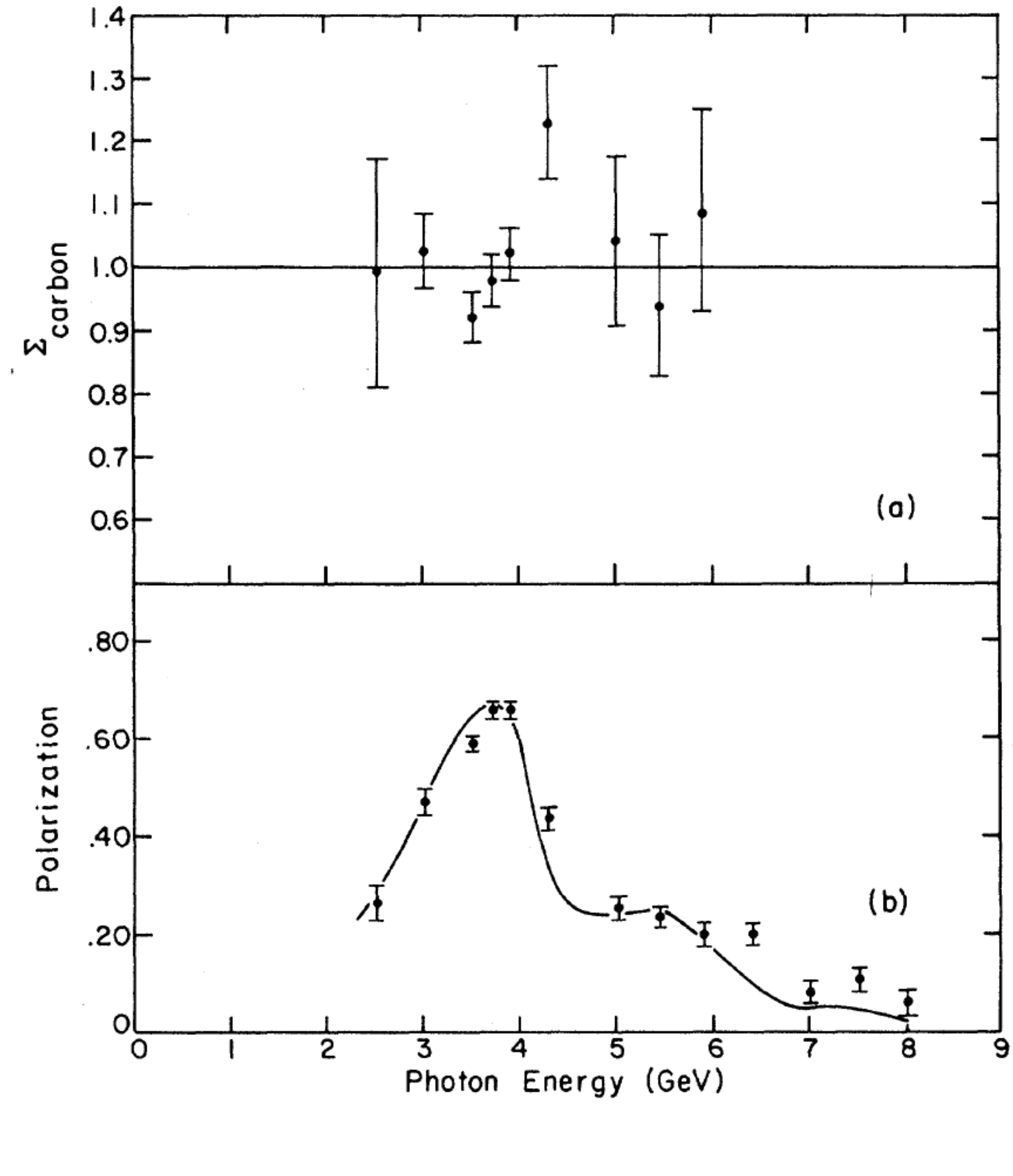

Figure 2: (a) The beam asymmetry, $\check{\Sigma}_A$, obtained for ρ production on carbon using the calculated polarization. (b) The counting rate asymmetry for carbon, $P_\gamma \check{\Sigma}_A$ for the horizontal orientation of the polarization. This is Figure 3 from reference [6].

