Measuring the Neutral Pion Polarizability

Proposal Submitted to PAC 48

Draft to GlueX Collaboration

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Abstract

This proposal presents our plan to make a precision measurement of the cross section for $\gamma\gamma \to \pi^0\pi^0$ via the Primakoff effect using the GlueX detector in Hall D. The aim is to significantly improve the data in the low $\pi^0\pi^0$ invariant mass domain, which is essential for understanding the low-energy regime of Compton scattering on the π^0. In particular, the aim is to obtain a first ever experimental determination of the neutral pion polarizability $\alpha_\pi - \beta_\pi$, which is one of the important predictions of chiral perturbation theory and a key test of chiral dynamics on the π^0. Our goal is to measure $\sigma(\gamma\gamma \to \pi^0\pi^0)$ to a precision of about 6%, which would determine the combination of $\alpha_{\pi^0} - \beta_{\pi^0}$ to a precision of 50%. We expect this experiment to run concurrently with the previously approved experiment to measure the charged pion polarizability (CPP) [1] in Hall D.

Contents

1 Introduction

Electromagnetic polarizabilities are fundamental properties of composite systems such as molecules, atoms, nuclei, and hadrons [2]. Whereas form factors provide information about the ground state properties of a system, polarizabilities provide information about the excited states of the system, and are therefore determined by the system's dynamics. Measurements of hadron polarizabilities provide an important test point for Chiral Perturbation Theory, dispersion relation approaches, and lattice calculations. Among the hadron polarizabilities, the neutral pion polarizability is important because it tests fundamental symmetries, in particular chiral symmetry and its realization in QCD. Indeed, the non-trivial (non-perturbative) vacuum properties of QCD result in the phenomenon of spontaneous chiral symmetry breaking, giving rise to the Goldstone boson nature of the pions. In particular, the Goldstone boson nature of the π^0 manifests itself most notably in its decay into $\gamma\gamma$ and also in its electromagnetic polarizability, which according to ChPT can be predicted to leading order in the expansion in quark masses.

Hadron polarizabilities are best measured in Compton scattering experiments where, in the case of nucleon polarizabilities, one looks for a deviation of the cross section from the prediction of Compton scattering from a structureless Dirac particle. In the case of pions, the long lifetime of the charged pion permits experiments of low energy Compton scattering using a beam of high energy pions scattering on atomic electrons. On the other hand, the short lifetime of the neutral pion requires an indirect study of low energy Compton scattering via measurements of the process $\gamma\gamma \to \pi^0\pi^0$, a method that can also be applied to the charged pion (CPP) and for which a proposal in Hall D is already approved [1].

Measurements of hadron polarizabilities are among the most difficult experiments performed in photo-nuclear physics. For charged hadrons, because of the Born term, the polarizability effect in the cross section can range from 10 to 20% depending on the kinematics. For neutral hadrons, where the Born term is absent, the polarizability effect will be much less than this. To set reasonable expectations for what can be accomplished in a measurement of this type, it is important to recognize that after 30 years of dedicated experiments using tagged photons at facilities across North America and Europe, the error on the proton electric polarizability is 4%, without doubt the paramount experimental achievement in this field. However, the error on the proton magnetic polarizability is 16% [3]. Absolute uncertainties provide a better gauge of a measurements sensitivity; for proton electric and magnetic polarizabilities the uncertainty in both is $\pm 0.4 \times 10^{-4}$ fm^3. Another level of precision to consider for setting expectations is the result COMPASS obtained for charged pion $\alpha - \beta$. COMPASS is also a Primakoff measurement. COMPASS achieved a relative error of 46% in $\alpha - \beta$ and an absolute error of $\pm 0.9 \times 10^{-4}$ fm^3. COMPASS cannot measure the neutral pion polarizability.

This proposal presents a plan to make a precision measurement of the $\gamma\gamma^* \to \pi^0\pi^0$ cross section using the GlueX detector in Hall D. The measurement is based on the Primakoff effect which allows one to access the low $W_{\pi^0\pi^0}$ invariant mass regime with the virtual photon γ^* provided by the Coulomb field of the target. The central aim of the measurement is to drastically improve the determination of the cross section in this domain, which is key for constraining the low energy Compton amplitude of the π^0 and thus for extracting its polarizability. At present, the only accurate measurements exist for invariant masses of the two π^0s above 0.7 GeV, far above the threshold 0.27 GeV. The existing data at low energy were obtained in $e^+e^- \to \pi^0\pi^0$ scattering in the early 1990's with the Crystal Ball detector at the DORIS-II storage ring at DESY [4].

Meanwhile, theory has made significant progress over time, with studies of higher chiral corrections [5, 6, 7] and with the implementation of dispersion theory analyses which serve to make use of the higher energy data [8, 9, 10, 11]. It is expected that the experimental data from this proposal, together with these theoretical frameworks, will allow for the most accurate extraction of the π^0 polarizabilities to date.

2 Theoretical predictions for the neutral pion polarizability

The low energy properties of pions are largely determined by their nature as the Goldstone Bosons of spontaneously broken chiral symmetry in QCD, and are described in a model independent way by the framework of Chiral Perturbation Theory (ChPT) (Gasser and Leutwyler [12]), which implements a systematic expansion in low energy/momentum and in quark masses. In particular the pions' low energy electromagnetic properties can serve as tests of their Goldstone Boson (GB) nature. One such a case is the $\pi^0 \to \gamma\gamma$ decay, which at the same time tests its GB nature and the chiral anomaly. Another case is low energy Compton scattering on pions: at low energy the Compton differential cross section can be expanded in powers of the photon energy and expressed in terms of the corresponding pion charge form factor (for charged pion) and the electric and magnetic polarizabilities, where the latter give the order ω^2 terms in the Compton cross section, being ω the photon energy. The polarizabilities appear as deviations of the pions from point like particles, and thus result from carrying out the chiral expansion to the next-to-leading order. For both charged an neutral pions the polarizabilities are fully predicted at leading order in quark masses, and thus represent a sensitive test of chiral dynamics. For the charged pions, at $O(p^4)$ ChPT predicts that the electric and magnetic polarizabilities (α_{π^+} and β_{π^+}) are related to the charged pion weak form factors F_V and F_A in the decay $\pi^+ \to e^+\nu\gamma$

$$\alpha_{\pi^+} = -\beta_{\pi^+} \propto \frac{F_A}{F_V} = \frac{1}{6}(l_6 - l_5), \tag{1}$$

where l_5 and l_6 are low energy constants in the Gasser and Leutwyler effective Lagrangian [12]. Using recent results from the PIBETA collaboration for F_A and F_V [13], the $O(p^4)$ ChPT prediction for the charged pion electric and magnetic polarizabilities is given by:

$$\alpha_{\pi^+} = -\beta_{\pi^+} = (2.78 \pm 0.1) \times 10^{-4}\ \mathrm{fm}^3. \tag{2}$$

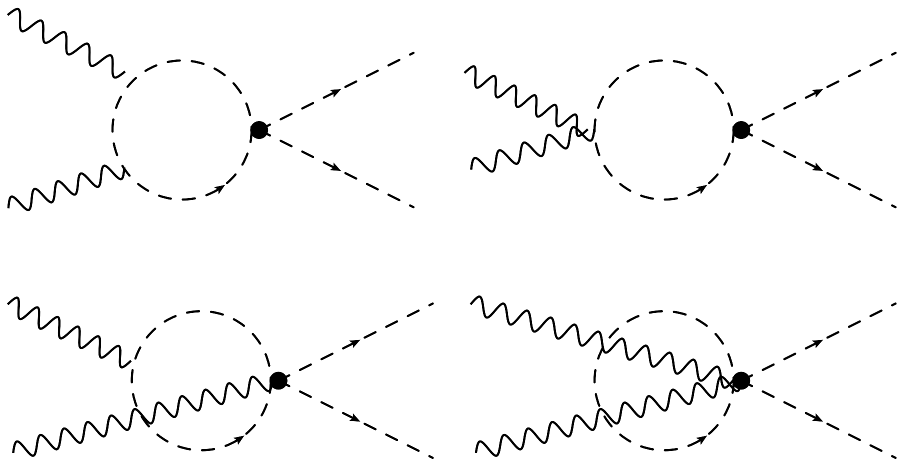


Figure 1: Diagrams contributing to Compton scattering off the π^0.

In the case of the neutral pion, the polarizabilities are determined by the one loop chiral contributions (see Fig. 1) which are calculable, free of unknown parameters, and given only in terms of the fine structure constant, the pion mass and the pion decay constant:

$$\begin{aligned} \alpha_{\pi^0} + \beta_{\pi^0} &= 0 \\ \alpha_{\pi^0} - \beta_{\pi^0} &= -\frac{\alpha}{48\pi^2 M_\pi F_\pi^2} \simeq -1.1 \times 10^{-4}\ \mathrm{fm}^3 \end{aligned} \tag{3}$$

However, there is a range of predictions beyond NLO and the experimental test of these important predictions is very challenging. In the first place, the polarizabilities drive the very low energy regime of Compton scattering on the π^0 as there is no Thomson term, so one would expect that it would be easier to determine them than in the charged pion case. However, in the first place Compton scattering on the π^0 is experimentally inaccessible due to its short lifetime, and therefore it is necessary to resort to the process $\gamma\gamma \to \pi^0\pi^0$ of this proposal. In addition, ChPT indicates that the polarizabilities are smaller in the case of the neutral pion, about a third of their value for the charged pion, i.e. somewhere between -1.7×10^{-4} fm^3 and -1.9×10^{-4} fm^3, depending on the model used to estimate higher order effects in the chiral expansion. The challenge is therefore to measure the cross section for $\gamma\gamma \to \pi^0\pi^0$ with sufficient accuracy at low invariant mass $W_{\pi\pi}$ so that one can infer the low-energy Compton amplitude and extract the polarizabilities. The demand for accuracy is required in order to allow for the extrapolation of the Compton amplitude from the kinematics of $\gamma\gamma \to \pi^0\pi^0$ to low energy Compton scattering, something that is at present impossible with the poor accuracy of the only available data from the Crystal Ball experiment [].

For this purpose, the theoretical foundations have been laid in works studying $\gamma\gamma \to \pi^0\pi^0$ both using ChPT (Bellucci et al [14, 5], Gasser et al [6], Aleksejevs and Barkanova [7]) and dispersion theory (Oller and Roca [8], Dai and Pennington [9, 10], Moussallam [11]). In particular, in ChPT at the next-to-next to leading order, which provides the higher order quark mass corrections to the polarizabilities, some of the low energy constants need to be fixed and for that a significantly more accurate measurement of the $\gamma\gamma \to \pi^0\pi^0$ cross section is needed than available presently.

Accurate measurements of the cross section near threshold combined with data for $W_{\pi\pi} > 0.6$ GeV will provide the necessary input for performing a full theoretical analysis, combining dispersion theory with and without inputs from ChPT at low energy. This is a well established method which has been used to analyze $\pi\pi$ scattering and also to the very problem of the $\gamma\gamma \to \pi^0\pi^0$ process, where numerous works have been steadily improved the theoretical dispersive analysis, to mention a few [15, 16, 8, 11, 17]. Through such an analysis it will be possible to determine, via combination with ChPT, the low energy Compton amplitude and extract the combination $\alpha_\pi - \beta_\pi$. The latter extraction represents a challenge as shown in Fig. 2, where the polarizabilities have a small direct effect on $\gamma\gamma \to \pi\pi$. Calculations by Dai and Pennington (Table II) [17] indicate that a 1.3% determination of $\sigma(\gamma\gamma \to \pi^0\pi^0)$ will determine the combination of $\alpha_{\pi^0} - \beta_{\pi^0}$ to a precision of 10%. In general, the determination of the accuracy one can get for $\alpha_\pi - \beta_\pi$ based on a more accurate measurement as the one proposed here is still an issue being currently studied theoretically, with J. L. Goity and A. Aleksejevs and S. Barkanova forming a group to take a lead on the project. At present a theoretical study based on the S-wave dominance below $W_{\pi\pi} \sim 0.8GeV$ and dispersion theory has allowed to represent the two Compton amplitudes A and B in the physical domain of the experiment. The study of the extrapolation to low energy Compton kinematics is under study, in particular the issues related to the stability of the dispersive analysis. This study is expected to provide a more accurate estimate on the sensitivity with which the experiment will allow for the determination of the polarizability $\alpha - \beta$.

3 Past Measurements

Past measurements of the $\gamma\gamma \to \pi^0\pi^0$ cross section are shown Fig. 3 and with theoretical curves in Fig. 2. The data can be summarized as follows:

1. In the early 1990's measurements were made in e^+e^- collisions at DESY with the Crystal Ball detector at the DORIS-II storage ring [4], which are the only available data for $W_{\pi\pi} <$ 0.6 GeV.

2. In 2008-2009, measurements were carried out by BELLE for 0.6 GeV $< W_{\pi\pi} <$ 4.0 GeV [18, 19, 20]. Two data sets were produced with different selection cuts on $|\cos\theta^*|$.

