

Partial wave analysis studies with
simulated
 $\eta^{(\prime)}\pi^0$ events in GlueX

Florida International University 2020

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1. Introduction

- Evidence for π_1 (1600) exotic meson with π beam
- Search for exotic π_1 (1600) via $\gamma p \rightarrow p\eta'\pi^0$ in GlueX via Partial Wave Analysis (PWA)
- Newly developed model of intensity for $\eta\pi$ photoproduction
- The GlueX experiment

2. Developing and testing methods for PWA using AMPTOOLS package

- Calculating moments of angular distribution using fitted partial waves
 - Compare moments from different methods

3. Summary

π_1 (1600) results from studies of $\eta'\pi$ system with π beam incident on a p target

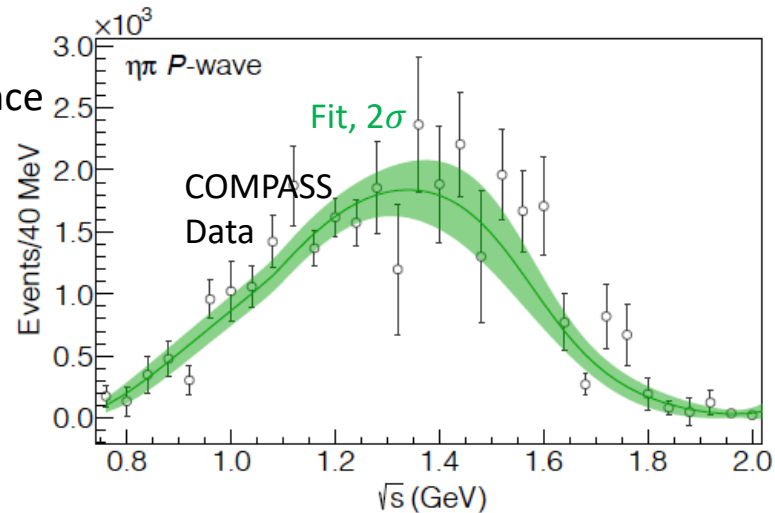
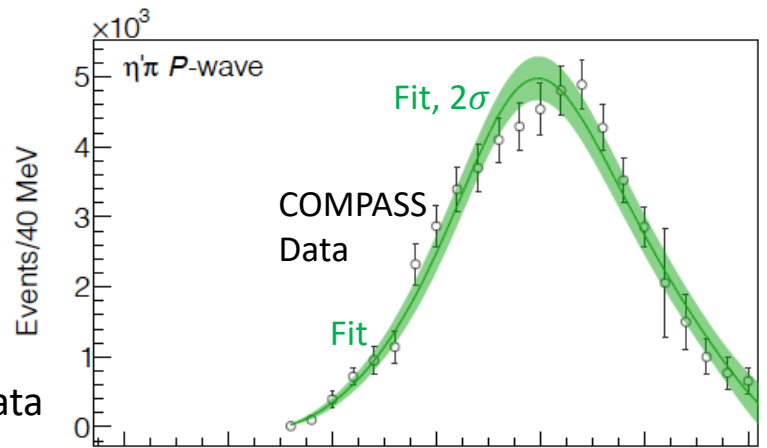
Evidence for exotic $1^G J^{PC} = 1^- 1^{+-}$ state π_1 (1600)

$G = C \cdot (-1)^I$, C operator followed by a rotation in isospin (I)

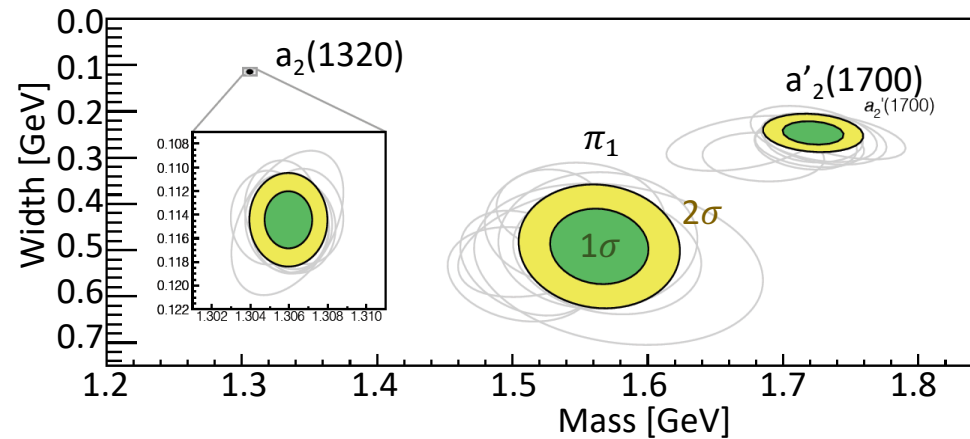
Several experiments suggest existence of π_1 :

- **VES**, $E_\pi = 37$ GeV/c (D. V. Amelin et al., Phys. Atom. Nucl. 68, 359 (2005))
- **E852**, $E_\pi = 18$ GeV/c (E. I. Ivanov et al. [E852 Collaboration], Phys. Rev. Lett. 86, 3977 (2001))
- **COMPASS**, $E_\pi = 191$ GeV/c (C. Adolph, et al. [COMPASS Collaboration], Phys. Lett. B740, 303 (2015))

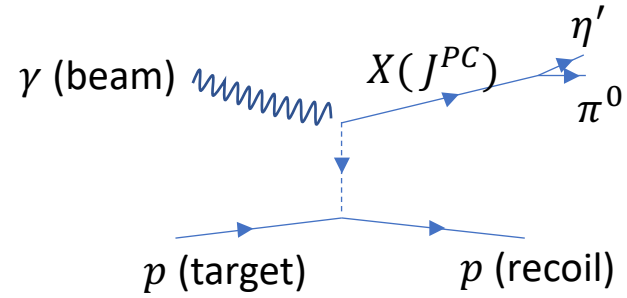
- Fit COMPASS data with amplitude model
- Extract resonance poles



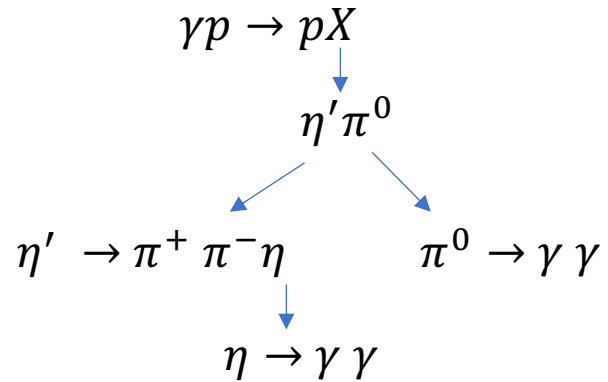
Joint Physics Analysis Center (JPAC)