B.2 DESY Experiment

Measured data for E_γ in the range of 1.4 to 3.3 GeV. Collimated photon beam produced using coherent bremsstrahlung on a diamond target. Photons incident on a liquid hydrogen target [15].

$$\gamma p \quad \rightarrow \quad p\pi^+\pi^-$$

The final-state p, π^+ and π^- were all detected in counter wire-plane telscopes with the pion detectors symmetric above and below the horizontal plane defined by the beam direction and the proton telescope.

Beam asymmetry ratio, Σ, defined as

$$\begin{aligned}\check{\Sigma}_A &= \frac{1}{P_\gamma}\frac{Y_\parallel - Y_\perp}{Y_\parallel + Y_\perp} \\ \check{\Sigma}_A &= \frac{\sigma_\parallel - \sigma_\perp}{\sigma_\parallel + \sigma_\perp}\end{aligned}$$

The cross sections, $\sigma_\parallel$ and $\sigma_\perp$ are the cross sections for parallel to the normal to the floor and perpendicular to the normal to the floor. The variables Y are the counting rates.

$$W^\pm(\theta,\psi) \quad \sim \quad \sin^2\theta\ (1 \pm P_\gamma\ \cos 2\psi)\ ,$$

$$Y \quad \sim \quad \sigma_\parallel\, W^+(\theta,\psi) + \sigma_\perp\, W^-(\theta,\psi)\,.$$

The angle θ is the polar angle, which is $\theta \approx \frac{\pi}{2}$ in this experiment. The azimuthal angle ψ is the angle of one of the pions with respect to the beam polarization. The experiment restricted this as

$$\begin{aligned}\psi_\parallel &= 0\ \pm\ 0.4\,\mathrm{rad} \\ \psi_\perp &= \frac{\pi}{2}\ \pm\ 0.4\,\mathrm{rad}\,.\end{aligned}$$