As mentioned above, several works have made use of dispersion theory methods (Oller and Roca [8],

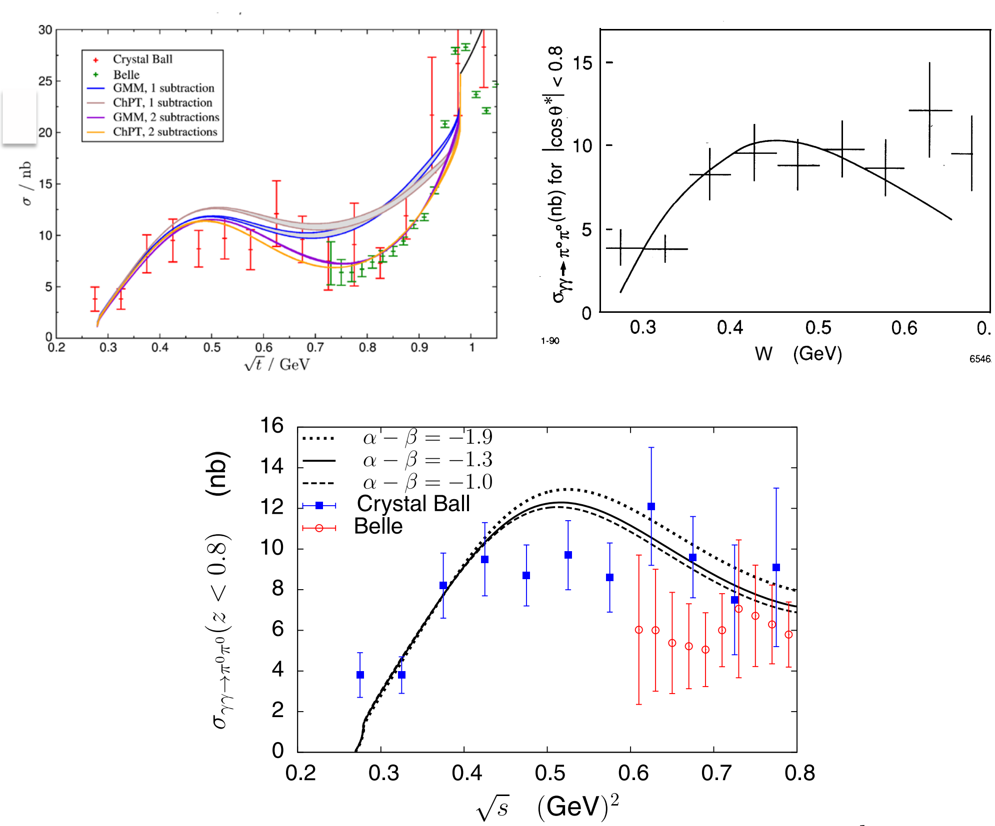
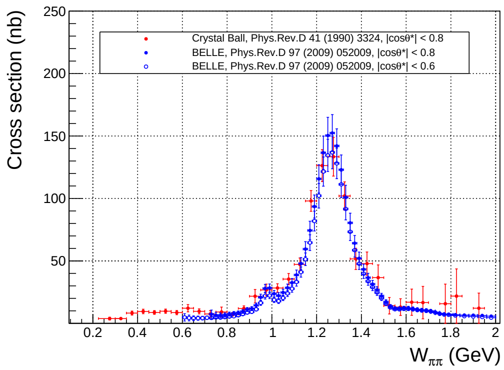


Figure 2: Left panel: experimental status; right panel: results from the 1990 XBall experiment. The lower panel shows the effect of π^0 polarizabilities on the cross section ($\sqrt{s} = W_{\pi\pi}$) [11] .

Dai and Pennington [17], and in particular Moussallam [11] who performed the dispersive analysis where one of the photons has non vanishing virtuality, which is particularly important for our case.) with those available data. In particular these methods give results for the cross section at small $W_{\pi\pi}$, but the poor accuracy of the data in that region does not serve as a useful constraint that could improve those analyses. On the other hand, the ChPT calculations carried out at NNLO (Bellucci et al [14, 5]) can only be fitted to the low $W_{\pi\pi}$ data, and thus the uncertainty in the determination of low energy constants is rather large. It is therefore expected that accurate data at low $W_{\pi\pi} < 0.6$ GeV will have a very big impact on both theoretical approaches, which together may allow for an accurate description of the low energy Compton amplitude, and for a first time experimental determination of the polarizability.



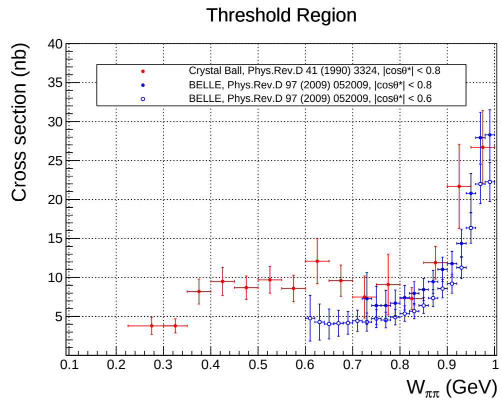


Figure 3: Data from Crystal Ball and BELLE. There are two data samples from BELLE experimental with different selection on $|\cos\theta^*|$. Left) Full range of $W_{\pi\pi}$. Right) Threshold region.

4 Experimental conditions

The measurement of the neutral pion polarizability is expected to run concurrently with the experiment to measure the charged pion polarizability (CPP) [1] in Hall D. Essentially all the optimizations for that experiment are expected to improve the sensitivity of this experiment also. We briefly summarize the configuration for CPP, which is compared in Table 1 to nominal GlueX running.

The diamond radiator will be adjusted to set the coherent peak of the photon beam between 5.5 and 6 GeV. This enhances the polarization significantly and also the tagging ratio. The experimental target will be placed upstream of the nominal GlueX target by 64 cm (z=1 cm in the Hall D coordinate system). These changes benefit the present experiment. In addition, the CPP experiment will add multi-wire proportional chambers downstream for muon identification, but these do not impact this measurement one way or another.

4.1 Expected signal

In order to estimate rates, resolution and acceptance due to the Primakoff reaction on lead, $\gamma^{208}\mathrm{Pb} \to \pi^0\pi^0\,\mathrm{Pb}$, we take the reaction process to be the same as for charged pion production and given in Eq. 8 of the Proposal for the Charged Pion Polarizability experiment [1], which is reproduced here for convenience:

$$\frac{d^2\sigma}{d\Omega_{\pi\pi}dW_{\pi\pi}} = \frac{2\alpha Z^2}{\pi^2}\frac{E_\gamma^4\beta^2}{W_{\pi\pi}}\frac{\sin^2\theta_{\pi\pi}}{Q^4}|F(Q^2)|^2\sigma(\gamma\gamma\to\pi^0\pi^0)(1+P_\gamma\cos 2\phi_{\pi\pi}). \tag{4}$$

Table 1: Configuration of the CPP experiment compared to nominal GlueX. We propose that this experiment run concurrently with CPP. Detectors not identified in the table are assumed to be operated under the same conditions as in the nominal configuration.

Configuration	GlueX I	CPP/NPP
Electron beam energy	11.6 GeV	11.6 GeV
Emittance	10^{-8}m rad	10^{-8}m rad
Electron current	150 nA	20 nA
Radiator thickness	50μm	50 μm diamond
Coherent peak	8.4 – 9.0 GeV	5.5 – 6.0 GeV
Collimator aperture	5 mm	5 mm
Peak polarization	35%	72%
Tagging ratio	0.6	0.72
Flux 5.5-6.0 GeV		11 MHz
Flux 8.4-9.0 GeV	20 MHz	
Flux 0.3-11.3 GeV	367 MHz	74 MHz
Target position	65 cm	1 cm
Target, length	H, 30 cm	^{208}Pb, 0.028 cm
Start counter	nominal	removed
Muon identification	None	Behind FCAL

The $\gamma\gamma$ cross section for charged pions has been substituted with the cross section for neutral pions, namely $\sigma(\gamma\gamma \to \pi^0\pi^0)$. In this expression, $\Omega_{\pi\pi}$ is the solid angle in the laboratory frame for the emission of the $\pi\pi$ system, $W_{\pi\pi}$ is the $\pi\pi$ invariant mass, Z is the atomic number of the target, β is the velocity of the $\pi\pi$ system, E_γ is the energy of the incident photon, $F(Q^2)$ is the electromagnetic form factor for the target with final-state-interactions (FSI) corrections applied, $\theta_{\pi\pi}$ is the lab angle for the $\pi\pi$ system, $\phi_{\pi\pi}$ is the azimuthal angle of the $\pi\pi$ system relative to the incident photon polarization, and P_γ is the incident photon polarization.[1]

The cross section for $\sigma(\gamma\gamma \to \pi^0\pi^0)$ has been measured by the Crystal Ball Collaboration [4], albeit with limited statistical precision. We have parameterized the cross section for $W_{\pi\pi} < 0.8$ GeV, which is of specific interest to this program as shown in Fig.4. Using this parameterization and Eq.4, we can calculate the photoproduction cross section on lead, which is shown in Fig.5. The integrated cross section is $0.30 \pm 0.05\mu$b/nucleus. The uncertainty comes from the model dependence and was obtained by comparing two different calculations using completely different parameterizations for the nuclear form factor on lead, $F(Q^2)$. For reference, we note that the cross

[1]The expression for the cross section in terms of invariant quantities can be found in Ref. [21].

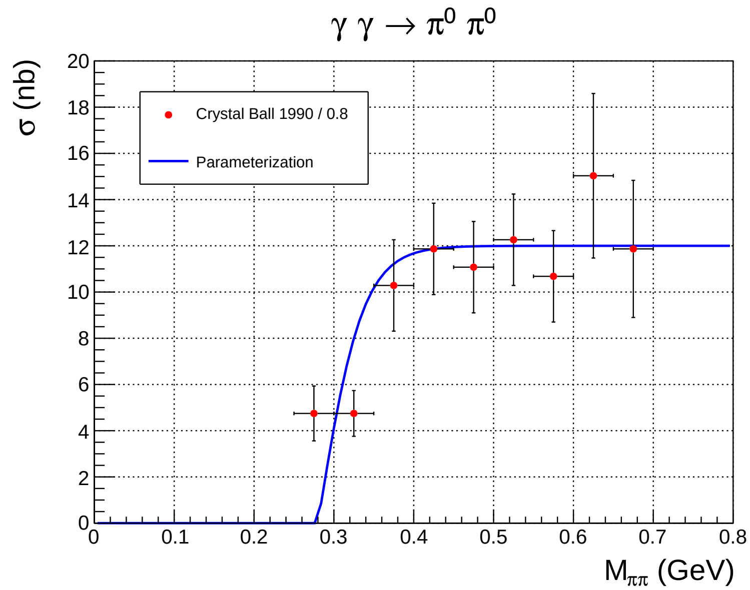


Figure 4: Parameterization of the $\sigma(\gamma\gamma \to \pi^0\pi^0)$ cross section as a function of the 2π invariant mass compared to the data from Crystal Ball [4].

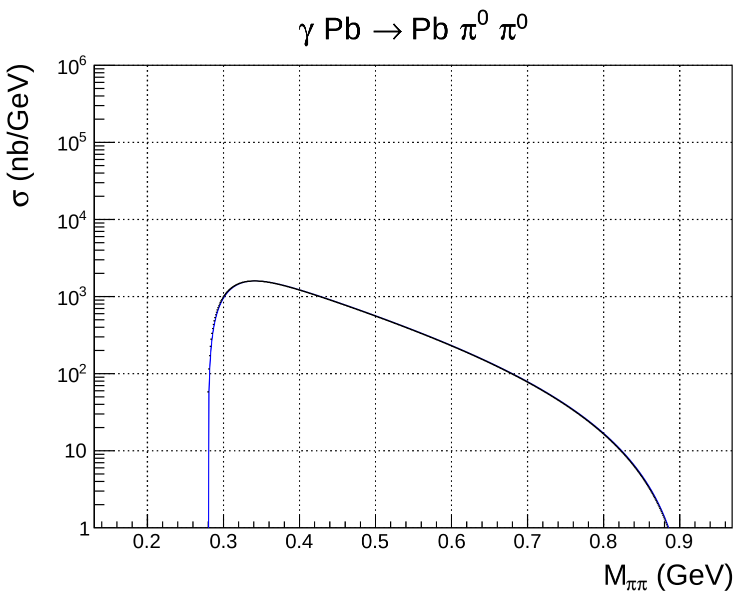


Figure 5: Primakoff cross section for $\gamma Pb \to Pb\,\pi^0\pi^0$ using the parameterization of $\sigma(\gamma\gamma \to \pi^0\pi^0)$ in the previous figure. The integrated cross section is $0.3\,\mu$b/nucleus.

section for charged pions ($\pi^+\pi^-$) production is 10.9 μb, a factor of 30 larger.

The number of neutral-pion-Primakoff-signal events produced during 20 PAC days is shown in Fig.6. The impacts of detector trigger, acceptance and resolution are discussed in the next section.

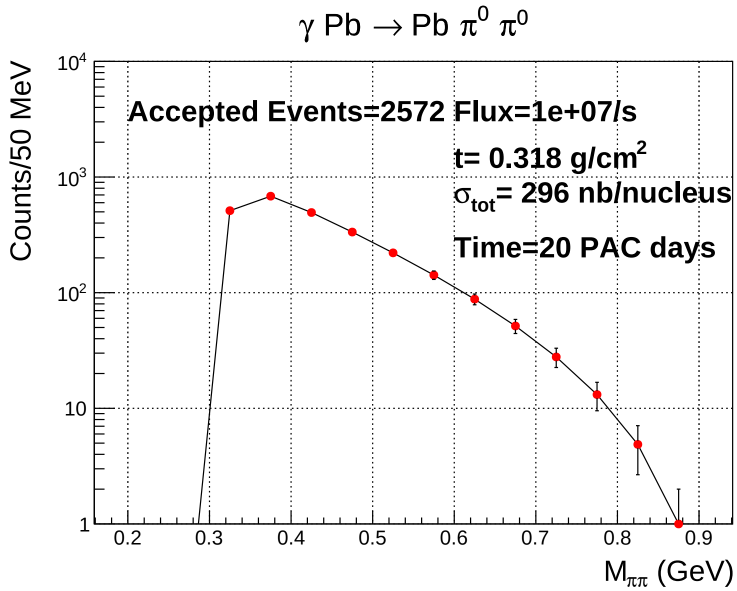


Figure 6: Estimated production rate for γPb $\rightarrow$ $\pi^0\pi^0$ Pb as a function 2π mass. For this calculation, it is assumed the detector has perfect resolution and has a linearly increasing efficiency from zero at threshold up to 0.6 at 0.34 GeV (see top right of Fig. 12).