t – channel exchange



Decay mode



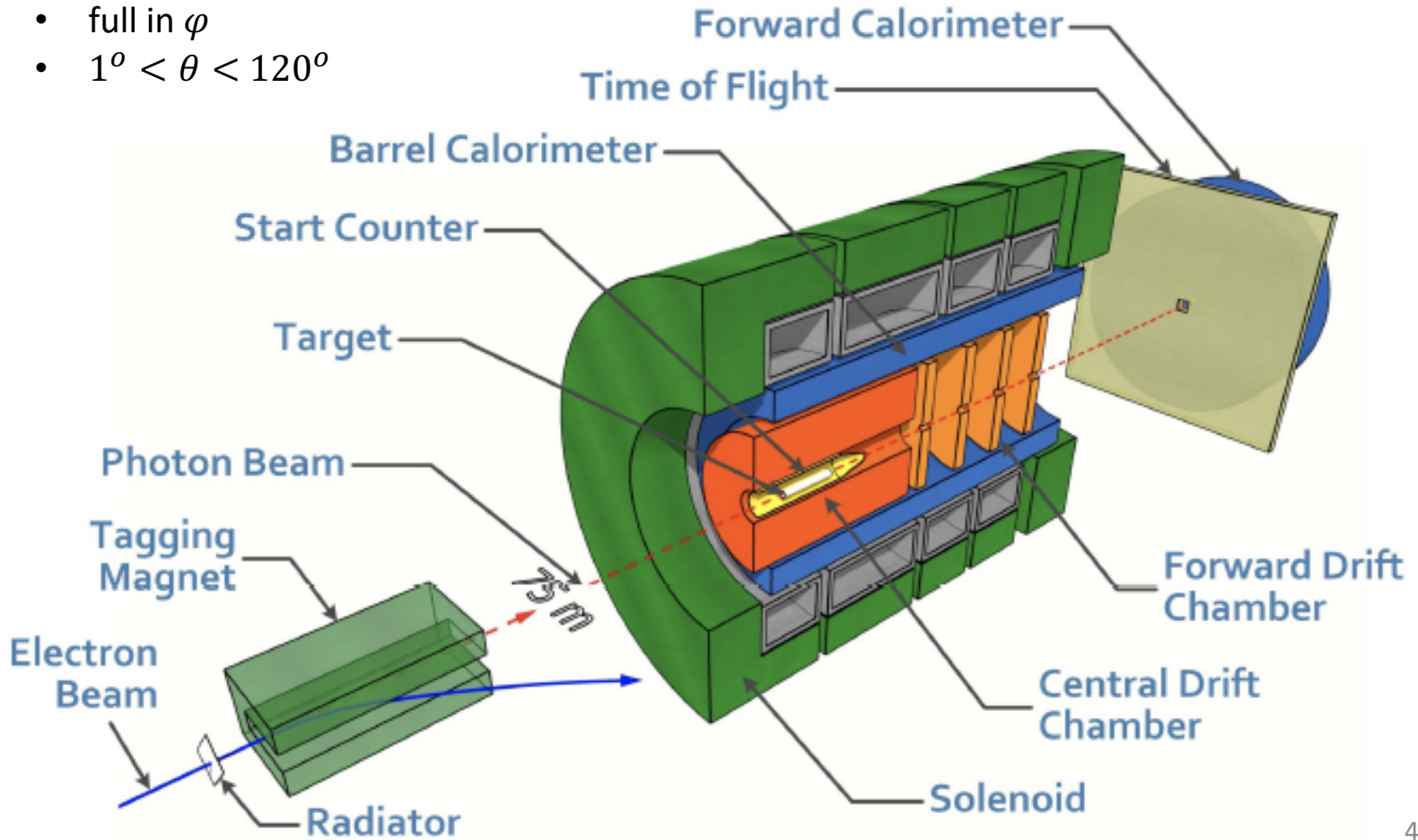
The GlueX experiment

Beam:

- Linearly polarized photon beam
- $E_\gamma \sim 9$ GeV

Angular coverage:

- full in φ
- $1^\circ < \theta < 120^\circ$



- Write intensity I in terms of partial wave amplitudes
- Introduce reflectivity basis to trade helicity λ for the reflectivity index $\epsilon=\pm 1$

$$I(\Omega, \Phi) = 2\kappa \sum_k \left\{ (1 - P_\gamma) \left| \sum_{l,m} [l]_{m;k}^{(-)} \text{Re}[Z_l^m(\Omega, \Phi)] \right|^2 + (1 - P_\gamma) \left| \sum_{l,m} [l]_{m;k}^{(+)} \text{Im}[Z_l^m(\Omega, \Phi)] \right|^2 \right. \\ \left. + (1 + P_\gamma) \left| \sum_{l,m} [l]_{m;k}^{(+)} \text{Re}[Z_l^m(\Omega, \Phi)] \right|^2 + (1 + P_\gamma) \left| \sum_{l,m} [l]_{m;k}^{(-)} \text{Im}[Z_l^m(\Omega, \Phi)] \right|^2 \right\}$$

$\Omega = (\theta, \varphi)$

l, m - spin, its projection

$\vec{\epsilon}'$ - γ polarization vector

P_γ - degree of polarization

$Z_l^m(\Omega, \Phi) \equiv Y_l^m(\Omega) e^{-i\Phi}$

κ - kinematical factors

Nucleon spin flip $k=1$, non-flip $k=0$

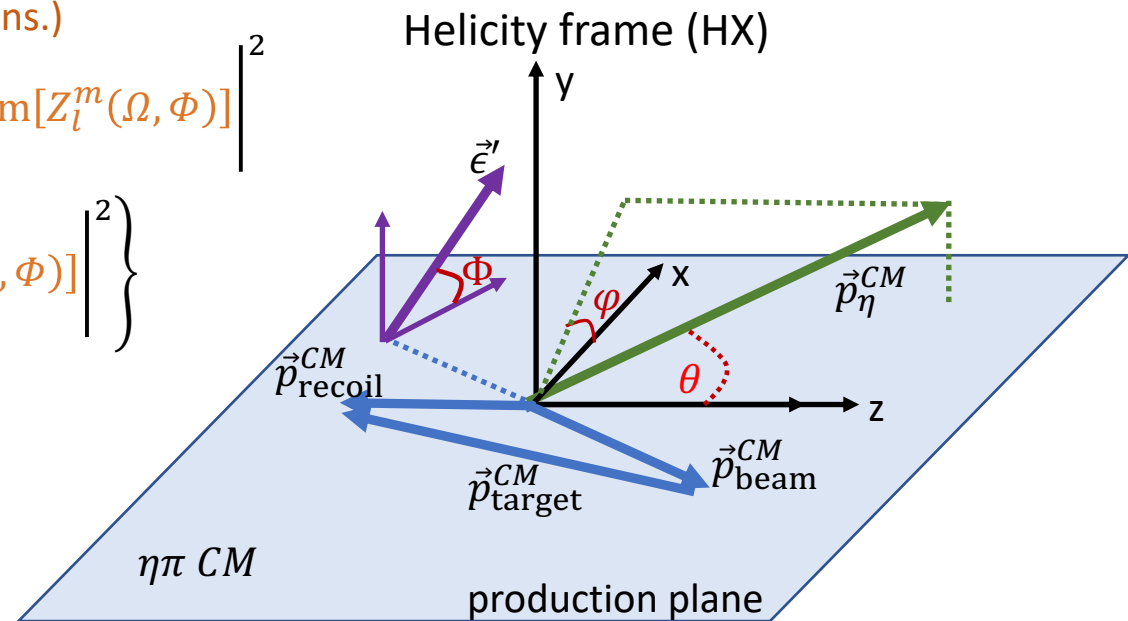
PWA

- Bin data in small bins of $m_{\eta\pi}$, t and E_γ with constant $[l]_{m;k}^{(-)}$, $[l]_{m;k}^{(+)}$
- Fit data using extended unbinned (in (θ, φ)) maximum likelihood method using AmpTools package (<https://github.com/mashephe/AmpTools>)

$$\ln L(l) = \sum_{i=1}^N \ln I(l, \theta, \varphi) - \int I(V, \theta, \varphi) \eta(\theta, \varphi) d\Omega$$