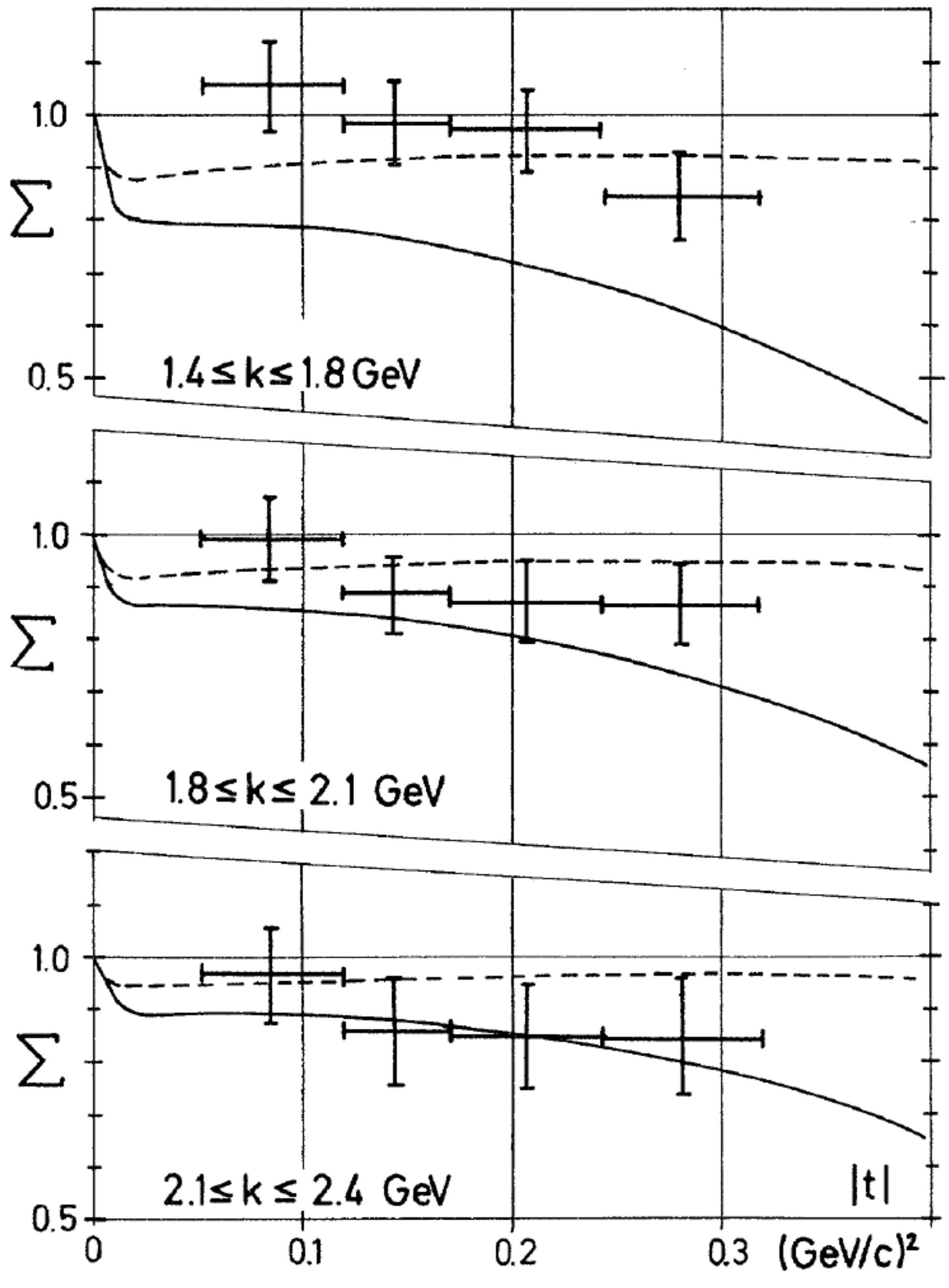

Figure 3: The beam asymmetry, $\check{\Sigma}_B$, as a function of t for three photon energies from reference [15]. This is Figure 3 from reference [15].

B.3 SLAC Data

The SLAC publications [13, 16, 14] present initial measurements of the spin density matrix elements (SDMEs) extracted using the Shilling Method [1]. Photons are generated via Compton backscattering at 2.8 GeV, 4.7 GeV and 9.3 GeV. The polarization is computed and found to be 94% and 92% at the two beam energies, respectively. They find that ρ^0 photoproduction is dominated by natural parity exchange, and that for $\mid t \mid< 0.4$GeV/c, that it conserves s-channel helicity conservation. In Figure 4 are reported values for both P_σ and $\check{\Sigma}_A$ taken from reference [13]. Figure 5 shows the nine measured SDMEs evaluated in each of the three systems.

Reference [16] provides numerical values, with errors for the SDMEs in all three reference frames in bins of t and 2.8 and 4.7 GeV photon beam energy. Reference [14] provides numerical values with errors for the SDMEs in the helicity frame for $E_\gamma = 9.3$ GeV.