4.2 Detector resolution

The response of the GlueX detector to neutral pion Primakoff events was simulated using the standard GlueX generation and reconstruction tools, but with the specific geometry for the CPP experiment. The schematic of the detector configuration is shown in Fig. 7. The primary differences between the standard GlueX geometry and CPP are summarized in Table 1. For this experiment, the main differences include a) coherent peak position at 5.5-6 GeV and re-positioning of the microscope to cover the coherent peak, b) solid ^{208}Pb at z=1cm, and c) Start counter removed. For the CPP experiment, the addition of muon identification chambers behind the FCAL is critical. However, for neutral pions this addition plays no role because the photons are detected in the FCAL. The GEANT3 simulation,[2] which is used for these studies, includes most changes except for the

[2]The event simulation has been updated to run with GEANT4 and yields similar results.

addition of the muon chambers, which are not needed. In addition, the microscope geometry has not been modified and we use the tagger hodoscope for that region in the simulation. The slightly reduced energy of the hodoscope relative to the microscope has little impact and the gaps between counters is ignored by simulating the tagged flux.

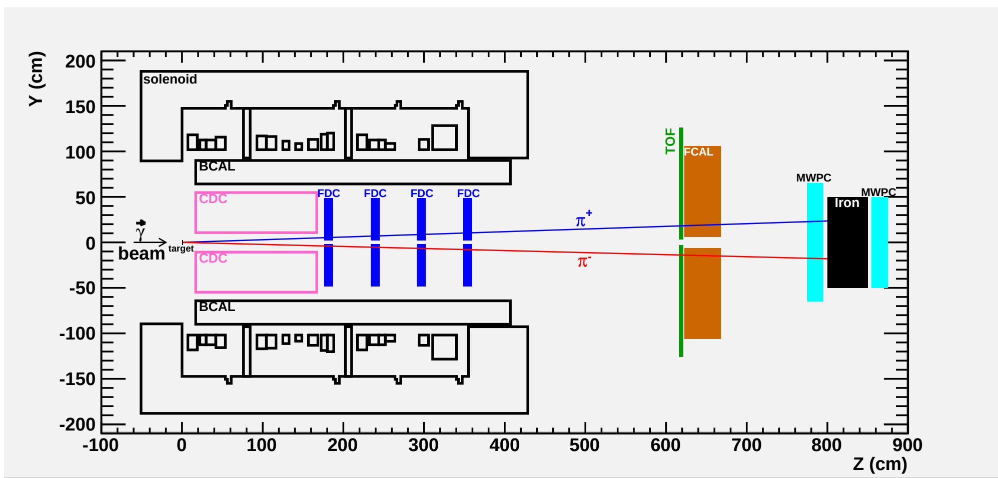


Figure 7: Diagram of the GlueX detector including the additional muon chambers for the CPP experiment.

The Primakoff signal was generated according to the cross section described in the previous chapter, using the *gen_2pi0_primakoff*, which is a modified version of the CPP event generator. By default, the production amplitudes are symmetrized between the two identical π^0's by AmpTools. One hundred thousand events were generated with $M_{\pi\pi}$ <0.5 GeV and with no background, fed to GEANT3 to track particles, and subsequently processed using *mcsmear* to simulate the detector response. The simulated events were then analyzed using the GlueX event filter to analyze the reaction $\gamma Pb \to \pi^0\pi^0$ with a missing Pb nucleus and constraining the detected photon pairs to the π^0 mass. Energy and momentum conservation is imposed on the reaction as well as the requirement that all photons originate from a common vertex (i.e. “vertex-P4”). The output of the reconstruction, both kinematically fit and “measured” quantities, were available for inspection.

In the following we show various reconstructed quantities as well as estimated resolutions. The distribution of generated photon energy and the unconstrained reconstructed momenta of the two pions are shown in Fig. 8. The missing mass, 2π mass and $-t$ distributions are shown in Fig. 9. The reconstructed momentum relative to its generated value is shown in Fig.10. The central peak of the kinematicall fit momentum is about 2%, similar to that for charged pions. However, there are long uniform tails that will effect the final reconstruction. The resolution of the azimuthal angle, $\phi_{\pi\pi}$, between the production and the photon polarization planes is quite poor owing to the fact that the pion pairs are produced at very shallow angles. Nevertheless it is sufficient to measure the

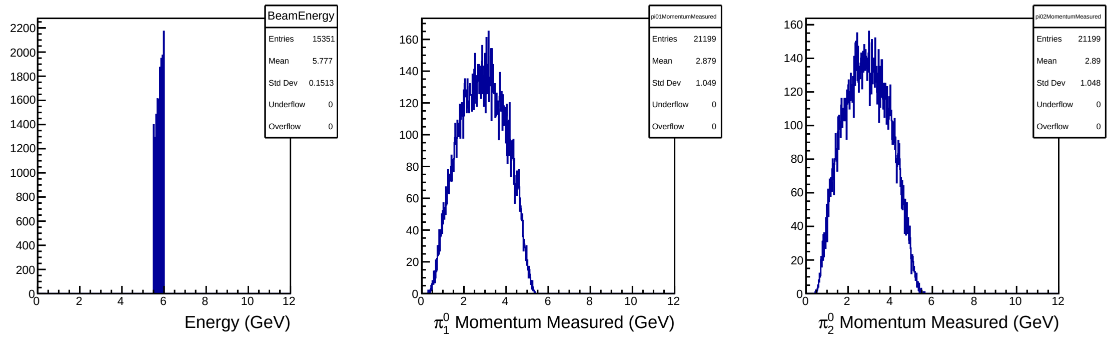


Figure 8: Left: Generated photon energies. Center: Reconstructed momentum distribution of one π^0. Right: Reconstructed momentum distribution of the second π^0.

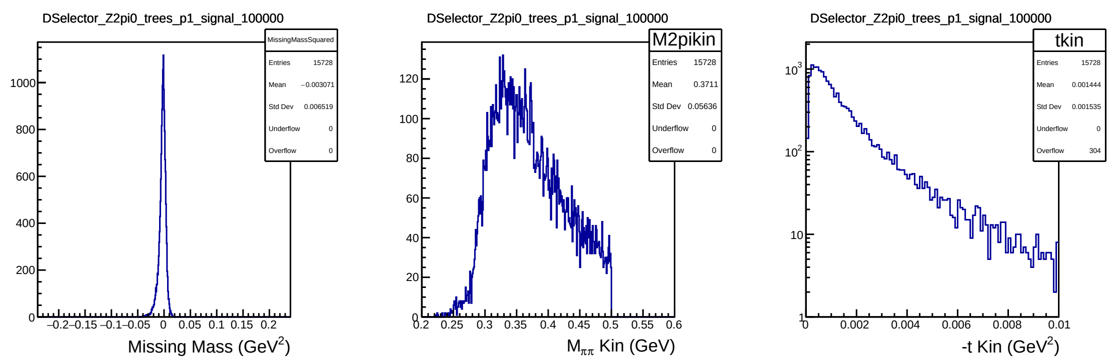


Figure 9: Left: Missing mass distribution minus the mass of the recoil nucleus. Center: Kinematically fit 2π mass distribution. Right: Kinematically fit -t distribution.

asymmetry due to the photon beam polarization. The resolution of the 2π invariant mass is shown

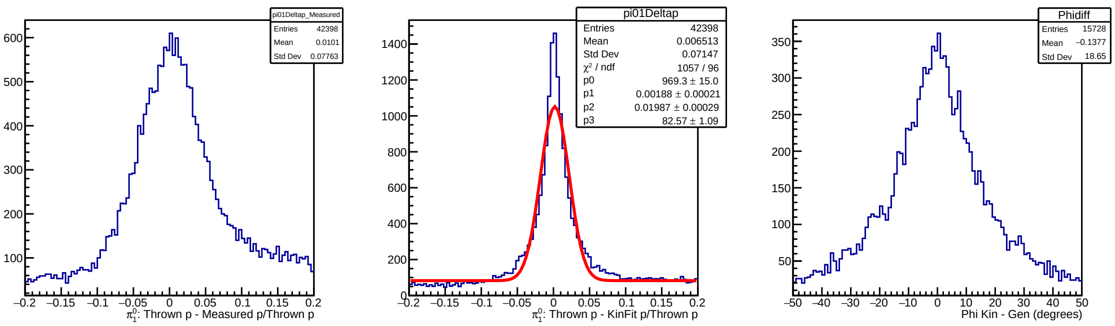


Figure 10: Left: Difference between measured and generated momentum. Center: Difference between kinematically fit and generated momentum. The central peak has a width of about 2%. Right: Difference between the kinematically fit azimuthal angle $\phi_{\pi\pi}$ and its generated value.

in Fig. 11, along with the resolution of Mandelstam $-t$, and the reconstructed time resolution. The mass resolution is about 8 MeV.

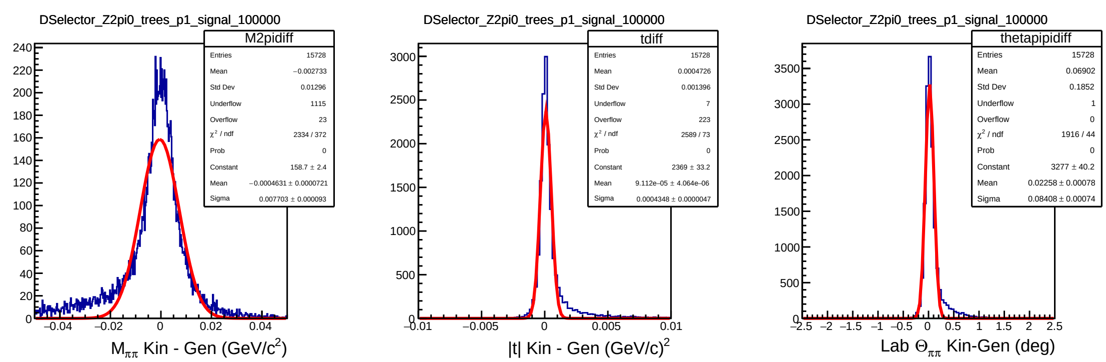


Figure 11: Left: Difference between kinematically fit and generated 2π mass. The central 2π-mass σ is about 8 MeV. Center: Difference between kinematically fit and generated -t. Right: Difference between kinematically fit and generated 2π polar angle. The resolution σ of the reconstructed angle is less than 0.1 degrees.

4.3 Trigger and acceptance

The Primakoff reaction will transfer all the energy of the beam into four photons, which are going forward. All this energy will be deposited in the FCAL, except for leakage down the beampipe.

We expect a simple trigger with an energy threshold in the FCAL should have very high efficiency for any events that can be reconstructed.

The acceptance of the signal events can be determined by comparing the kinematically fit to the generated distributions. The generated and kinematically fit 2π mass, $\phi_{\pi\pi}$ and $-t$ distributions are shown in Fig. 12. The reconstruction was described in the previous section. The acceptance is quite high at about 60%. However, there is also significant slewing due to resolution in most variables of interest. The main effect of resolution in the 2π mass happens at threshold and the distortions above that are not very great. The relatively poor resolution in $\phi_{\pi\pi}$ results in dilution of the measured azimuthal dependence, which will need to be adjusted based on simulation. Finally the $-t$ resolution softens the measured t-slope due to the smearing of high rate regions down to low rate regions.