$\eta(\theta, \varphi)$ - acceptance

- Minimize $-\ln L$ using MINUIT, to find $[l]_{m;k}^{(-)}$, $[l]_{m;k}^{(+)}$



Generate $2 \cdot 10^6$ ($p\eta\pi^0$) events with AmpTools with the wave set $[l]_{m;k}^{(\epsilon)} = \{S_0^{(+)}, P_{0,1}^{(+)}, D_{0,1,2}^{(+)}\}_{k=0}$ and $M \geq 0$

- Generated resonances are
- a_0 (980 MeV)
 - π_1 (1600 MeV) (**exotic**)
 - a_2 (1320 MeV)
 - a_2' (1700 MeV)

$$P_\gamma = 0.3$$

1. Fit intensity with generated wave set to find partial waves using AmpTools.

2. Implement and test calculation of moments of angular distribution in terms of $[l]_{m;k}^{(\epsilon)}$ using the following expressions in terms of the $\eta'\pi^0$

SDMEs $\rho_{mm'}^{(\epsilon)\alpha, ll'}$:

$$H^0(LM) = \sum_{ll', mm'} \left(\frac{2l'+1}{2l+1}\right)^{1/2} C_{l'l'0L0}^{l0} C_{l'm'LM}^{lm} \rho_{mm'}^{0, ll'}$$

$$\rho_{mm'}^{0, ll'} = \sum_{\epsilon} \kappa \sum_k \left(|l]_{m;k}^{(\epsilon)} |l']_{m';k}^{(\epsilon)*} + (-1)^{m-m'} |l]_{-m;k}^{(\epsilon)} |l']_{-m';k}^{(\epsilon)*} \right)$$

• • •

$$H^1(LM) = - \sum_{ll', mm'} \left(\frac{2l'+1}{2l+1}\right)^{1/2} C_{l'l'0L0}^{l0} C_{l'm'LM}^{lm} \rho_{mm'}^{ll'}$$

where $C_{l'l'0L0}^{l0}$ and $C_{l'm'LM}^{lm}$ denote the Clebsch-Gordan coefficients, $0 \leq L \leq 4$ and $0 \leq M \leq L$.

Non-zero odd L moments \leftrightarrow presence of exotic wave. If $M=0$, H^2 and H^3 are 0.