$\|t\|$	0.02–0.05	0.05–0.08	0.08–0.12	0.12–0.18	0.18–0.25	0.25–0.40	0.40–0.80
ρ^0_{00}	$.02 \pm .01$	$-.06 \pm .02$	$-.01 \pm .02$	$.03 \pm .02$	$-.02 \pm .02$	$.00 \pm .03$	$.03 \pm .05$
ρ^0_{1-1}	$-.02 \pm .03$	$.06 \pm .03$	$-.06 \pm .03$	$.01 \pm .03$	$-.08 \pm .03$	$-.10 \pm .03$	$-.16 \pm .04$
$\mathrm{Re}\rho^0_{10}$	$.00 \pm .02$	$.04 \pm .02$	$.04 \pm .02$	$.03 \pm .02$	$.07 \pm .02$	$.06 \pm .02$	$.10 \pm .03$
ρ^1_{00}	$.03 \pm .02$	$-.05 \pm .03$	$.03 \pm .05$	$-.05 \pm .04$	$.01 \pm .04$	$-.05 \pm .05$	$-.05 \pm .08$
ρ^1_{11}	$-.06 \pm .04$	$.06 \pm .04$	$-.05 \pm .05$	$-.02 \pm .04$	$.02 \pm .05$	$.05 \pm .05$	$-.06 \pm .06$
ρ^1_{1-1}	$.48 \pm .04$	$.38 \pm .05$	$.48 \pm .05$	$.48 \pm .05$	$.49 \pm .05$	$.41 \pm .06$	$.57 \pm .07$
$\mathrm{Re}\rho^1_{10}$	$.03 \pm .03$	$.03 \pm .03$	$0.05 \pm .03$	$-.01 \pm .03$	$-.13 \pm .03$	$-.02 \pm .03$	$-.15 \pm .04$
$\mathrm{Im}\rho^2_{1-1}$	$-.50 \pm .04$	$-.42 \pm .05$	$-.57 \pm .04$	$-.48 \pm .04$	$-.49 \pm .05$	$-.42 \pm .06$	$-.64 \pm .10$
$\mathrm{Im}\rho^2_{10}$	$-.05 \pm .03$	$.02 \pm .03$	$.03 \pm .02$	$.03 \pm .03$	$.08 \pm .03$	$.03 \pm .03$	$.06 \pm .05$
P_σ	$.93 \pm .09$	$.80 \pm .11$	$.93 \pm .11$	$1.00 \pm .01$	$.97 \pm .12$	$.86 \pm .12$	$1.17 \pm .15$

Table 4: SDMEs and P_σ for $9.3\,GeV$ photons from reference [14] for the reaction $\gamma p \to p\pi^+\pi^-$. The units of the momentum transfer, $|t|$ are $(\mathrm{GeV/c})^2$.

$\|t'\|$	0.0–0.4	0.4–1.0
ρ^0_{00}	-0.01 ± 0.02	$-0.01 \pm .03$
ρ^0_{1-1}	-0.02 ± 0.03	-0.02 ± 0.06
$\mathrm{Re}\rho^0_{10}$	0.03 ± 0.02	-0.01 ± 0.05
ρ^1_{00}	0.04 ± 0.11	0.38 ± 0.18
ρ^1_{11}	0.05 ± 0.08	0.14 ± 0.19
ρ^1_{1-1}	$0.28 \pm .11$	0.61 ± 0.24
$\mathrm{Re}\rho^1_{10}$	0.10 ± 0.08	-0.10 ± 0.15
$\mathrm{Im}\rho^2_{1-1}$	-0.39 ± 0.12	-0.69 ± 0.24
$\mathrm{Im}\rho^2_{10}$	-0.05 ± 0.07	0.37 ± 0.17
P_σ	0.52 ± 0.25	0.84 ± 0.51

Table 5: SDMEs and P_σ for $20\,GeV$ photons from reference [17] for the reaction $\gamma p \to p\omega$. The units of the momentum transfer, $|t'|$ are $(\mathrm{GeV/c})^2$.

We have used equation 24 together with the reported values of ρ^0 and ρ^1 in the reference to calculate the beam asymmetry $\check{\Sigma}_A$ for these data. For $E_\gamma = 2.8$ GeV and 4.7 GeV, SDMEs were reported in all three systems. In Table 7 we report the values of $\check{\Sigma}_A$ for $E_\gamma = 2.8$ GeV in each of