5 Backgrounds

We first classify the various backgrounds and then describe them in more detail one at a time. We note that two important backgrounds for the charged-pion polarizability experiment do not contribute in this experiment: First, coherent ρ^0 photo-production is absent in this experiment because the ρ^0 decay into the $\pi^0\pi^0$ channel is prohibited by I-spin conservation. Second, $\mu^+\mu^-$ production is also not a factor for the neutral pion case. We have the following categories of backgrounds:

- Coherent production: In this case, the target remains intact. Generically, one may classify the two-pion production according to the sketches in Fig. 13. The left-hand diagram represents the exchange of a virtual photon with the nucleus, i.e. the Primakoff mechanism. This mechanism is very long range, approximately 100 fm, and is affected minimally by the effects of shadowing or absorption. This is the signal for the experiment and our goal is to determine its cross section. The right-hand diagram represents the exchange of a strongly interacting particle (or trajectory) and effectively results in the production of pions at the surface of the nucleus. We note that for the neutral pion production, pion exchange is not allowed due to charge conjugation conservation, while for the charged pion case, single pion exchange is related to the axial anomaly ($\gamma\pi^0 \to \pi^+\pi^-$). When the interaction leaves the nuclear target intact, the reaction is referred to as "nuclear coherent" and this is our most serious background.
- Incoherent production: When the interaction produces two pions in the quasi-elastic scattering off a single nucleon, the scattered target usually fragments into particles that range out in the target and are unobservable experimentally. This reaction occurs at larger $-t$ and is generally kinematically distinct from the signal. The $\pi^0\pi^0$ momentum relative to the photon

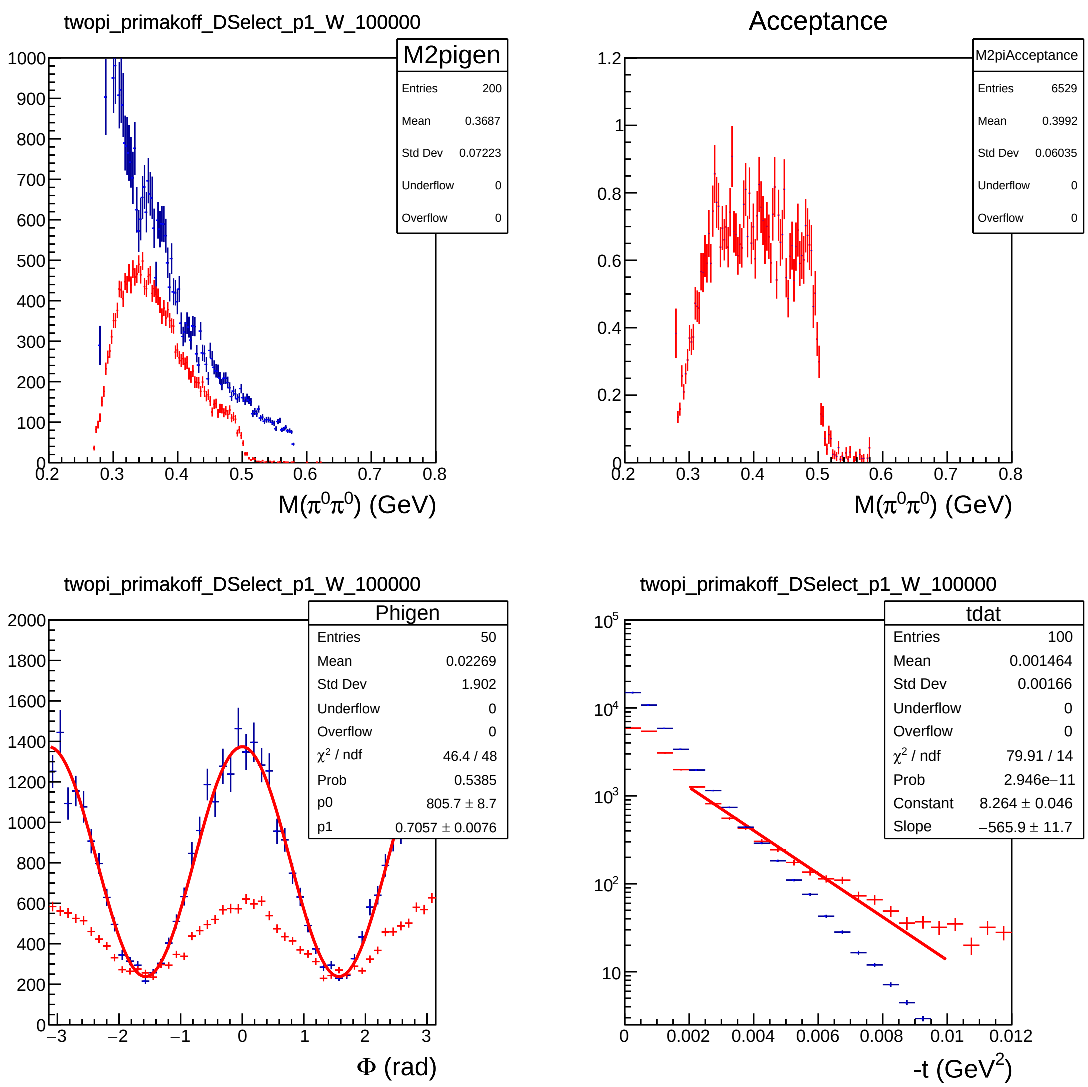


Figure 12: Top left: Generated and kinematically fit 2π mass distribution. Top right: Acceptance as a function of 2π mass. Events above $M_{\pi\pi}$ >0.5 GeV were eliminated from the analysis. The acceptance is about 60%. Bottom left: Generated and kinematically fit azimuthal angle $\phi_{\pi\pi}$. Bottom right: Generated and kinematically fit $-t$ distribution. There is significant slewing of the measured distribution to large $-t$.

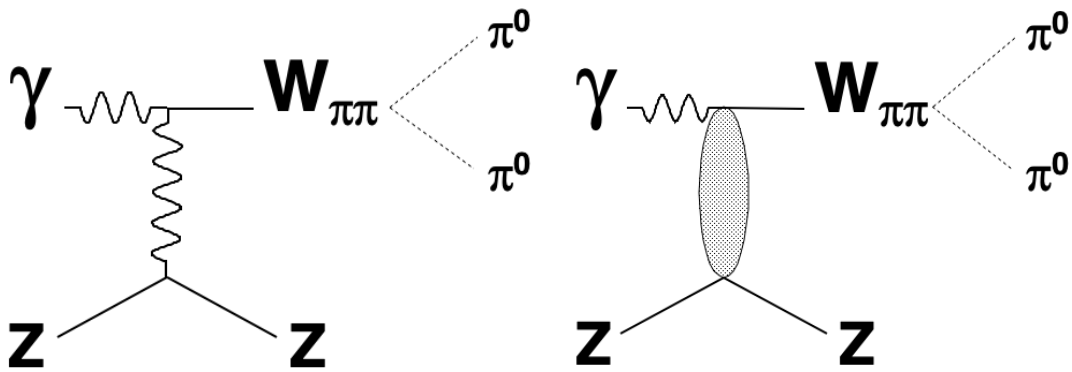


Figure 13: Sketch of coherent two-pion production. Left) Signal: Primakoff mechanism, Right) Backgrounds: Other production mechanisms.

polarization plane does differentiate between the Primakoff and incoherent production.

- Any reaction that may be confused with the signal within the experimental resolution or limited acceptance: An example of this type of reaction is Primakoff production of η mesons, where the $\eta \to \pi^0\pi^0\pi^0$ is mis-reconstructed as a two-pion final state.

5.1 Coherent backgrounds

The largest coherent background is from the $f_0(500)(J^{PC} = 0^{++})$, also called the σ meson, and the $f_0(980)$ photo-production. The width of the $f_0(980)$ is fairly narrow and does not contribute directly to the strength near threshold. The $f_0(500)$width is much broader, from 400 to 700 MeV, with significant overlap in the invariant mass region of interest. Since the $f_0(500)$ is a scalar particle with the same spin-parity as the $\gamma\gamma \to \pi^0\pi^0$ final state near threshold, the azimuthal distribution of the π^0 momentum or the $\pi^0\pi^0$ c.m. momentum relative to the photon polarization plane does not differentiate between coherent $f_0(500)$ production and the Primakoff reaction. This is similar to the Primex-π^0 experiment, where the dominant background was nuclear coherent π^0 photo-production. The approach used in the Primex analysis was to measure the π^0 angular distribution, effectively the t-distribution, then use theoretical calculations of the angular distributions to separate out contributions from Primakoff and nuclear coherent. The analysis of the $\pi^0\pi^0$ (NPP) reaction will approximately parallel what was done for the Primex-π^0 analysis.

We parameterize the σ meson as detailed in Appendix B and assume that the production amplitude can be factorized as

$$\mathcal{A} \;=\; \mathcal{A}_t(t)\,\mathcal{A}_W(m_{\pi\pi})\,\mathcal{A}_\tau(\Phi,\phi,\theta), \qquad (5)$$

where the last factor represents the angular distribution that results in a dependence on the di-pion azimuthal angle, $\phi_{\pi\pi}$, of the form $\mathcal{A}_\tau \propto (1 + \mathcal{P}\cos 2\phi_{\pi\pi})$. The mass dependence is given by the S-wave phase shifts that dominate the mass region below 0.8 GeV. We use the approximate description given in Appendix B.1.[3]

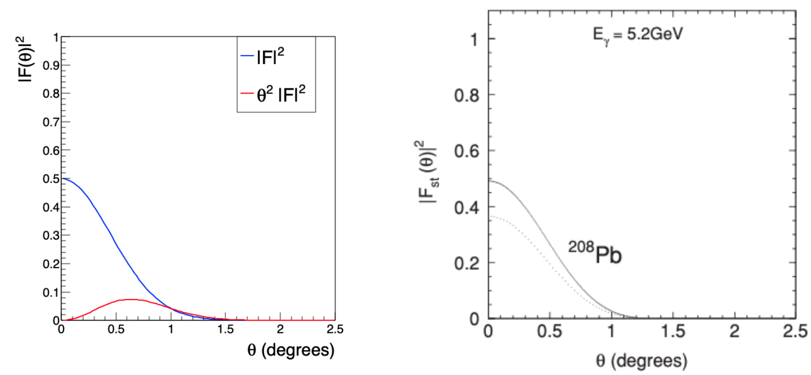


Figure 14: Left) Approximation to strong form factor for lead, Right) Figure 6 from Ref. [22] showing the calculated strong form factor for single π^0 production off a lead target.

We assume the $-t$ dependence of the σ has a similar form as for single π^0 production, namely $\mathcal{A}_t(t) \propto \sin\theta_{\pi\pi} \times F_{st}(t)$. The $\sin\theta_{\pi\pi}$ comes from the spin-flip required at forward angles to produce a 0^+ system from a spin-zero target. The factor $F_{st}(t)$ is the strong form factor for the target, which is approximated to match calculations for the single π^0 production (Fig. 6 from Ref.[22]). Our Gaussian approximation to the form factor is shown in Fig. 14 along side the calculation for single π^0 production. Efforts are underway to calculate the strong form factor for this reaction.[4]. The Primex data showed that the nuclear coherent process is highly suppressed for heavy nuclei, as shown in Fig.23. The reason for the suppression is π^0 absorption in the nuclear interior, making the coherent production primarily a surface effect, i.e. proportional to A and not A^2. For NPP it is expected that suppression of the nuclear coherent will be approximately twice stronger than that seen in Primex because two pions are produced in NPP as compared to a single π^0 in Primex.

5.2 Fits to the scattering angle

The Primakoff reaction is peaked at very small scattering angle $\theta_{\pi\pi} \sim m^2_{\pi\pi}/2E^2_\gamma \sim 0.1 - 0.2°$ for masses between 0.35 and 0.5 GeV, as shown in Fig. 15a. Nevertheless it is still an order-of-magnitude larger than the scattering angle for single π^0 production. The angular distribution for the nuclear

[3]More detailed studies may require including contributions from the D-wave and S-wave, I=2, amplitudes.
[4]S. Gevorkyan, private communication.

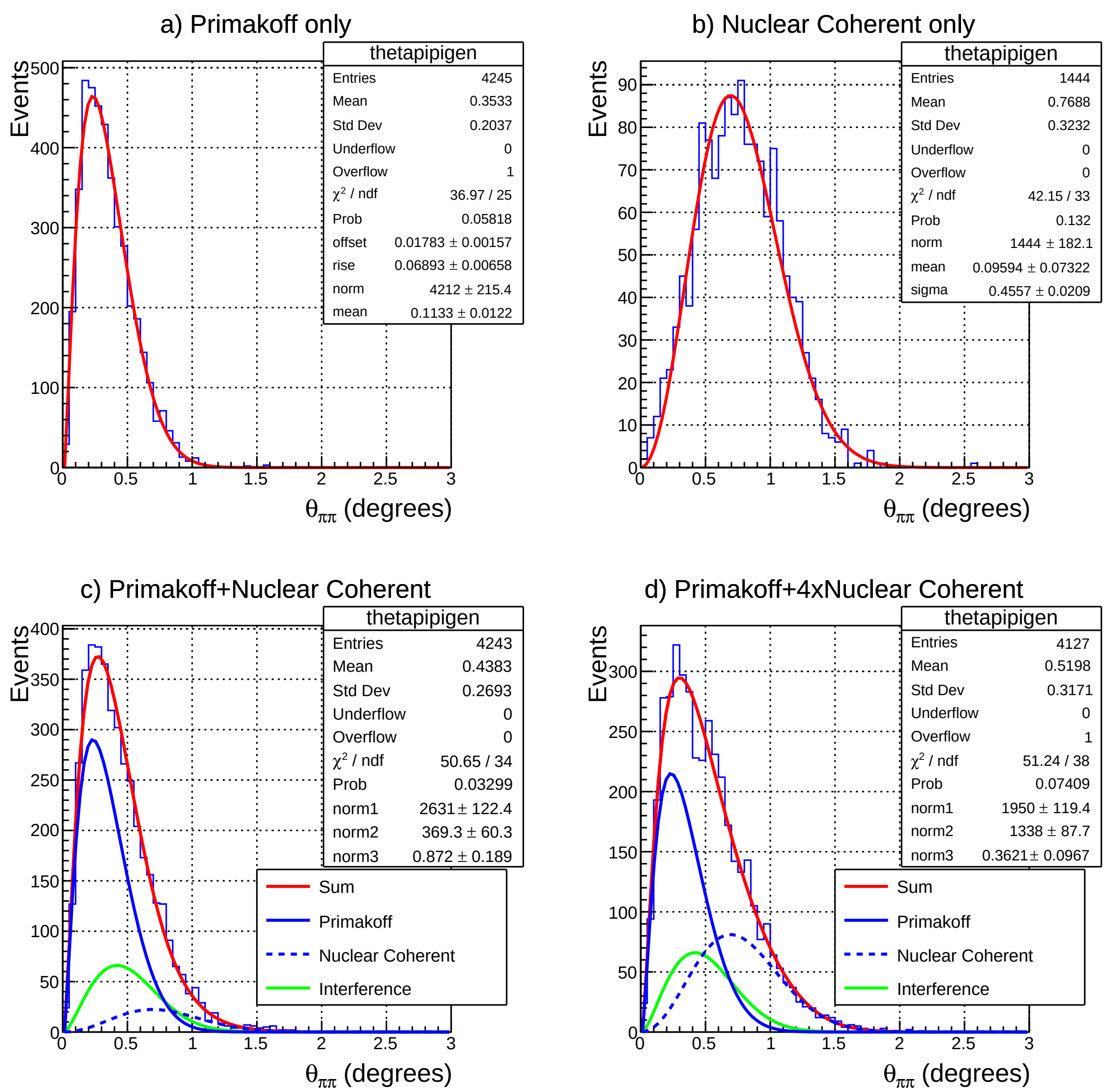


Figure 15: Angular distributions for a) Primakoff reaction only, b) Nuclear coherent reaction only, c) Sum of Primakoff and nuclear coherent, d) Same as c) with the nuclear coherent amplitude multiplied by 2. The curves represent fits to the coherent sum of the two components.

