Σ Asymmetry Extraction in the Presence of Small Broken Symmetries

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Results

Neglecting drifts in the instrumental asymmetries, the experimental asymmetry is

$$\mathcal{E}(\phi_i) = \langle P \rangle \Sigma \cdot \cos(2\phi_i)\left[1 - \frac{\Delta P}{2}\Sigma \cdot \cos(2\phi_i)\right]$$

where $\langle P \rangle$ is the average magnitude of perp and para polarizations, and ΔP is the difference. It's not clear yet how significant ΔP is for Jane's high statistics π^0 mass bin. But we may have to add a $(\cos 2\phi)^2$ term to get a good fit probability.

Instrumental asymmetries are contained in the sum of $Y_\perp(\phi_i) + Y_\parallel(\phi_i)$. If they do not drift then

$$\mathrm{Sum}(\phi_i) = 2Y^0 \langle A(\phi_i) \rangle \left\{1 + \frac{\Delta P}{2}\Sigma \cdot \cos 2\phi_i\right\}$$

Note that the Σ asymmetry signal leaks into the sum if $\Delta P \neq$ zero. If $\Delta P \Sigma$ is known, it could be removed from $\mathrm{Sum}(\phi_i)$ to provide a clean representation of how ϕ-dependent the acceptance is.

The randoms-subtracted yields for perp and para are

$$Y_\perp(\phi_i) = Y^\circ\left[1 + P_\perp \Sigma \cdot \cos(2\phi_i)\right] A_\perp(\phi_i)$$

$$Y_\parallel(\phi_i) = Y^\circ\left[1 - P_\parallel \Sigma \cdot \cos(2\phi_i)\right] A_\parallel(\phi_i)$$

where

Y° is the ϕ-independent yield*

$A_{\perp,\parallel}$ are the ϕ-dependent instrumental asymmetries.

$P_{\perp,\parallel}$ are the magnitudes of the polarization (don't use me for any sign conventions)

For now, I assume no misalignments so the perp phase shift is zero, and para is offset by 90° such that $\cos(2\phi + \frac{\pi}{2}) = -\cos(2\phi)$.

* In our case, we define $Y_\perp^\circ = Y^\circ$, and force para to have the same normalization by multiplying $Y_\parallel(\phi_i)$ by $\frac{Y_\perp^\circ}{Y_\parallel^\circ}$. The statistical error should not be ignored.

Expt'l Asymmetry

The expt'l asymmetry is

$$\mathcal{E}(\phi_i) = \frac{Y_\perp(\phi_i) - Y_\parallel(\phi_i)}{Y_\perp(\phi_i) + Y_\parallel(\phi_i)}$$

$$= \frac{(1 + P_\perp \Sigma \cos 2\phi_i) A_\perp(\phi_i) - (1 - P_\parallel \Sigma \cos 2\phi_i) A_\parallel(\phi_i)}{(1 + P_\perp \Sigma \cos 2\phi_i) A_\perp(\phi_i) + (1 - P_\parallel \Sigma \cos 2\phi_i) A_\parallel(\phi_i)}$$

$$= \frac{(1 + P_\perp \Sigma \cos 2\phi) - (1 - P_\parallel \Sigma \cos 2\phi)(1 + \delta(\phi_i))}{(1 + P_\perp \Sigma \cos 2\phi) + (1 - P_\parallel \Sigma \cos 2\phi)(1 + \delta(\phi_i))}$$

defining $\frac{A_\parallel}{A_\perp} = 1 + \delta(\phi_i)$

$$= \frac{(1 + P_\perp \Sigma \cos 2\phi) - (1 - P_\parallel \Sigma \cos 2\phi) - (1 - P_\parallel \Sigma \cos 2\phi)\delta(\phi_i)}{(1 + P_\perp \Sigma \cos 2\phi) + (1 - P_\parallel \Sigma \cos 2\phi) + (1 - P_\parallel \Sigma \cos 2\phi)\delta(\phi_i)}$$

$$= \frac{(P_\perp + P_\parallel)\Sigma \cos 2\phi_i - (1 - P_\parallel \Sigma \cos 2\phi)\delta(\phi_i)}{2 + (P_\perp - P_\parallel)\Sigma \cos 2\phi + (1 - P_\parallel \Sigma \cos 2\phi)\delta(\phi_i)}$$

$$= \frac{\left(\frac{P_\perp + P_\parallel}{2}\right)\Sigma \cos 2\phi - \frac{1}{2}(1 - P_\parallel \Sigma \cos 2\phi)\delta\phi_i}{1 + \left(\frac{P_\perp - P_\parallel}{2}\right)\Sigma \cos 2\phi + \frac{1}{2}(1 - P_\parallel \Sigma \cos 2\phi)\delta(\phi_i)}$$

$$= \langle P \rangle \Sigma \cos 2\phi \left[1 - \frac{\Delta P}{2} \Sigma \cos 2\phi_i + \ldots \right]$$

where the ellipsis denotes terms 1st order in $\delta(\phi_i)$ whose importance is hard to assess. (They will inject noise, but are unlikely to generate $\cos(2\phi_i)$-like signals.)

Null Asymmetry

The sum of perp and para yields cancels the Σ asymmetry signal to 1st order. It is therefore a good way to quantify the instrumental asymmetries.

$$Sum(\phi_i) = Y_\perp(\phi_i) + Y_\parallel(\phi_i)$$

$$= Y^0 \Big[A_\perp(\phi_i) + P_\perp \Sigma \cos 2\phi_i A_\perp(\phi_i) + A_\parallel(\phi_i) - P_\parallel \Sigma \cos 2\phi_i A_\parallel(\phi_i) \Big]$$

$$= Y^0 \Big[A_\perp(\phi_i) + A_\parallel(\phi_i) + \big(P_\perp A_\perp(\phi_i) - P_\parallel A_\parallel(\phi_i)\big) \Sigma \cdot \cos 2\phi_i \Big]$$

$$= 2Y^0 \Big[\langle A \rangle + \frac{P_\perp A_\perp(\phi_i) - P_\parallel A_\parallel(\phi_i)}{2} \Sigma \cdot \cos 2\phi_i \Big]$$

If $A_\perp(\phi_i) = A_\parallel(\phi_i)$, this simplifies to

$$= 2Y^0 \langle A \rangle \left\{ 1 + \frac{\Delta P}{2} \Sigma \cdot \cos 2\phi_i \right\}$$

Note that the Σ asymmetry signal leaks into the sum if $\Delta P \neq$ zero.