$\|t\|$	0.02–0.06	0.06–0.15	0.15–0.60
ρ^0_{00}	$.00 \pm .07$	$.02 \pm .06$	$.20 \pm .07$
ρ^0_{1-1}	$.16 \pm .08$	$.06 \pm .06$	$-.05 \pm .07$
$\mathrm{Re}\rho^0_{10}$	$-.03 \pm .05$	$.01 \pm .04$	$.01 \pm .06$
ρ^1_{00}	$-.08 \pm .13$	$-.13 \pm .11$	$-.01 \pm .14$
ρ^1_{11}	$.09 \pm .12$	$.14 \pm .10$	$.05 \pm .10$
ρ^1_{1-1}	$.38 \pm .14$	$.29 \pm .12$	$.54 \pm .13$
$\mathrm{Re}\rho^1_{10}$	$.04 \pm .08$	$-.11 \pm .08$	$-0.02 \pm .10$
$\mathrm{Im}\rho^2_{1-1}$	$-.19 \pm .14$	$-.29 \pm .14$	$-.21 \pm .13$
$\mathrm{Im}\rho^2_{10}$	$.01 \pm .09$	$.10 \pm .08$	$.12 \pm .09$
P_σ	$.9 \pm .3$	$.7 \pm .3$	$1.1 \pm .3$

Table 6: SDMEs and P_σ for $9.3\,GeV$ photons from reference [14] for the reaction $\gamma p \to p\omega$. The units of the momentum transfer, $|t|$ are $(\mathrm{GeV/c})^2$.

the three systems. These are expected to be the same, so we also report the average value, but given the correlations in the data, it is unclear what the error should be on the average. We do the same for $E_\gamma = 4.7$ GeV in Table 8.

$\|t\|\ (GeV/c)^2$	Σ			
	Helicity	Adair	Gottfried-Jackson	Average
0.02-0.05	0.968 ± 0.124	0.966 ± 0.121	0.975 ± 0.117	0.969
0.05-0.08	0.835 ± 0.148	0.836 ± 0.148	0.821 ± 0.139	0.831
0.08-0.12	0.967 ± 0.158	0.970 ± 0.156	0.974 ± 0.146	0.970
0.12-0.18	0.855 ± 0.181	0.853 ± 0.179	0.862 ± 0.156	0.857
0.18-0.25	1.075 ± 0.229	1.087 ± 0.215	1.077 ± 0.184	1.080
0.25-0.40	0.571 ± 0.172	0.580 ± 0.158	0.668 ± 0.161	0.606
0.40-0.80	1.148 ± 0.418	1.137 ± 0.396	1.141 ± 0.404	1.142

Table 7: The beam asymmetry, $\check{\Sigma}_A$, for $E_\gamma = 2.8$ GeV computed from the SDME's reported in reference [16] and [14]. The asymmetry has been computed in each of the three reference systems using equation 24. The final column is the average of the three.

$\|t\|\ (GeV/c)^2$	$\check{\Sigma}_A$			
	Helicity	Adair	Gottfried-Jackson	Average
0.02-0.05	1.124 ± 0.132	1.138 ± 0.132	1.177 ± 0.131	1.146
0.05-0.08	0.847 ± 0.169	0.852 ± 0.168	0.822 ± 0.141	0.840
0.08-0.12	1.129 ± 0.142	1.119 ± 0.140	1.122 ± 0.140	1.123
0.12-0.18	1.012 ± 0.143	1.015 ± 0.144	0.956 ± 0.124	0.995
0.18-0.25	0.925 ± 0.164	1.164 ± 0.173	0.919 ± 0.146	1.003
0.25-0.40	1.156 ± 0.208	0.857 ± 0.189	1.140 ± 0.163	1.051
0.40-0.80	1.236 ± 0.330	1.246 ± 0.346	1.257 ± 0.310	1.246

Table 8: The beam asymmetry, $\check{\Sigma}_A$, for $E_\gamma = 4.7$ GeV. The caption is the same as Table 7.