coherent process on lead is shown in Fig. 15b, which peaks at an angle of about $\theta_{\pi\pi} \sim 0.7^\circ$. The coherent sum of both processes, e.g. Fig. 15c and d, is a distribution that is considerably wider than the Primakoff distribution alone, but a fit is required to determine the Primakoff signal. The amplitude of the NC component in Fig. 15d was generated with an amplitude that is twice that in Fig. 15c. In order to extract the Primakoff component, we proceed as follows: We first parameterize the shapes of the Primakoff and NC components separately, as shown with the curves in Fig. 15a and b and the parameters for those shapes are fixed. Two distributions generated using both the Primakoff and NC amplitudes are shown in the bottom plots of Fig. 15c and d and then fit to the sum of the two distributions plus an interference term. We let the normalization of each component vary as well as the size of the interference term. The resulting fitted parameters are shown on the figures. From this exercise we conclude that the determination of 2000 signal Primakoff events can be determined with a statistical uncertainty of about 5%.

5.3 Incoherent backgrounds

The inelastic and incoherent reactions that might contribute to the data include

(i) nuclear coherent production of η followed by $\eta \to \pi^0\pi^0\pi^0 \to \gamma\gamma\gamma\gamma(\gamma\gamma)$, where two of the six decay photons go unobserved

(ii) $\gamma N \to N\pi^0\pi^0$

The first reaction is an inelastic, coherent process, and as such could produce a significant rate for a heavy nuclear target. Rejecting events that do not satisfy our elastic selection and the constraint that the four photons must reconstruct to two π^0's mass should eliminate most of these events. Additional gammas in the final state can also be used to suppress this background. The second reaction is an incoherent process, and is small relative to coherent processes. The Primex analysis showed that incoherent reactions generally peak at angles larger than about 1° and had a small effect on the extraction of the Primakoff π^0 cross sections. Likewise, we expect that incoherent processes to be a small issue for this experiment. The $\pi^0\pi^0$ momentum relative to the photon polarization plane does differentiate between Primakoff and incoherent production.

5.4 Analysis of existing data

We investigated the challenges of reconstructing $2\pi^0$ final states with a missing recoil proton using the 2017 GlueX data taken with a Hydrogen target and the 2019 PrimEx-Eta data with Beryllium and Helium targets.

5.4.1 Hydrogen target

We selected and reconstructed events that matched the topology of the reaction $\gamma p \to \gamma\gamma\gamma\gamma\,(p)$ with a missing proton. A kinematic fit was performed that conserved energy and momentum and imposed a vertex constraint at z=65 cm (CL $> 10^{-6}$). We note that even though the vertex was fixed at 65 cm to perform the fit, the actual target extends from 50 to 80 cm. Several other nominal selections were imposed to clean up the event sample, including no charged tracks and no missing energy. No constraints were imposed on the π^0 mass in order to study backgrounds. Accidental background subtractions were performed to obtain the resulting mass distributions.

The invariant mass distributions of two photon pairs each show a strong π^0 peak, as shown in the top of Fig. 16. There are background events that fall under the two π^0 peaks, which requires further study, nevertheless, using the selection of photon pairs that reconstruct to the π^0, we can plot the $2\pi^0$ mass spectrum (bottom of Fig. 16). The mass spectrum has recognizable features, in particular the prominent $f_2(1270)$ that decays to $\pi^0\pi^0$ 85% of the time. The structure at $M_{\pi\pi}$ ~0.8 GeV appears too low for the $f_0(980)$ and is present in a location where the Crystal Ball data [4] shows a low yield. The yield for $M_{\pi\pi}$ <0.5 GeV is consistent within a factor of two of the relative yield compared to the f_2(1270 peak in the Crystal Ball data. This analysis demonstrates that these neutral events can be analyzed in our detector under significant more challenging circumstances than we anticipate for the Primakoff experiment. In particular, for the Primakoff experiment, we will have a point nuclear target that will allow valid geometrical constraints and limit the amount of missing momentum in the reaction. This will make the kinematic fitting more effective.

It is evident from top plot in fig. 16 that a cut on the invariant mass of one reconstructed π^0 will reduce the background on the other π^0 significantly. This is shown in fig. 17 where a cut on the invariant of one π^0 significantly reduces the background in the other while keeping the main signal mostly undisturbed.

These photons are detected by the lead-glass calorimeter and are the main contribution to the resolution of the reconstructed pi^0 mass. A lead-tungstate calorimeter with a substantially better energy resolution would yield a significant improvement in the signal to noise ration as the width of the reconstructed π^0 would be smaller by about a factor of 2.

5.4.2 Helium and beryllium targets

The PrimEx-Eta experiment collected valuable data on light nuclear targets (^{4}He and ^{9}Be) in 2019. Analysis of the two neutral pion system photoproduction on these nuclei gives a good estimation of the main background sources, signal to background levels, and the Hall-D detector resolution for the main kinematic variables of two neutral pion photoproduction process. The total PrimEx-Eta luminosity corresponds to approximately one day on 5 % rad. len. beryllium target and 18 days on a 4 % rad. len. helium target at 200 nA electron beam current and a 10^{-4} rad. len. thick amorphous

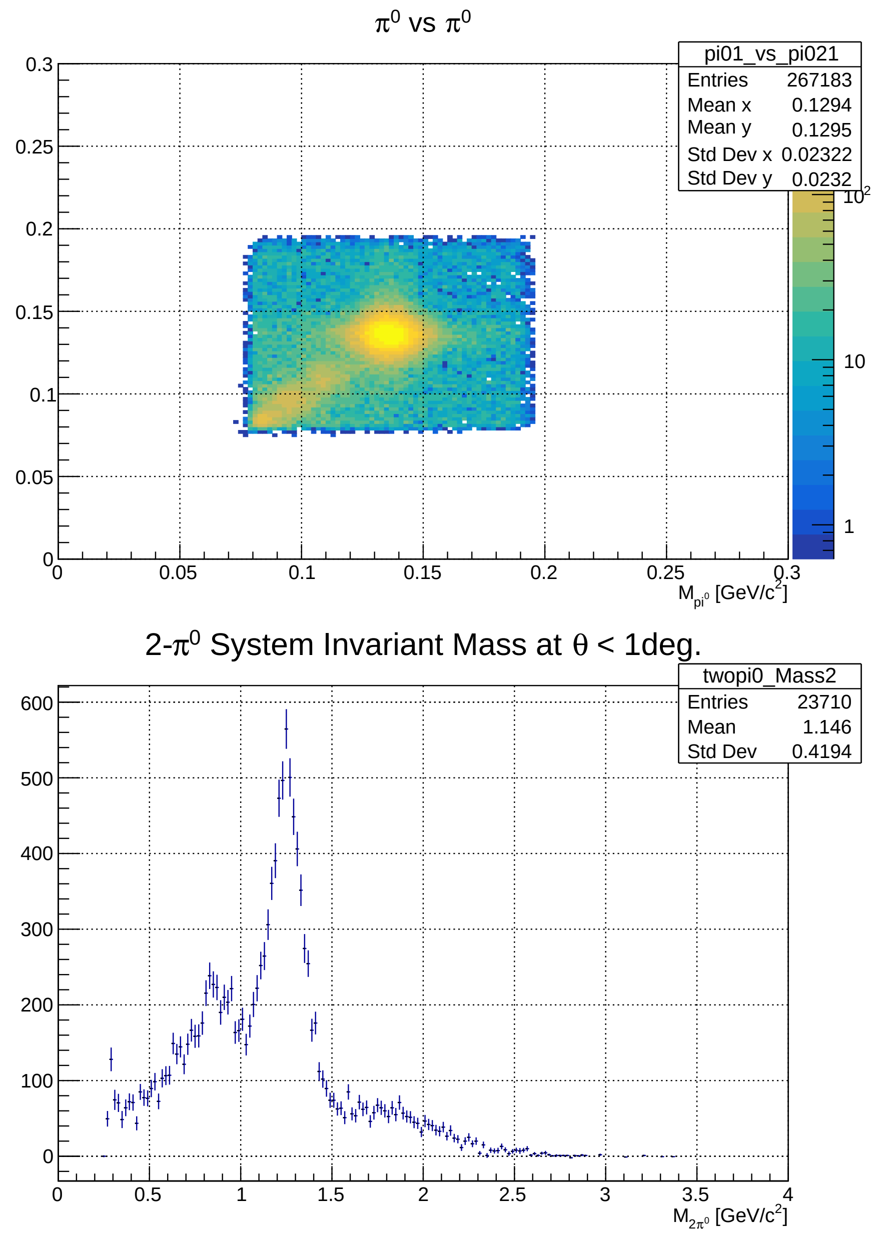


Figure 16: Experimental distributions from the 2017 GlueX data set analyzed as $\gamma p \to \gamma\gamma\gamma\gamma\,(p)$ with a missing proton. Top: Two photon invariant mass of one pair vs the two photon invariant mass of the second pair. Bottom: 2π mass distribution selecting events with the reconstructed photon pair masses close to the π^0 mass as shown above. The plot also requires that the angle of the two pion system be less than 1 degree.

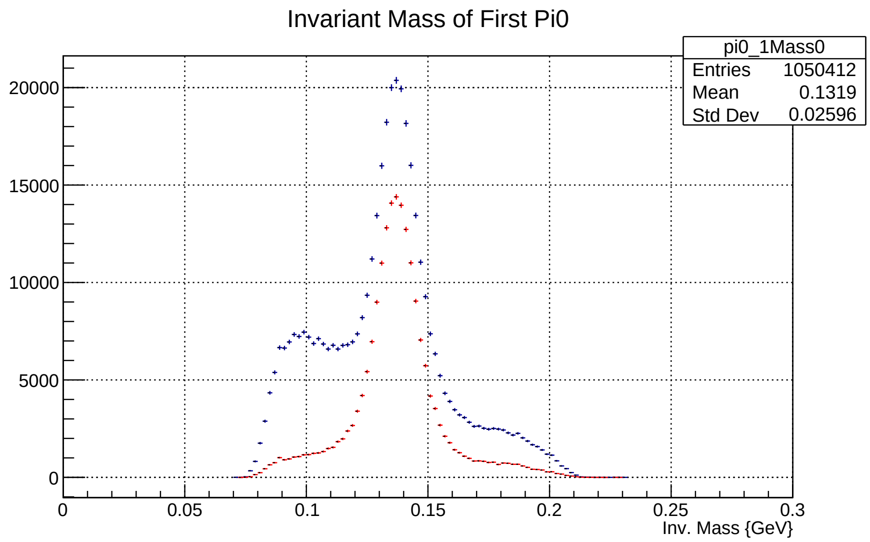


Figure 17: Invariant mass of the two photon system with(red) and without(blue) a cut on the invariant mass of the second pair of photons.

tagger radiator. The Beryllium target has a thickness of only 1.5 cm (compare to 30 cm liquid Helium and Hydrogen targets), which allows constraining interaction point (important for the neutral pions reconstruction without any additional vertex information from the tracking system). First we identified the two neutral pion exclusive photoproduction process using the energy ratio of two pions to the initial beam energy with the expected recoil energy subtracted. Fig. 18 shows this distribution for pions detected in FCAL and time accidentals and out-of-target beam interaction subtracted. We first required exactly four showers to be detected in FCAL and no extra showers in BCAL and COMCAL, a minimum shower energy of 0.5 GeV, and no neutral signals in TOF. The number of the signal events here is about 900, the width of the observed signal with pion kinematic fit to the mass is about 3 %, and the signal to background ratio value is promising. Fig. 19 shows two dimensional distribution of those events: elasticity vs invariant mass. One can see the horizontal line of the exclusive production events and vertical line of $K_{short} \rightarrow \pi^0\pi^0$ decays, which are separated from each other. Presence of $K_{short} \rightarrow \pi^0\pi^0$ decays in the data is really beneficial for the Primakoff analysis since it allows tuning the detector resolution in Monte-Carlo and make an assessment of the level of this value agreement with the data, which is essential for the successful cross-section fitting procedure and systematic uncertainty control.