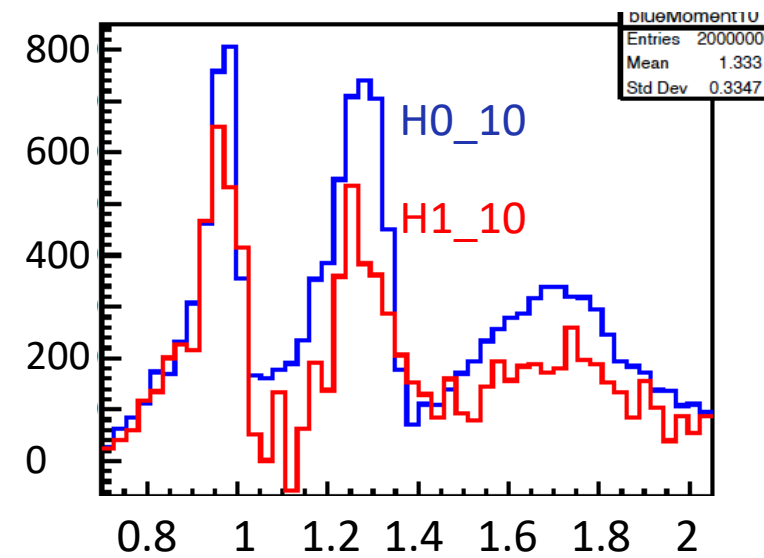
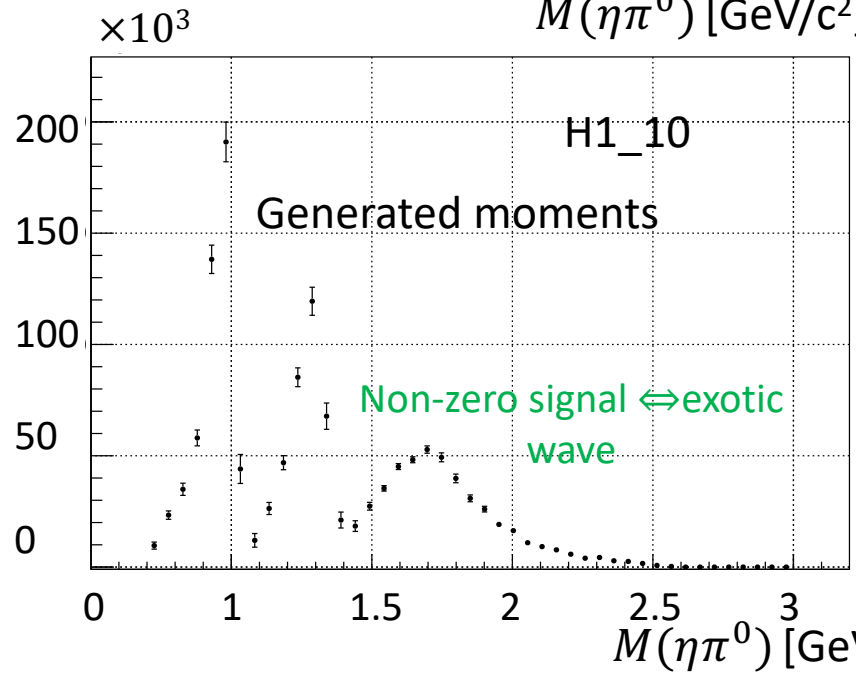
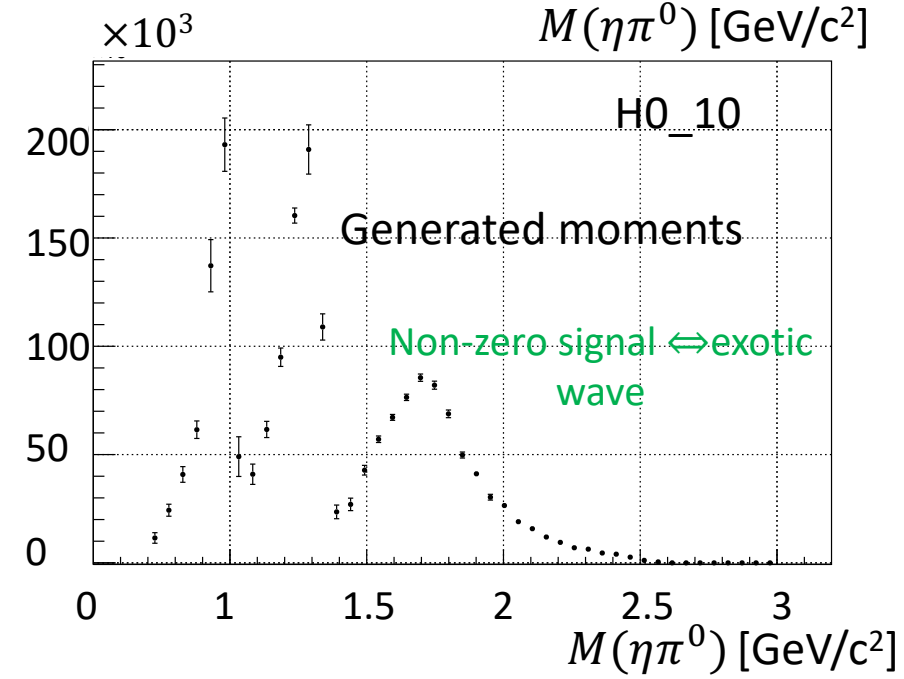
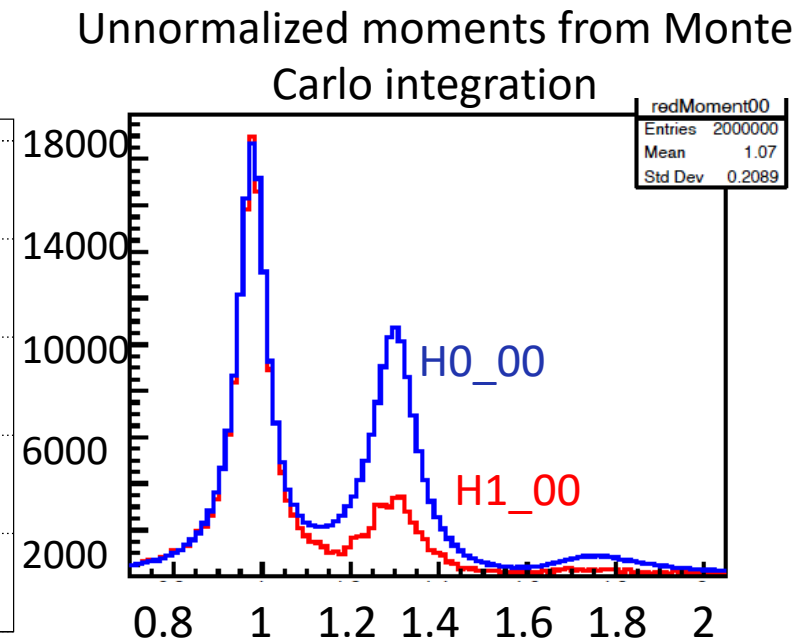