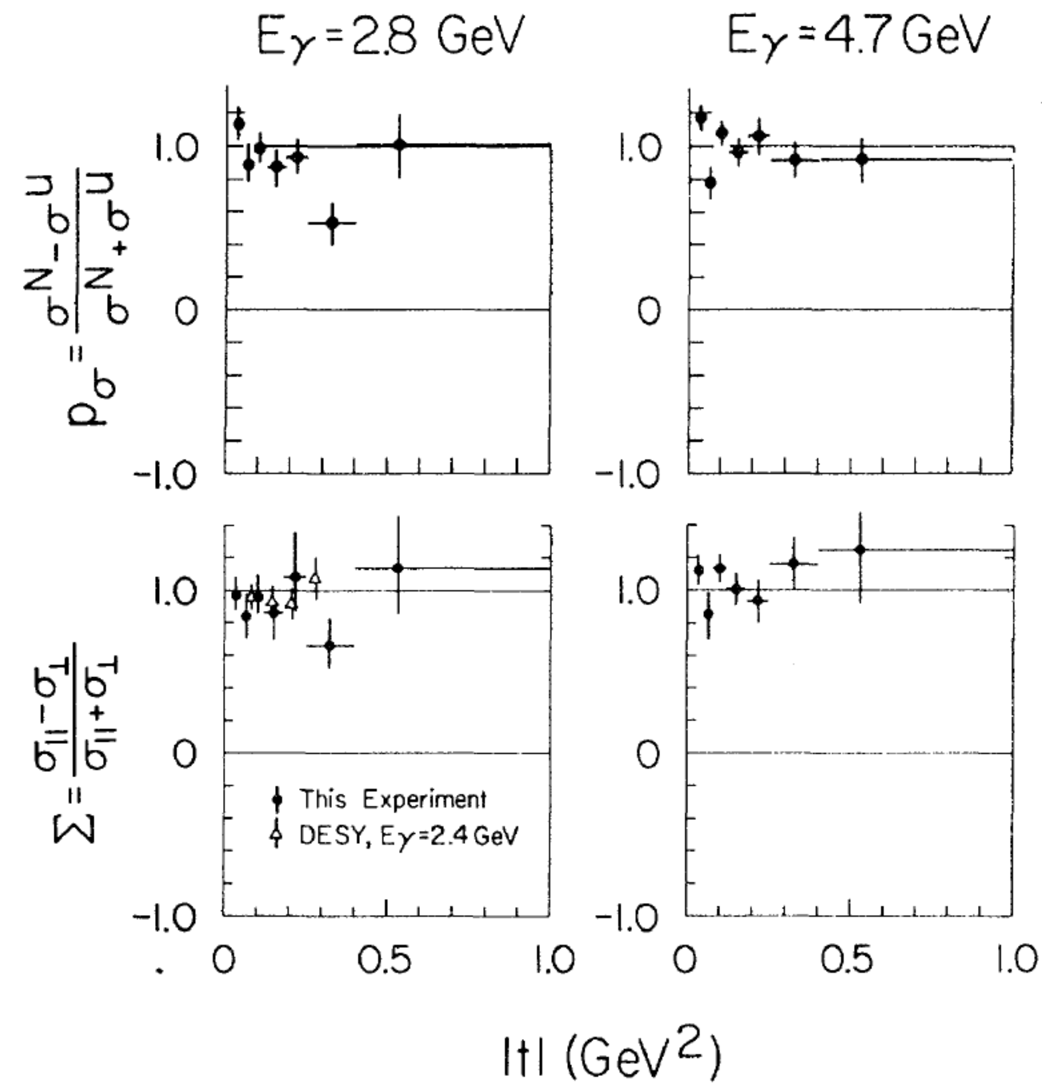

Figure 4: The measurements from reference [13] for P_σ and $\check{\Sigma}_A$ at 2.8 and 4.7 GeV.