Including BCAL showers in the neutral pion reconstruction increases the acceptance (especially for large invariant mass region) and number of observed events by an order of magnitude. For the beryllium target this increases the number of exclusive events to ∼10 K and for the helium target to ∼200 K events. Fig. 20 shows two π^0 invariant mass distribution with the energy within 10%

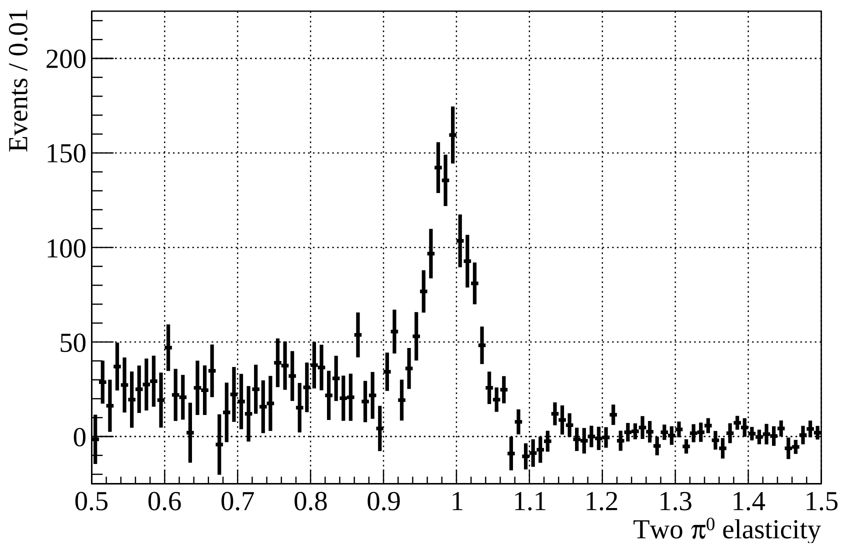


Figure 18: Two neutral pion elasticity (energy ratio to the expected value for the exclusive production) for the Beryllium target.

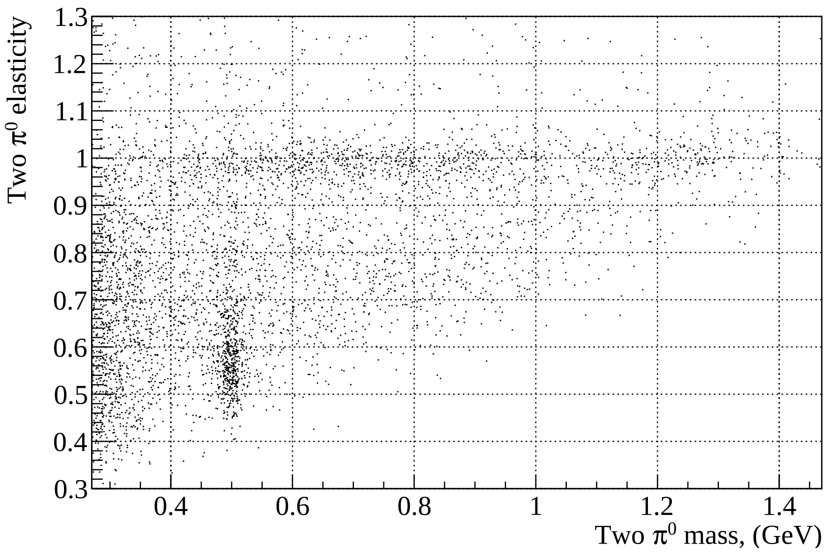


Figure 19: Two neutral pion elasticity (energy ratio to the expected value for the exclusive production) vs invariant mass of two pions for the Beryllium target.

of the expected for the exclusive production with BCAL included for low production angle events (below one degree). The f_2 meson peak is clearly seen. Fig. 21 shows elasticity distribution for both helium and beryllium targets with BCAL reconstructions included (time accidentals and "empty" target background subtracted).

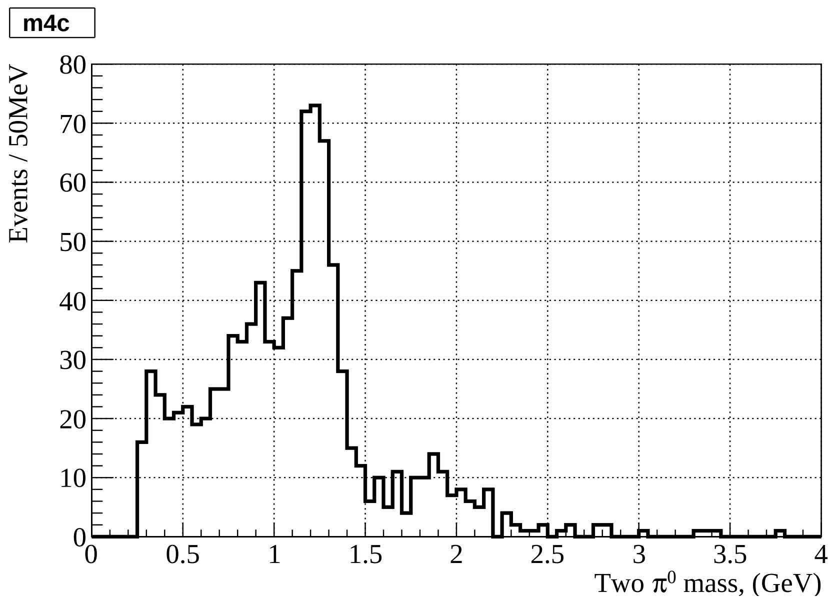


Figure 20: Two neutral pion invariant mass for the exclusive events (within 10% energy), the Beryllium target and production angle below one degree.

To conclude this section, we wish to highlight the good detector resolution for two π^0 production kinematics variables, the presence of the calibration process (K_{short}) in the data and controllable level of backgrounds observed for light nuclear targets exposition.

6 Photon beam flux

6.1 Photon beam flux accounting with the GlueX pair spectrometer

The photon beam flux can be directly extracted by analyzing the pair spectrometer (PS) data with the thin beryllium converter installed in the beam in from of it. The absolute normalization of the PS performed with the total absorption counter (TAC) during the dedicated run.

The systematics from the photon beam flux accounting by pair spectrometer is originated from

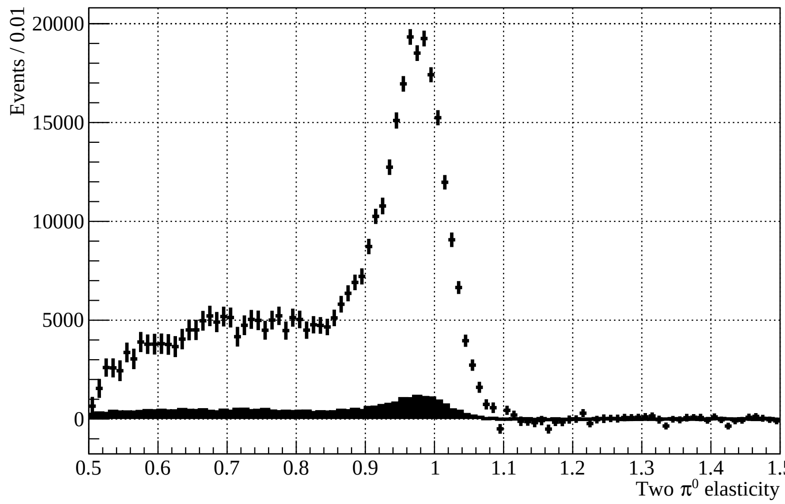


Figure 21: Two pion elasticity distribution with BCAL included in the analysis. "Empty" target and time accidentals are subtracted. Open histogram - Helium target, ∼200 K events in the elastic peak; solid histogram - Beryllium target, ∼10 K events in the peak.

few main contributions: overall spectrometer calibration with TAC quality; accuracy of the Monte-Carlo simulation of this process; long term stability of the spectrometer performance; and change of conditions between low intensity beam (TAC calibration) and production intensity. There are few other less significant contributions. GlueX PS acceptance [23] shown on Fig. 22. For the proposed experiment PS magnetic field should be reduced to cover the beam energy range $5 - 6\,GeV$. The methodology and accuracy of the PS analysis is the same as in PrimEx-D experiment, currently running in Hall-D, and has value $\sim 1 - 1.5\,\%$ [24].

6.2 Cross section verification with the exclusive single π^0 photoproduction

The extracted cross section can also be normalized on or independently from PS analysis verified with the π^0 radiative decay width extraction. Fig. 23 shows exclusive single π^0 photoproduction yield at forward angle obtained by the PrimEx experiment and used for π^0 radiative decay width extraction. The photon beam flux in PrimEx was 0.725×10^{12} for 4.9-5.5 GeV bremsstrahlung spectrum part on 5% rad. len. lead target. The distance between calorimiter and target was $\sim 7.3\,m$ and the central square part of the calorimeter, used in analysis was $\sim 70 \times 70\,cm$. These conditions have to be compared with the proposed experiment conditions: 20 days of 10^7 collimated beam photon/sec (i.e. 20 times more than PrimEx lead target beam flux), the distance between

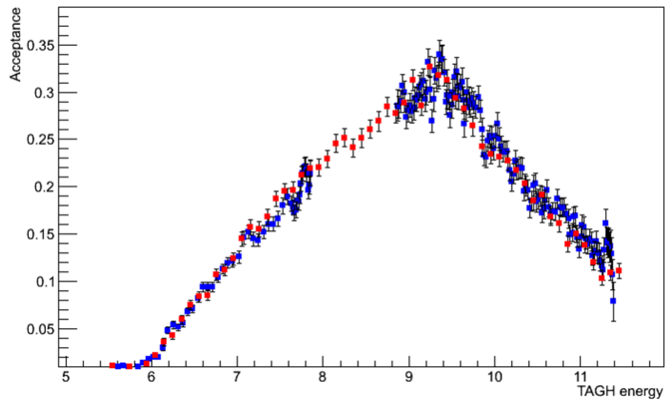


Figure 22: GlueX PS acceptance extracted from TAC data analysis (blue points); red points – Monte-Carlo simulation

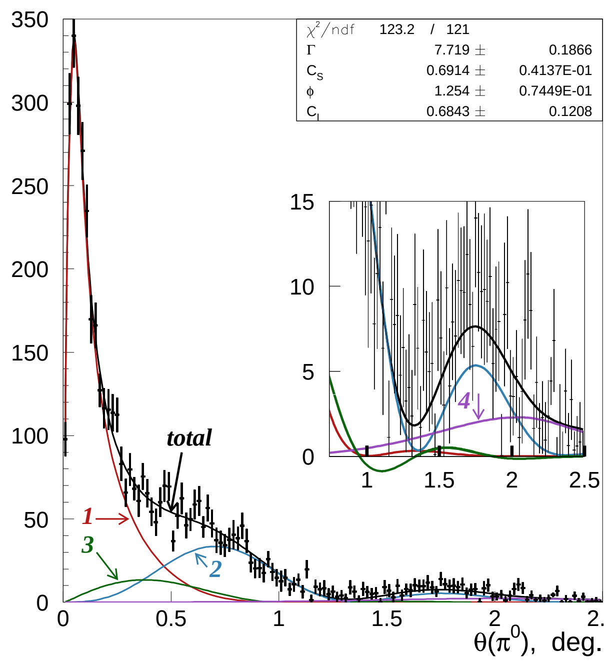


Figure 23: Exclusive π^0 production yield at forward angle on lead target observed in the PrimEx experiment [25]. Curves show production mechanisms input: 1 – Primakoff, 2 – strong coherent, 3 – interference of first two mechanisms, 4 – strong incoherent

target and FCAL $\sim 6.2\,m$ and active calorimeter part diameter $\sim 2\,m$. The central hole with one calorimeter modules layer around which should be excluded from the analysis for PrimEx case was $\sim 8 \times 8\,cm$ and for FCAL $\sim 20 \times 20\,cm$, which is decreasing FCAL acceptance at forward angle. Comparison of these experimental conditions allows us expecting an order of magnitude higher exclusive single π^0 photoproduction statistics. Thus PrimEx statistical uncertainty for lead will be decreased from $\sim 2.5\,\%$ down to $\sim 1\,\%$. For the systematical uncertainty, in PrimEx it was $\sim 2.1\,\%$ and has two major contributions: yield extraction ($\sim 1.6\,\%$) and photon beam flux accounting ($\sim 1.0\,\%$). The first contribution is partly statistically driven and reduces with increasing of statistics; and the second one cancels out since it is the same photon beam flux for the single and double exclusive π^0 photoproduction. The main factors increasing systematics for the proposed experiment are: the angular resolution of FCAL is about a factor of two worse than for PWO crystals used in the PrimEx analysis; and the magnetic field is not swiping out charged background like it was in PrimEx. As a result we can expect slightly worse systematical uncertainty than in PrimEx and statistical precision of $\sim 1\,\%$, i.e. total error $2.5 - 3.5\,\%$ for π^0 radiative width extraction (excluding absolute photon beam flux accounting, target number of atoms and partly FCAL trigger efficiency contributions to the systematics which are canceling out). The expected total beam flux uncertainty for such a normalization should also include the PrimEx total error of the π^0 radiative width, which was recently reported as 1.5% [26]. All this gives $\sim 3 - 4\,\%$ error for photon beam flux from normalization to the re-extracted π^0 radiative decay width.

6.3 Muon pair production

In addition to these normalization channels, production of muon pairs, which has a known cross section, can be used as a measurement of photon flux. Since the experiment will be running concurrently with the Charged Pion Polarizability (CPP) experiment, the photon flux on target will be the same by definition. CPP plans to use muon pair creation by beam photons as its main normalization channel, and so those measurements will be available for normalization of the neutral pion channel as well. In the case of CPP, the GlueX track finding and fitting efficiency will have to be determined for muon pairs, but any systematic error in that determination will largely cancel when applied to charged pion pairs. That will not be the case for the neutral pion channel and will have to be taken into account when evaluating systematic errors due to this method of normalization. In any case, muon pair production should provide a useful check on the other methods mentioned above.