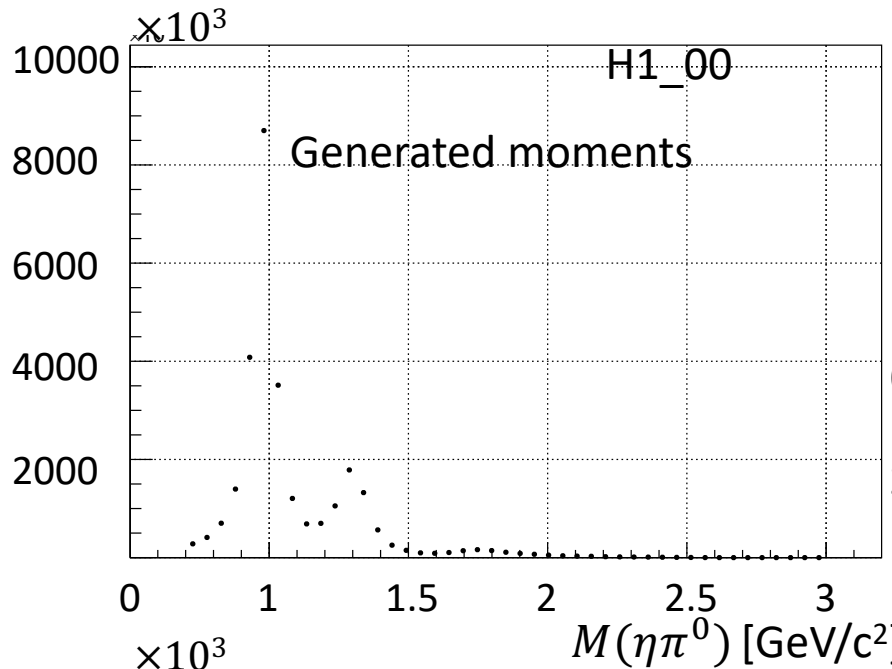
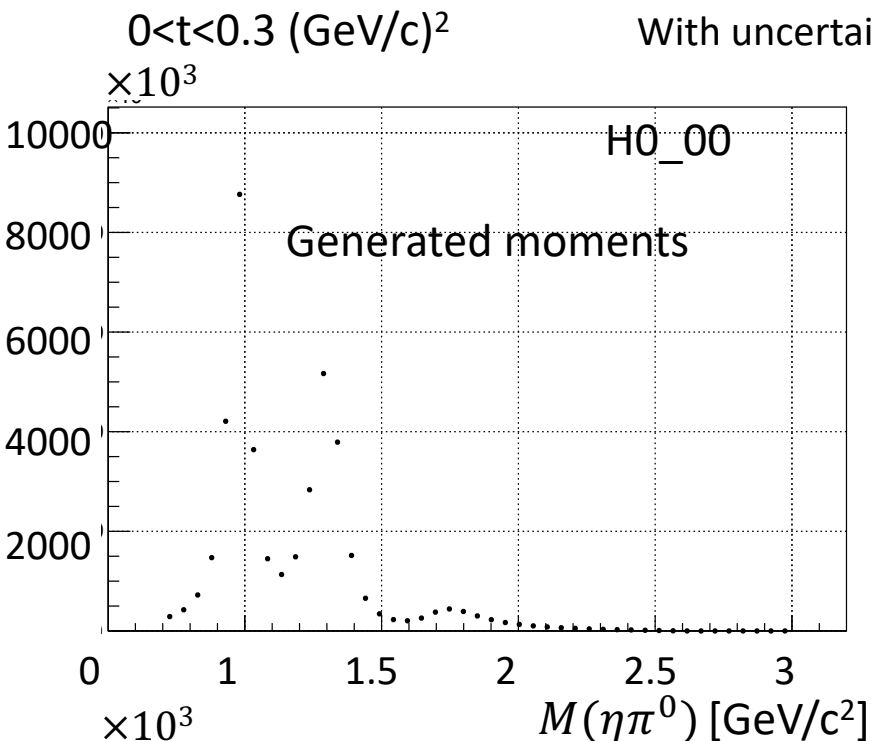
3. Compare to moment distributions obtained by Monte Carlo integration:

$$H^0(LM) = \frac{P_\gamma}{2} \int_0^\pi I(\Omega, \Phi) d_{M0}^L(\theta) \cos M\phi,$$

$$H^1(LM) = \int_0^\pi I(\Omega, \Phi) d_{M0}^L(\theta) \cos M\phi \cos 2\Phi$$

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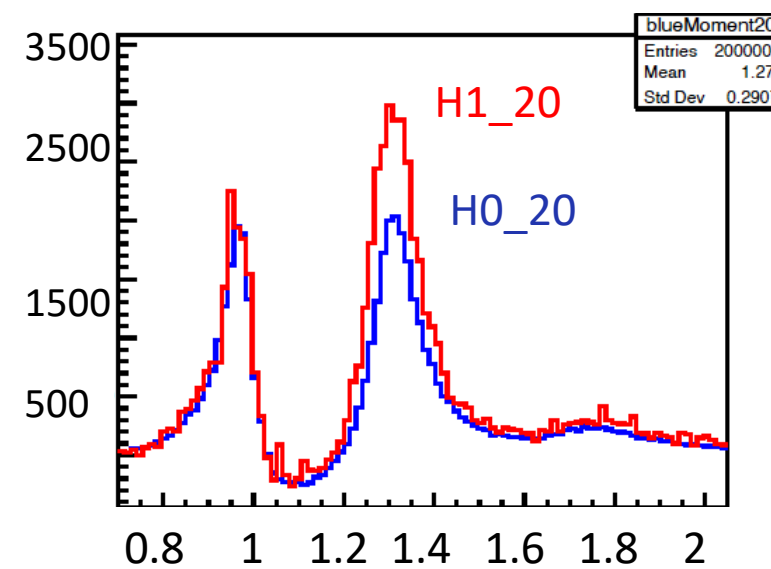
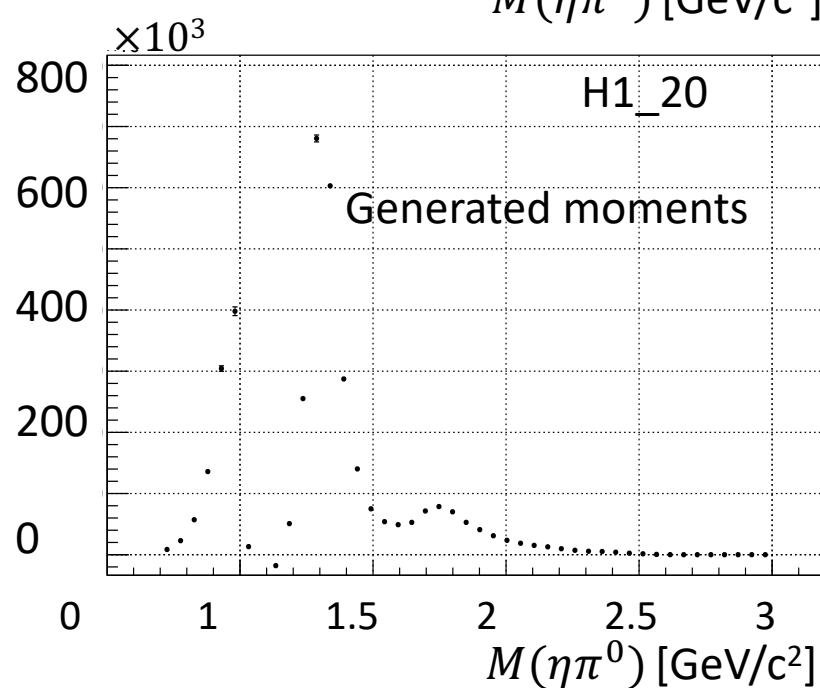
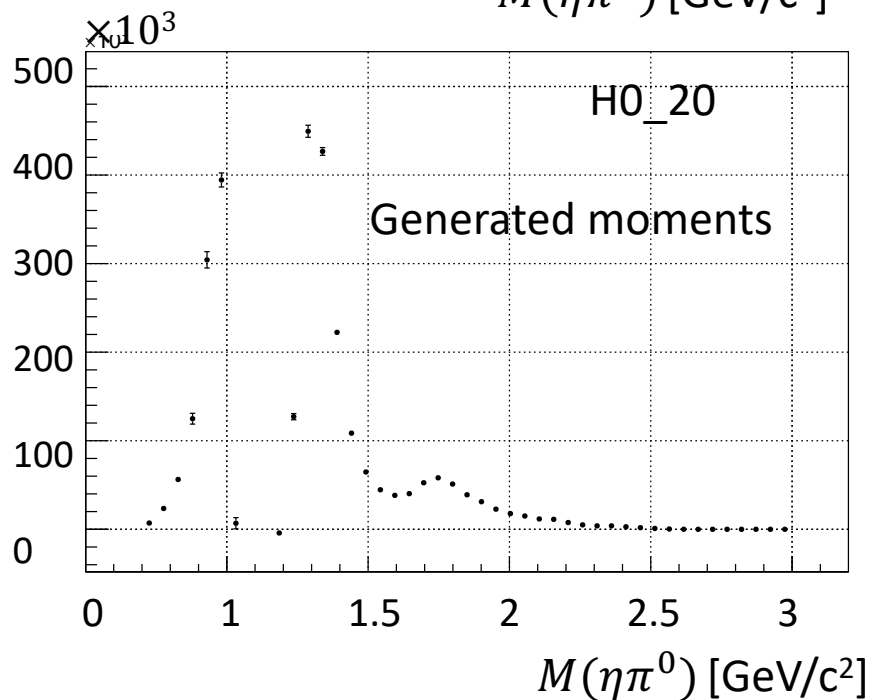
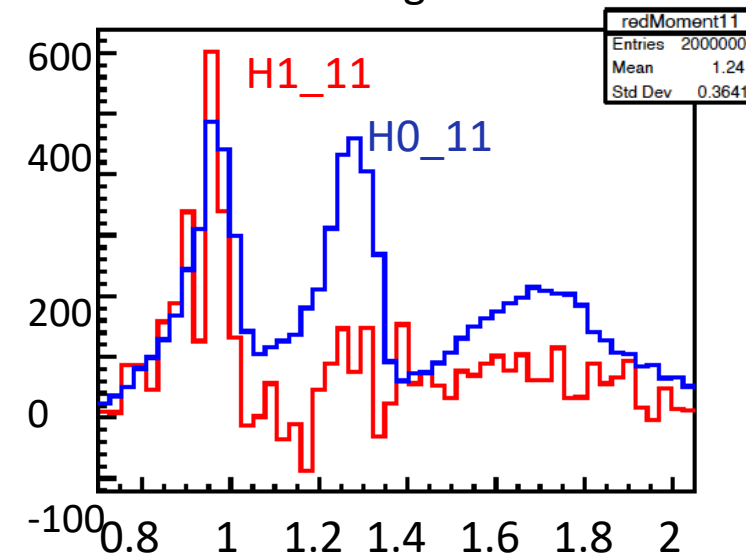
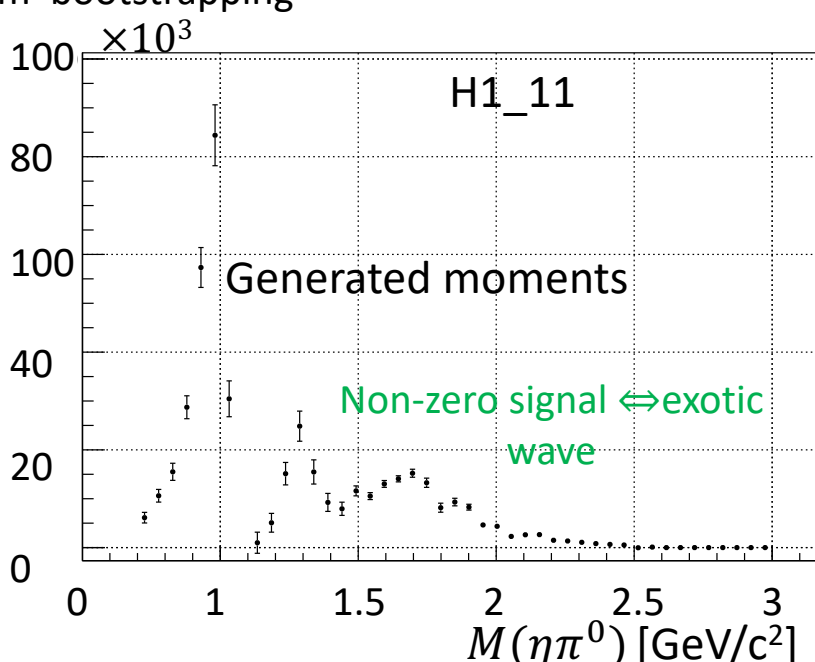
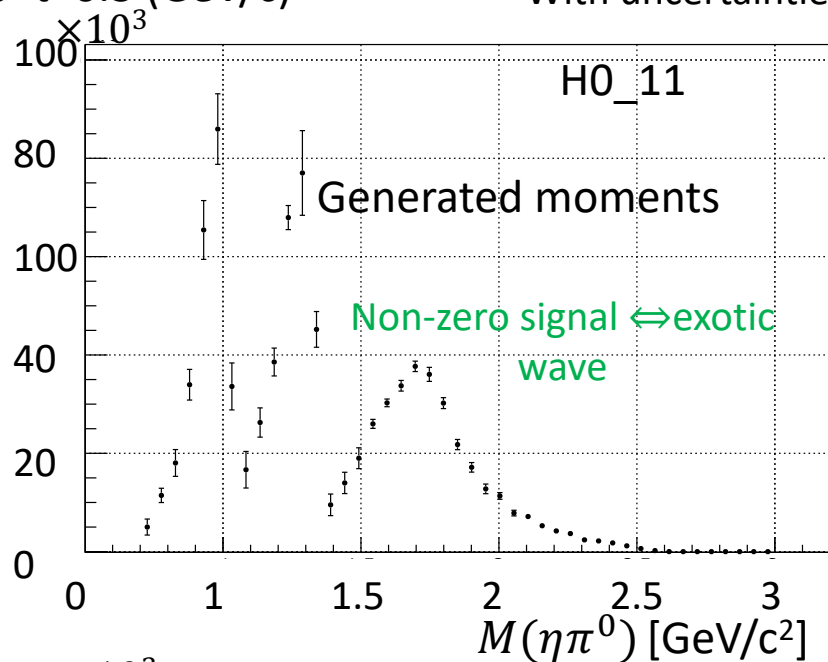
with $\int_0^\pi = (1/\pi P_\gamma) \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi \int_0^{2\pi} d\Phi$ and $d_{M0}^L(\theta)$ is Wigner d-function.