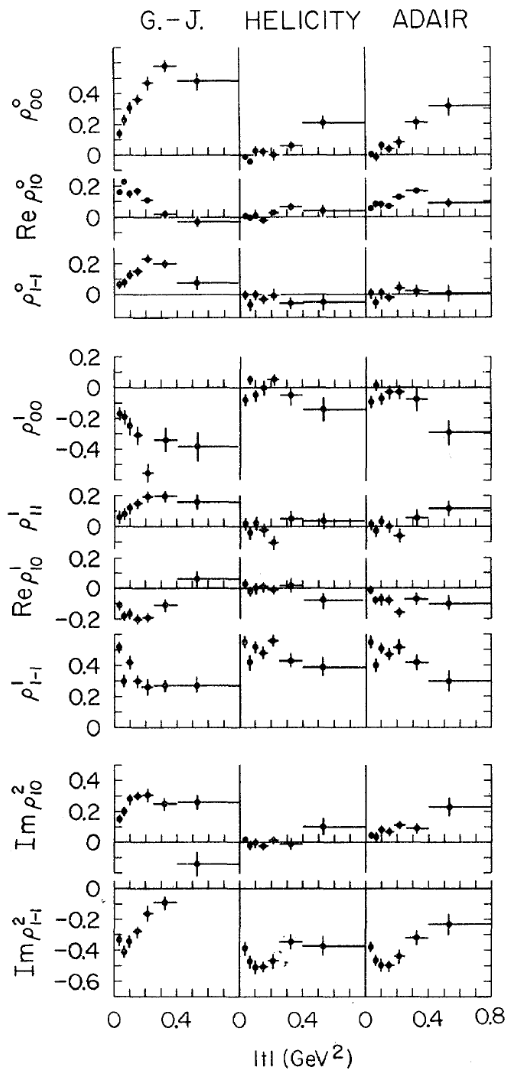

Figure 5: The SDMEs evaluated in the three frames for $E_\gamma = 4.7\,GeV$.

B.4 SLAC Hybrid Facility Photon Collaboration

Measurements of ρ^0 photoproduction were carried out by the SLAC Hybrid Facility Photon Collaboration [17]. They used linearly polarized photons produced by laser backscattering on 30 GeV electrons. This produced 20 GeV photons with 52% linear polarization which are incident on protons in a liquid-hydrogen bubble chamber. In addition to measuring the ρ' in the four-pion channel, the $\rho(770)$ was measured in the reaction

$$\gamma p \quad \rightarrow \quad p\pi^+\pi^- \, .$$

	$0.0 < \|t'\| < 0.4\,(GeV/c)^2$	$0.4 < \|t' < 1.0\,(GeV/c)^2$
ρ^0_{00}	-0.01 ± 0.02	-0.01 ± 0.03
ρ^0_{11}	0.505 ± 0.01	0.505 ± 0.015
ρ^0_{1-1}	-0.02 ± 0.03	-0.02 ± 0.06
ρ^1_{11}	0.05 ± 0.08	0.14 ± 0.19
ρ^1_{1-1}	0.28 ± 0.11	0.61 ± 0.24
$\check{\Sigma}_A$	0.68 ± 0.28	1.55 ± 0.66
P_σ	0.52 ± 0.19	0.84 ± 0.44

Table 9: The SDMEs needed to compute the beam asymmetry, $\check{\Sigma}_A$, presented in the helicity system for 20 GeV photons. The data are taken from Table II of reference [17]. The values of $\check{\Sigma}_A$ are computed from the SDMEs using equation 24.

As a comment on this data set, the value of ρ^1_{1-1} seems quite small for lower t bin and too large for the higher one. In addition, the errors seem underestimated. These lead to the quite low values of both $\check{\Sigma}_A$ and P_σ. For s-channel helicity conservation, one expects that the value of ρ^1_{1-1} should be $\frac{1}{2}$.

Justin pointed out that we can obtain an average value of $\check{\Sigma}_A$ for $E_\gamma = 20$ GeV from data presented in reference [18]. They report an expected photon polarization of $P_\gamma = 0.52$ from their laser back-scattered photons. Assuming $\Sigma = 1$, they extracted an average polarization of $\bar{P}_\gamma = 0.49 \pm 0.02$. They do not give an error in the expected polarization, but assuming that it is the same 0.02, we get that $\check{\Sigma}_A = 0.94 \pm 0.05$.

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