Table 2: Uncertainties in the extraction of π^0 polarizabilities $\alpha_{\pi^0} - \beta_{\pi^0}$.

	Source	Uncertainty
1	Signal extraction	5 %
2	Flux normalization	1.5 %
3	Background subtraction	<1 %
4	Detector acceptance and efficiency	3.5 %
5	Total systematic error	3.9%
6	Total error on cross section	6.3%
7	Projected error in $\alpha - \beta$	49%

7 Errors and Sensitivity

We summarize the anticipated errors in the determination of the π^0 polarizability. We assume 20 days of running on a 5% radiation length ^{208}Pb target, 10^7 photons/s, and nominal acceptance for $\pi^0\pi^0$. Table 2 summarizes the estimated statistical and systematic errors. In the following we describe each of these contributions in detail:

1. Primakoff signal extraction from the angular distribution (Fig. 15), which also contains nuclear coherent contributions to the cross section. This uncertainty is approximately 5%.

2. Flux normalization. We have several methods for determining the flux (Section 6). The Primex-D experiment expects an uncertainty of 1.5% and we use that as our estimate here.

3. Subtraction of backgrounds other than nuclear coherent. These backgrounds are not expected to be significant ($< 1\%$) and studies are on-going.

4. Detector acceptance and efficiency. We can measure the detector acceptance times efficiency for the process $\gamma \mathrm{Pb} \to \pi^0 \mathrm{Pb}$ with an accuracy of 3.5% (Section 6.2), which should allow us to reduce the systematic uncertainty in the acceptance calculation for the process of interest to this level.

5. Total systematic error (items 2-4): combining the systematic errors in quadrature gives 3.9%.

6. Error on cross section (quadrature sum of items 1 and 5): 6.3%.

7. The current estimate by Dai and Pennington (Table II in Ref. [17]) indicates that a 13% determination of $\sigma(\gamma\gamma \to \pi^0\pi^0)$ will determine the combination $\alpha_{\pi^0} - \beta_{\pi^0}$ to a precision of 100%, i.e., $\Delta(\alpha_{\pi^0} - \beta_{\pi^0}) \sim 7.7\Delta(\sigma)$. From here we estimate that our uncertainty on

$\Delta(\alpha_{\pi^0} - \beta_{\pi^0}) \sim 49\%$. We note that the basis for extracting the polarizabilities may be improved in the near future and theoretical effort is being directed specifically toward this goal.

Table 3: Approved beam request and running conditions for CPP. NPP would run concurrently.

Running condition	
Days for production running	20
Days for calibrations	5
Target	^{208}Pb
Photon intensity in coherent peak	10^7 photons/s
Edge of coherent peak	6 GeV

8 Summary and beam request

We have investigated the possibility of determining the neutral pion polarizabilities $\alpha_{\pi^0} - \beta_{\pi^0}$, a quantity for which there are no existing measurements. Our proposal is to extract the polarizability from a measurement of the cross section of the Primakoff reaction $\gamma\text{Pb} \rightarrow \pi^0\pi^0\text{Pb}$. We propose to make this measurement using data taken simultaneously with the CPP[1] experiment in Hall D. Table 3 summarizes the approved beam request for the CPP experiment. The existing GlueX detector has sufficient resolution and high acceptance for this process. We expect to collect approximately 2500 signal events during the approved 20 PAC days. The anticipated statistical uncertainties on the signal represent a significant improvement over existing data as shown in Fig. 24. Using the estimate by Dai and Pennington [17] we expect to be able to make the first extraction of the $\alpha_{\pi^0} - \beta_{\pi^0}$ polarizability with an uncertainty of 49%.

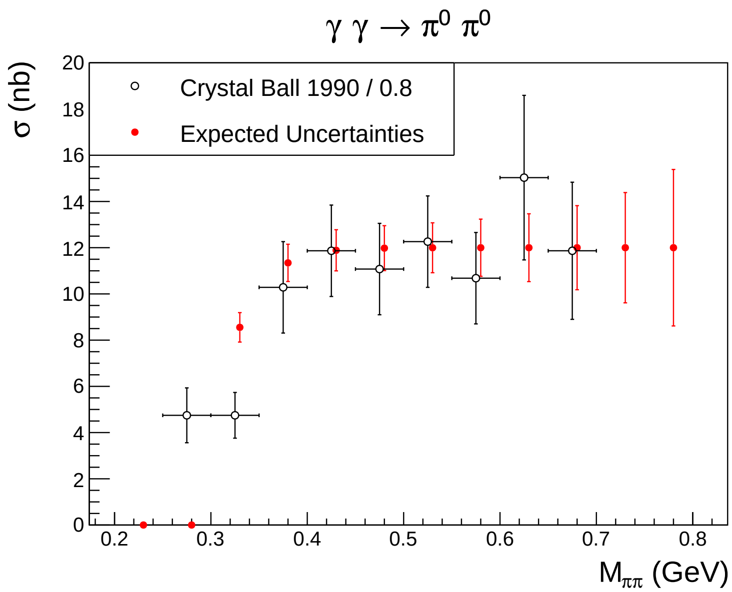


Figure 24: Estimated statistical uncertainties on determining $\sigma(\gamma\gamma \to \pi^0\pi^0)$ during 20 PAC days running simultaneously with the approved CPP experiment. The data points from the single previous Cristal Ball measurement [4] are shown for comparison.

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A More on theoretical predictions

The scattering amplitude for $\gamma\gamma^* \to \pi^0\pi^0$ is given in terms of the Compton tensor, whose low energy expansion in the Compton scattering channel $\gamma\pi^0 \to \gamma\pi^0$ is given in terms of the electric and magnetic polarizabilities of the π^0. For the case of interest with one real photon, the Compton tensor is given in by two amplitudes, namely:

$$\begin{aligned} T_{\mu\nu} &= -(A(s,t,u) + \frac{1}{4}B(s,t,u))(\frac{1}{2}\, s\, g_{\mu\nu} - k_\nu q_\mu) && (6) \\ &+ \frac{1}{4s}B(s,t,u)((s-q^2)p_{-\mu}p_{-\nu} - 2(k\cdot p_-\, q_\mu p_{-\nu} + q\cdot p_- k_\nu p_{-\mu} - g_{\mu\nu} k\cdot p_-\;\, q\cdot p_-) && (7) \end{aligned}$$

Here $s = W_{\pi\pi}^2$ is the invariant mass squared of the two π^0s, k the momentum of the beam photon, q the momentum of the virtual photon, and p_- the $p_- = p_1 - p_2$ the momentum difference between the two pions.

The limit of interest for the polarizabilities is:

$$\begin{aligned} \alpha_\pi &= -\frac{\alpha}{2M_\pi}(A(s,t,u) - \frac{2}{s}M_\pi^2 B(s,t,u))|_{s=0,t=u=M_\pi^2} \\ \beta_\pi &= \frac{\alpha}{2M_\pi}A|_{s=0,t=u=M_\pi^2} && (8) \end{aligned}$$

where α_π β_π are the electric and magnetic polarizabilities respectively.

The low energy limit is analyzed in ChPT. At the lowest significant order, i.e., one loop, the π^0 polarizabilities are entirely given in terms of known quantities, namely:

$$\alpha_{\pi_0} = -\beta_{\pi_0} = -\frac{\alpha}{96\pi^2 M_\pi F_\pi^2} \simeq -0.55\times 10^{-4}\ \text{fm}^3 \qquad (9)$$

The positive magnetic susceptibility indicates that the π_0 is diamagnetic, and naturally the negative electric polarizability tells that it behaves as a dielectric.

There are higher order corrections in the chiral expansion to the above prediction corresponding to a two-loop calculation, which is undefined up to two low energy constants $h_\pm$ in the notation of Ref. [5], expected to be significant for the corrections.

The amplitudes A and B are constrained by unitarity and analiticity to satisfy dispersion relations. In particular below $s \sim 0.8$ GeV^2 the dominant contributions are for the pair of pions in an S-wave. The rather well established S-wave phase shifts thus allow for implementing dispersion relations [15, 8, 11, 9, 10, 17]. In this proposal the model by Donoghue and Holstein [15] for implementing the dispersive representation using S-wave final state interaction was adopted. The model implements twice subtracted dispersion relations for the isospin 0 and 2 components of the amplitude A with the addition of t- and u-channel resonance exchanges for both A and B. The four subtraction constants require the experimental input of the cross section to be measured by the proposed experiment.

A summary of useful theory results is the following:

1) representation of the Compton amplitudes:

$$
\begin{aligned}
s\,A(s,t,u) &= -\frac{2}{3}(f_0(s)-f_2(s)) + \frac{2}{3}(p_0(s)-p_2(s)) - \frac{s}{2}\sum_{V=\rho,\omega} R_V\left(\frac{t+M_\pi^2}{t-M_V^2}+\frac{u+M_\pi^2}{u-M_V^2}\right) \\
B(s,t,u) &= -\frac{1}{8}\sum_{V=\rho,\omega} R_V\left(\frac{1}{t-M_V^2}+\frac{1}{u-M_V^2}\right) \\
R_V &= \frac{6M_V^2}{\alpha}\frac{\Gamma(V\to\pi\gamma)}{(M_V^2-M_\pi^2)^3}
\end{aligned}
\qquad (10)
$$

where $V=\rho,\ \omega$,

$$
\begin{aligned}
p_I(s) &= f_I^{\rm Born}(s) + p_I^A(s) + p_I^\rho(s) + p_I^\omega(s) \\
p_0^A(s) = p_2^A(s) &= \frac{L_9^r + L_{10}^r}{F_\pi^2}\left(s + \frac{M_A^2 - M_\pi^2}{\beta(s)}\log\frac{1+\beta(s)+s_A/s}{1-\beta(s)+s_A/s}\right) \\
p_0^\rho(s) &= \frac{3}{2}R_\rho\left(\frac{M_\rho^2}{\beta(s)}\log\frac{1+\beta(s)+s_\rho/s}{1-\beta(s)+s_\rho/s}\right) \\
p_2^\rho(s) &= 0 \\
p_0^\omega(s) = -\frac{1}{2}p_0^\omega(s) &= -\frac{1}{2}R_\omega\left(\frac{M_\omega^2}{\beta(s)}\log\frac{1+\beta(s)+s_\omega/s}{1-\beta(s)+s_\omega/s} - s\right),
\end{aligned}
\qquad (11)
$$

where $\beta(s)=\sqrt{\frac{s-4M_\pi^2}{s}}$, M_A the mass of the A_1 resonance. The f_Is are given by the dispersive representation:

$$
f_I(s) = p_I(s) + \Omega_I(s)\left(c_I + d_I\,s - \frac{s^2}{\pi}\int_{4M_\pi^2}^\infty p_I(s')\mathrm{Im}(\Omega_I^{-1}(s'))\frac{ds'}{(s'-s)s'^2}\right), \qquad (12)
$$

with the Omnès function:

$$
\Omega_I(s>4M_\pi^2) = e^{i\phi_I(s)}\exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^\infty \frac{\phi_I(s')-\phi_I(s)}{s'-s}\frac{ds'}{s'} + \frac{\phi_I(s)}{\pi}\log\frac{4M_\pi^2}{s-4M_\pi^2}\right). \qquad (13)
$$

the phases ϕ_I are related to the corresponding $\pi\pi$ S-wave phase shifts according to:

$$
\begin{aligned}
\phi_0(s) &= \theta(M-\sqrt{s})\delta_0^0(s) + \theta(\sqrt{s}-M)(\pi-\delta_0^0(s)) \\
\phi_2(s) &= \delta_0^2(s),
\end{aligned}
\qquad (14)
$$

where M is the mass of the f_0 resonance.

The values used for the parameters entering the representations above are:

$$\begin{aligned} L_9^r + L_{10}^r &= 1.43 \pm 0.27 \times 10^{-3} \\ s_i &= 2(M_i^2 - M_\pi^2) \\ R_\omega &= 1.35/\mathrm{GeV}^2; \quad R_\rho = 0.12/\mathrm{GeV}^2 \end{aligned} \tag{15}$$

and the $\pi\pi$ phase shifts are well approximated up to $\sqrt{s} \sim 1.5$ GeV by the parametrization:

$$\delta_0^I(s) = \arcsin\left(\frac{\Gamma_I}{2\sqrt{(\sqrt{s}-M_I)^2 + \frac{\Gamma_I^2}{4}}}\right) + \sum_{n=0}^{N} a_n \ (\sqrt{s})^n \tag{16}$$

where we include one single resonance for each $I = 0, 2$.