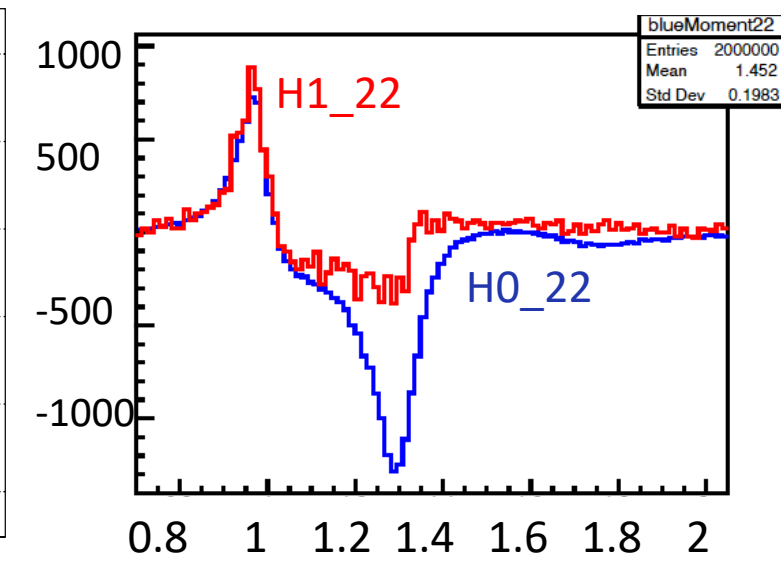
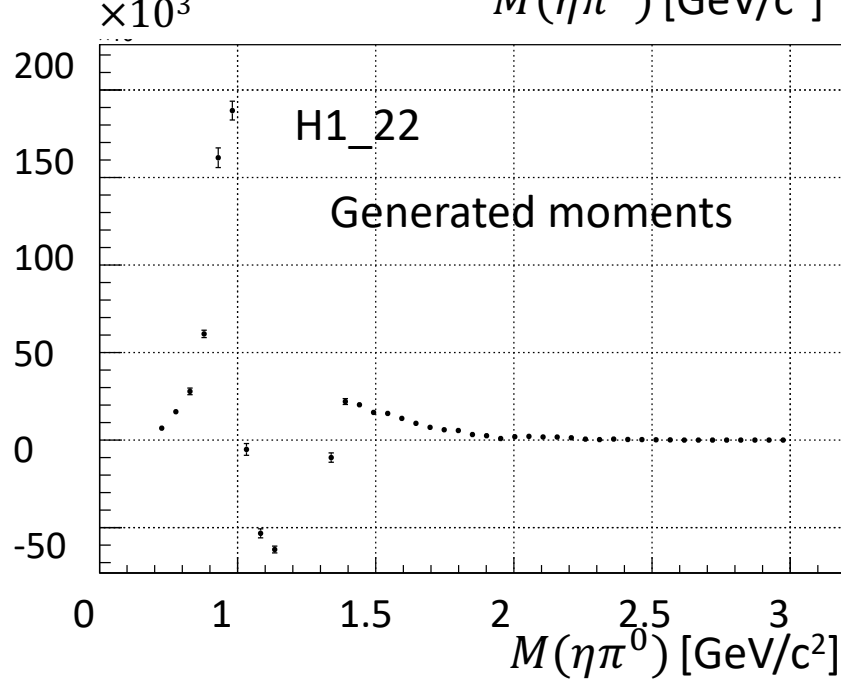
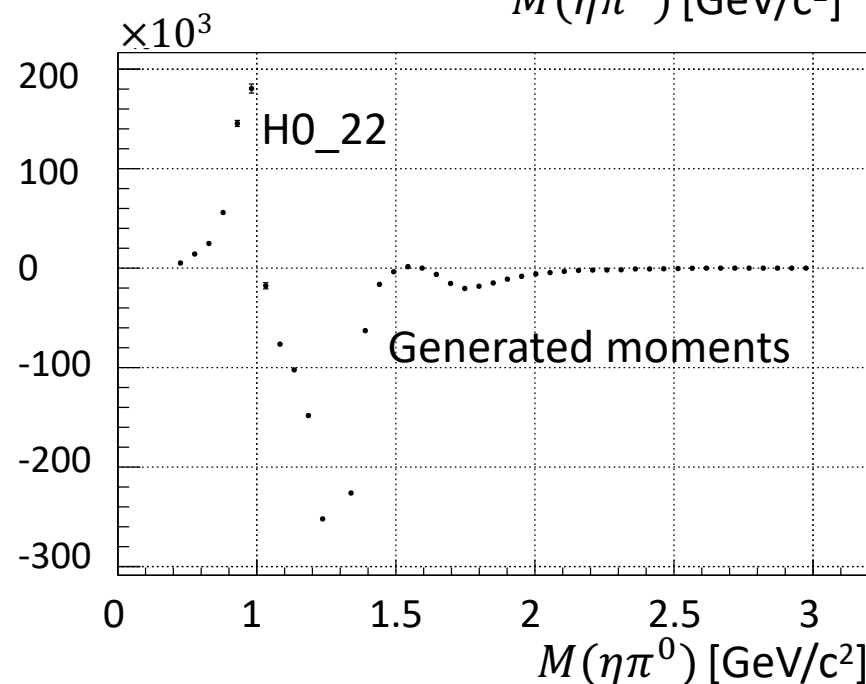
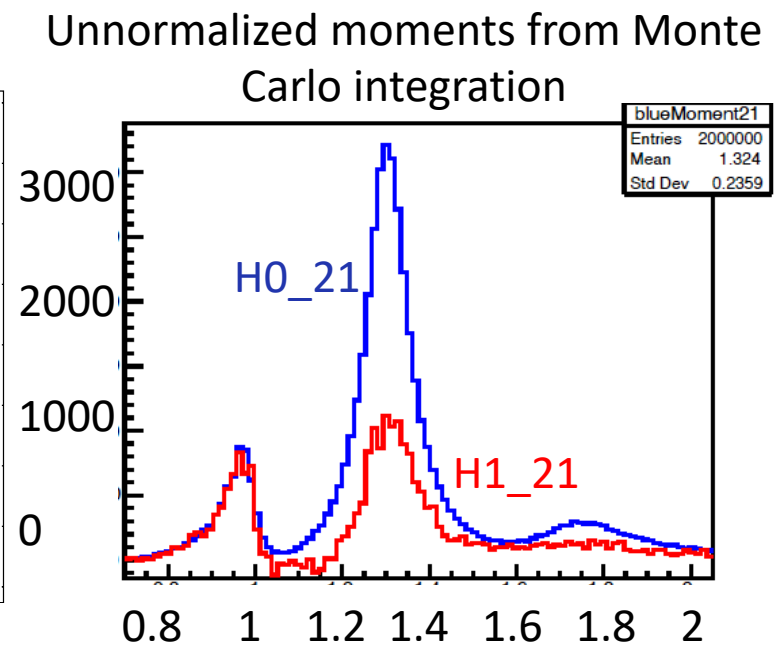
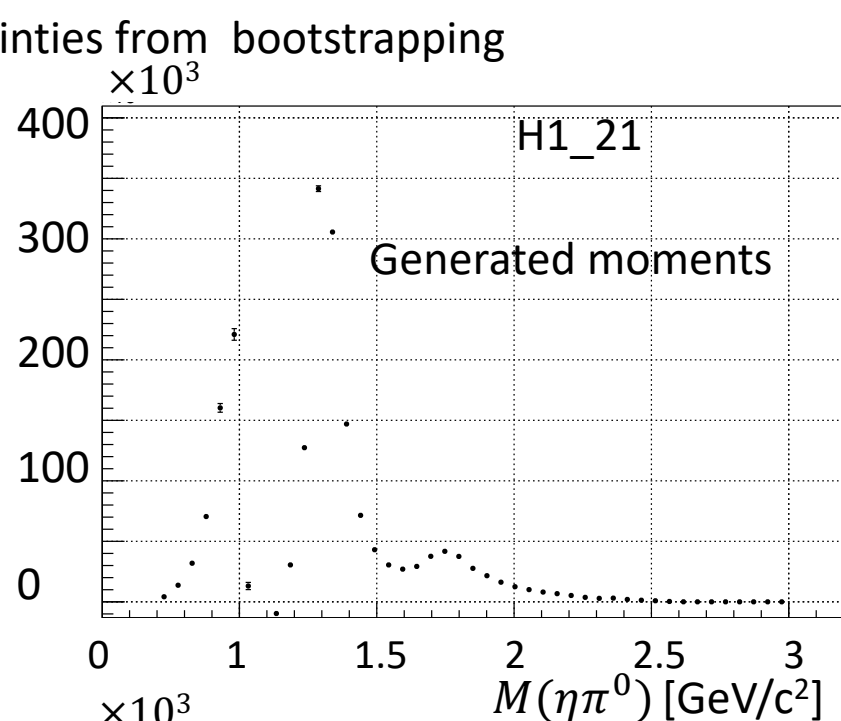
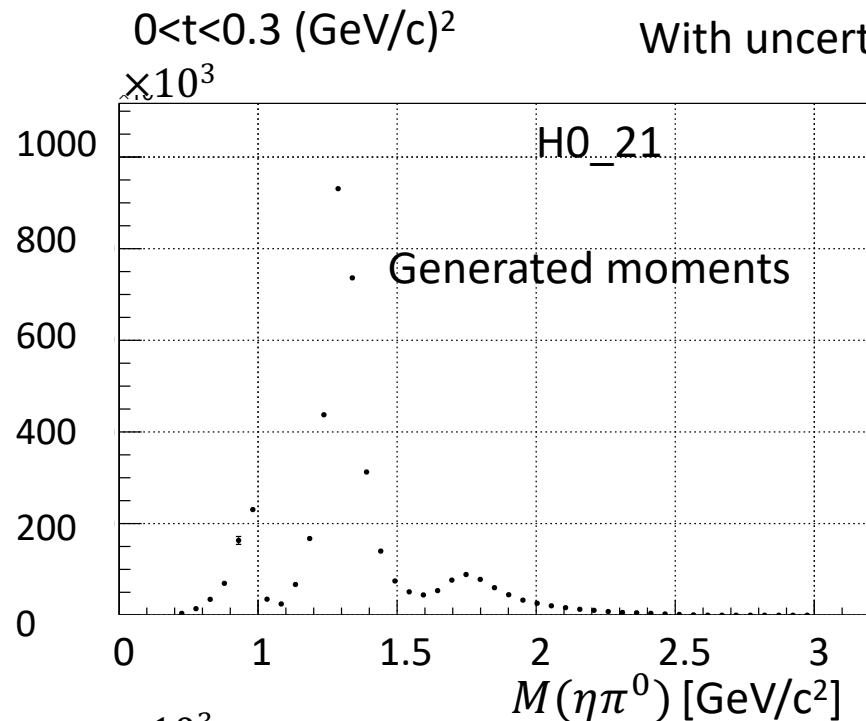