For the available data we need only up to $N = 3$ for $I = 0$, with the result:

$$\begin{aligned} M_0 = 0.994 \text{ GeV}; &\qquad \Gamma_0 = 0.0624 \text{ GeV} \\ a_0 = -1.439; &\qquad a_1 = 6.461/\mathrm{GeV}; \quad a_2 = -5.529/\mathrm{GeV}^2; \quad a_3 = 2.022/\mathrm{GeV}^3 \end{aligned} \tag{17}$$

For the case $I = 2$ one finds that the resonance term is not needed at all and a good fit is provided with $N = 3$ with the result:

$$a_0 = -0.878; \qquad a_1 = -0.611/\mathrm{GeV}; \quad a_2 = -0.083/\mathrm{GeV}^2; \quad a_3 = 0.115/\mathrm{GeV}^3 \tag{18}$$

The $\gamma\gamma \to \pi_0\pi_0$ in the S-wave approximation valid up to about $\sqrt{s} \sim 0.9$ GeV is given by:

$$\begin{aligned} \sigma_{\gamma\gamma\to\pi^0\pi^0}(|\cos\theta| < Z)(s) &= \frac{\pi\alpha_{EM}^2}{s^2}\frac{Z}{2}\sqrt{s(s-4M_\pi^2)} \\ &\times \ (|\ A(s)s - M_\pi^2 B(s)\ |^2 \\ &+ \ \frac{1}{s^2}\left(M_\pi^4 - \frac{1}{16}(\frac{Z^2}{3}s(4M_\pi^2 - s) + 4(s-2M_\pi^2)^2)\right) |\ B(s)\ |^2) \end{aligned} \tag{19}$$

Fitting to the Cristal Ball data[4] the parameters $c_0,\ d_0,\ c_2,\ d_2$ can be estimated, giving in the corresponding units:

$$\begin{aligned} c_0 &= -0.529 \\ d_0 &= -2.033 \\ c_2 &= 0.953 \\ d_2 &= -1.271. \end{aligned} \tag{20}$$

B Parameterization of the nuclear coherent production

We consider the reaction $\gamma A \to m_{\pi\pi} A$, where $m_{\pi\pi} \to \pi\pi$ is a dipion system. The 2π system is treated as a particle with mass $m_{\pi\pi}$, which is produced with four-momentum transfer t. The cross section for a three-body final state can be written as [27]:

$$\begin{aligned} d\sigma &= \frac{1}{4\mathcal{F}}\, d\phi_3\, |\mathcal{A}|^2 && (21)\\ \mathcal{F} &= p_\gamma^{cm}\sqrt{s} && (22)\\ d\phi_3 &= \frac{4}{(4\pi)^5}\frac{p_\sigma^{cm}}{\sqrt{s}} d\Omega_\sigma^{cm}\, p_\pi^\sigma dm_{\pi\pi} d\Omega_\pi^\sigma && (23)\\ \frac{dt}{d\Omega_\sigma^{cm}} &= \frac{dt}{d\cos\theta_\sigma^{cm} d\phi_\sigma^{cm}} = \frac{2\, p_\gamma^{cm} p_\sigma^{cm}}{d\phi_\sigma^{cm}} && (24) \end{aligned}$$

The center-of-mass energy (cm) energy and the momentum transfer are represented by the commonly used variables s and t. Other variables are subscripted by particle name and their superscripts indicate the reference frame. Thus p_γ^{cm} is the incident photon momentum, p_σ^{cm} is the scattered momentum, and Ω_σ^{cm} corresponds to the solid angle of the σ, all in the cm frame. The momentum of the pions in the σ rest frame is denoted by p_π^σ and Ω_π^σ denotes the solid angle of one of them. Thus the cross section can be written as

$$\frac{d\sigma}{dt dm_{\pi\pi} d\phi_\sigma^{cm} d\Omega_\pi^\sigma} = \frac{1}{2(4\pi)^5}\frac{p_\pi^\sigma}{(p_\gamma^{cm})^2 s} |\sum_i \mathcal{A}^i|^2, \qquad (25)$$

where the index i runs over the number of resonances or mechanisms included in the calculation. We will assume that we can parameterize each production amplitude as a factorized product

$$\mathcal{A}^i = \mathcal{A}_t(t)^i\, \mathcal{A}_W(m_{\pi\pi})^i\, \mathcal{A}_\tau(\Phi, \phi, \theta)^i. \qquad (26)$$

For simplicity, we will drop the superscript i since for the moment we are considering single production mechanism. The function $\mathcal{A}_\tau(\phi_{\pi\pi}, \phi_\pi, \theta_\pi)$ contains the angular dependence of the produced pions, where (θ_π, ϕ_π) are the decay angles in the rest frame of the 2π system, which is flat for S-wave production. Azimuthal symmetry is broken by the photon polarization, where $\phi_{\pi\pi}$ is the angle between the plane of photon polarization and the production plane. The amplitudes are given by Eq. 41 and lead to a cross section dependence of the form $\mathcal{A}_\tau \propto (1 + \mathcal{P}\cos 2\phi_{\pi\pi})$.

The primary background in this mass region is given by the $f_0(500)(J^{PC} = 0^{++})$ also called the σ. The σ has the same angular structure as the Primakoff reaction and can only be identified through its dependence on t and $m_{\pi\pi}$. Our parameterization of the mass dependence for the σ meson is described in Section B.1.

We assume the $-t$ dependence of the σ has a similar form as for single π^0 production, namely $\mathcal{A}_t(t) \propto \sin\theta_{\pi\pi} \times F_{st}(t)$. The $\sin\theta_{\pi\pi}$ comes from the spin-flip required at forward angles to produce a 0^+ system from a spin-zero target. The factor $F_{st}(t)$ is the strong form factor for the target, which is approximated to match calculations for the single π^0 production (Fig. 6 from Ref.[22]).

B.1 Parameterization of the s-wave amplitude

There is considerable strength in the 2π channel coming from s-wave production, which is due to the now established $f_0(500)$ meson. It is also commonly referred to as the σ meson. We assume the amplitude for σ production is governed by the $\pi\pi$ J=0, I=0 phase shifts. We parameterize the $m_{\pi\pi}$ dependence as

$$\mathcal{A}_W(m_{\pi\pi}) \sim \frac{m_{\pi\pi}}{2k} \sin\delta_0 e^{i\delta_0} \left(\alpha_1 + \alpha_2 m_{\pi\pi}^2\right) + \cos\delta_0 e^{i\delta_0} \left(\alpha_3 + \alpha_4 m_{\pi\pi}^2\right), \tag{27}$$

where δ_0 is the s-wave phase shift for $I = 0$ and α_i (i=1, 2, 3, 4) are empirical constants to be obtained from data. The first term is due to "compact source" production of the pion pair (see Eq. 5 from Ref. [28]) and the second term is due to production due to an "extended source," for example pion rescattering (see Eq. 5 from Ref. [29] and Eq. 9 from Ref. [30]). We use the parameterization for the s-wave phase shifts from Appendix D of Ref. [31]:[5]

$$\tan\delta_0 = \frac{2k}{m_{\pi\pi}} \left(A_0^0 + B_0^0 k^2 + C_0^0 k^4 + D_0^0 k^6\right) \left(\frac{4m_\pi^2 - s_0^0}{M_{\pi\pi}^2 - s_0^0}\right), \tag{28}$$

where we use the same notation as the reference with A_0^0 =0.225, B_0^0 =12.651 GeV^{-2}, C_0^0= -43.8454 GeV^{-4}, D_0^0=-87.1632 GeV^{-6}, and s_0^0=0.715311 GeV^2. We have converted the constants to units of GeV and evaluated the parameters for $a_0^0 = 0.225\, m_\pi^{-1}$, and $a_0^2 = -0.0371\, m_\pi^{-1}$. These fits are only valid below $m_{\pi\pi} < 0.9$ GeV because they do not properly include the $f_0(980)$.

The empirical constants in Eq. 27 were determined by fitting $|\mathcal{A}_W|^2$ to the S-wave contribution to the photoproduction cross section[6] measured by CLAS for $E_\gamma = 3-3.8$ GeV [32] for $-t = 0.4-0.5$ GeV^2. The fits are for $m_{\pi\pi} = 0.3-0.95$ GeV, which is our region of interest. All four parameters are needed to obtain a good representation to the central values of the data, although the uncertainty band in the data allow for a wide range of parameters. Assuming that the constants are real and relatively independent of energy and $-t$, we take the average of the fitted constants for our parameterization ($\alpha_1 = 8.4 \pm 1.4$, $\alpha_2 = -4.1 \pm 2.2$, $\alpha_3 = 2 \pm 1.1$, $\alpha_4 = 8 \pm 1.1$).

[5]See also Eq. 44 of Ref. [28].

[6]The data are available through the Durham HEP Databases, http://durpdg.dur.ac.uk/.

C Angular distribution in the helicity basis

C.1 Photon density matrix in the helicity basis

The linear polarization of the photon can be expressed as (Ref.[33] Eq. 18-19):

$$\begin{aligned} \rho(\gamma) &= \tfrac{1}{2}I + \tfrac{1}{2}\vec{P}_\gamma \cdot \vec{\sigma}, \text{ where} && (29)\\ \vec{P}_\gamma &= \mathcal{P}(-\cos 2\phi_{\pi\pi}, -\sin 2\phi_{\pi\pi}, 0) && (30) \end{aligned}$$

and $\vec{\sigma}$ are the Pauli matrices. The angle $\phi_{\pi\pi}$ is the angle between the polarization vector of the photon and the production plane and $\mathcal{P}$ represents the degree of linear polarization. Multiplying out these factors gives the expression for the photon density matrix in the helicity frame as (Ref.[34] Eq. 219):

$$\rho_{\epsilon,\epsilon'}(\gamma) = \frac{1}{2}\begin{pmatrix} 1 & -\mathcal{P}e^{-2i\phi_{\pi\pi}} \\ -\mathcal{P}e^{2i\phi_{\pi\pi}} & 1 \end{pmatrix} \qquad (31)$$

C.2 Parity constraints

We consider the reaction $a + b \to c + d$, where the spin of each particle is denoted by s_j, their helicity by λ_j and their intrinsic parity by η_j. If parity is conserved, there are relations between amplitudes with opposite helicities, which are given in Jacob and Wick [35] Eq. 43 and Ref.[33] Eq. 20 [7] (see also Ref. [36] Eq. 4.2.3):

$$^{\lambda_a}V^{\lambda_d\lambda_b}_{\lambda_c} = \left(\frac{\eta_c\eta_d}{\eta_a\eta_b}\right)(-1)^{s_c+s_d-s_a-s_b}(-1)^{(\lambda_c-\lambda_d)-(\lambda_a-\lambda_b)}\ {}^{-\lambda_a}V^{-\lambda_d-\lambda_b}_{-\lambda_c} \qquad (32)$$

C.3 S-wave production

For the case of S-wave production of two pions via the $f_0(500)$ or σ meson off an spinless target we have the following constraint:

$$^{\lambda_\gamma}V^{\lambda_Z\lambda_Z}_{\lambda_\sigma} = {}^{\epsilon}V^{00}_0 = \left(\frac{\eta_c+}{-+}\right)(-1)^1(-1)^{-1}\ {}^{-\epsilon}V^{00}_0 = -\eta_c\ {}^{-\epsilon}V^{00}_0 \qquad (33)$$

For convenience, we have separated out the parity of the scattered state η_c. The 2π intensity distribution (see Ref.[34] Eq. 220-223 and also Eqs. 264) is given by the following expression after

[7] We thank Adam Szczepaniak for clarifying the connection between these papers.

dropping the superscripts related to the target helicities and collapsing the sums over external and internal spins because both the target and resonance are 0^+ objects:

$$\begin{aligned} \mathcal{I} &= \sum_{\epsilon\epsilon'} {}^{\epsilon}V_0\, Y_0^0\, \rho_{\epsilon\epsilon'}\, {}^{\epsilon'}V_0^*\, Y_0^{0*} && (34) \\ &= \tfrac{1}{2}|Y_0^0|^2 \begin{pmatrix} {}^{1}V_0 & {}^{-1}V_0 \end{pmatrix} \begin{pmatrix} 1 & -\mathcal{P}e^{-2i\phi_{\pi\pi}} \\ -\mathcal{P}e^{2i\phi_{\pi\pi}} & 1 \end{pmatrix} \begin{pmatrix} {}^{1}V_0^* \\ {}^{-1}V_0^* \end{pmatrix} && (35) \\ &= \tfrac{1}{2}|Y_0^0|^2 \left[|{}^{1}V_0|^2 - \mathcal{P}\, {}^{1}V_0\, {}^{-1}V_0^*\, e^{-2i\phi_{\pi\pi}} - \mathcal{P}\, {}^{1}V_0^*\, {}^{-1}V_0\, e^{2i\phi_{\pi\pi}} + |{}^{-1}V_0|^2 \right] && (36) \end{aligned}$$

Noting that ${}^{1}V_0 = -\eta_c\, {}^{-1}V_0$, we obtain the following expression:

$$\mathcal{I} = \tfrac{1}{4\pi}|{}^{1}V_0|^2 \left(1 + \eta_c \mathcal{P}\cos 2\phi_{\pi\pi}\right), \qquad (37)$$

where $\phi_{\pi\pi}$ is the angle of the polarization vector relative to the production plane. For the case of σ production, $\eta_c = +1$, but for the case of π^0 production we have the opposite sign, $\eta_c = -1$. For the Primakoff production of $\pi^+\pi^-$ in S-wave, $\eta_c = (-1)(-1)(-1)^0 = +1$. See Ref. [1] Eq. 8.

The intensity distribution in Eq. 34 may be written in a more convenient form for use with AmpTools, namely

$$\begin{aligned} \mathcal{I} &= (\tfrac{1-\mathcal{P}}{4})|A_+|^2 + (\tfrac{1+\mathcal{P}}{4})|A_-|^2 && (38) \\ A_\pm &= Y_0^0({}^{1}V_0 \pm {}^{-1}V_0\, e^{2i\phi_{\pi\pi}}) && (39) \\ A_\pm &= Y_0^0\, {}^{1}V_0\, (1 \mp \eta_c\, e^{2i\phi_{\pi\pi}}), && (40) \end{aligned}$$

which can be written more symmetrically taking advantage of an arbitrary phase as

$$A_\pm = Y_0^0\, {}^{1}V_0\, (e^{-i\phi_{\pi\pi}} \mp \eta_c\, e^{i\phi_{\pi\pi}}). \qquad (41)$$