$0 < t < 0.3 \text{ (GeV}/c^2\text{)}$

With uncertainties from bootstrapping

Unnormalized moments from Monte Carlo integration

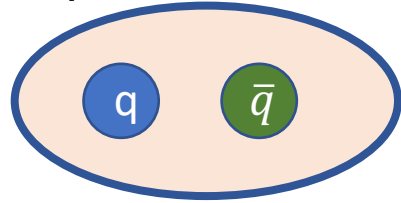




1. First studies of polarized moments in $\eta\pi^0$ photoproduction at GlueX using generated data sample
2. Have implemented and tested calculation of moments in terms of fitted partial waves
 - Results agree with moments from Monte Carlo integration.
3. Future plans:
 - Implement fitting of intensity to extract moments in AMPTOOLS.
 - Search for exotic π_1 via amplitude analysis of GLUEX $\gamma p \rightarrow p\eta'\pi^0$ data.

Backup slides

Mesons in standard quark model



Classified as J^{PC} multilets:

$$\vec{J} = \vec{L} + \vec{S}$$

$$P = (-1)^{L+1} \rightarrow \text{Spherical harmonics } (-1)^l \\ \times \text{Product of individual parities of } q, \bar{q} \text{ } (-1)$$

$$C = (-1)^{L+S} \rightarrow \text{Orbital angular momentum } (-1)^l \\ \times \text{Flip of spin wavefunctions } (-1)^{S+1} \\ \times \text{interchanging } q \text{ and } \bar{q} \text{ } (-1)$$

J - total angular momentum

S - total quark spin

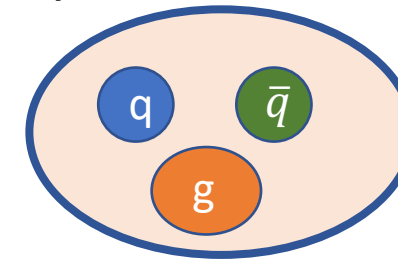
L - orbital angular momentum between $q\bar{q}$ pair

P - parity

C - charge conjugation

$J^{PC} = \mathbf{0}^{--}$, **odd** $^{-+}$ and **even** $^{+-}$ “**exotic**” quantum numbers are not available.

Hybrid mesons



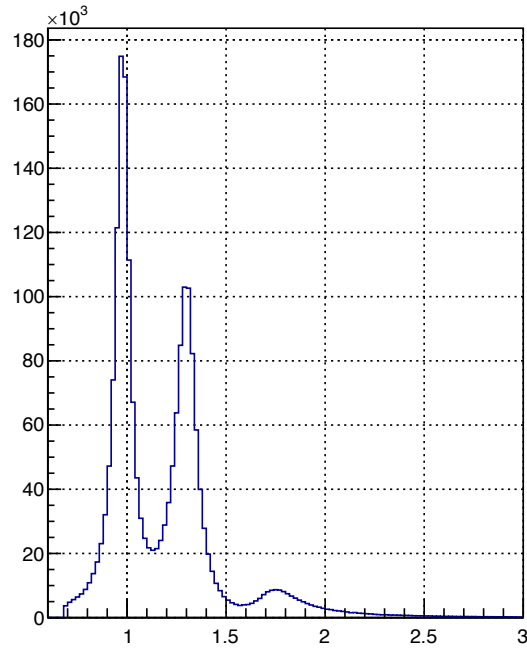
Quark anti-quark pair coupled to valence gluon.

“**Exotic**” J^{PC} are also available.

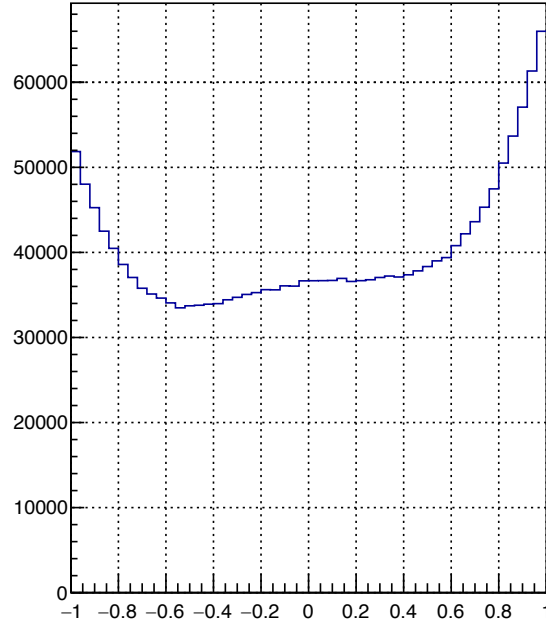
Predicted by lattice QCD (quantum chromodynamics) calculations (Phys. Rev. D 88, 094505 (2013)).

Primary motivation of the GLUEX is the search for light hybrid mesons.

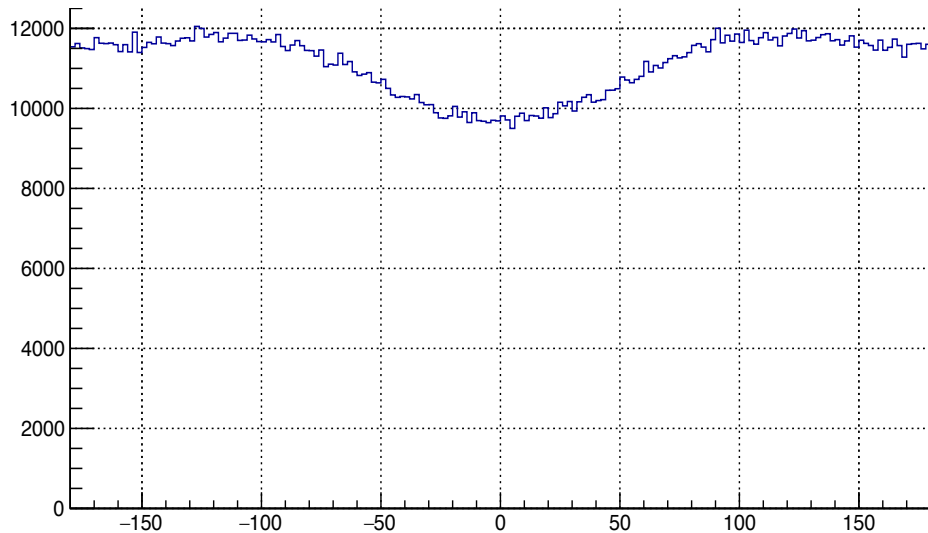
Generated $2 \cdot 10^6$ ($p\eta'\pi^0$) events with AmpTools



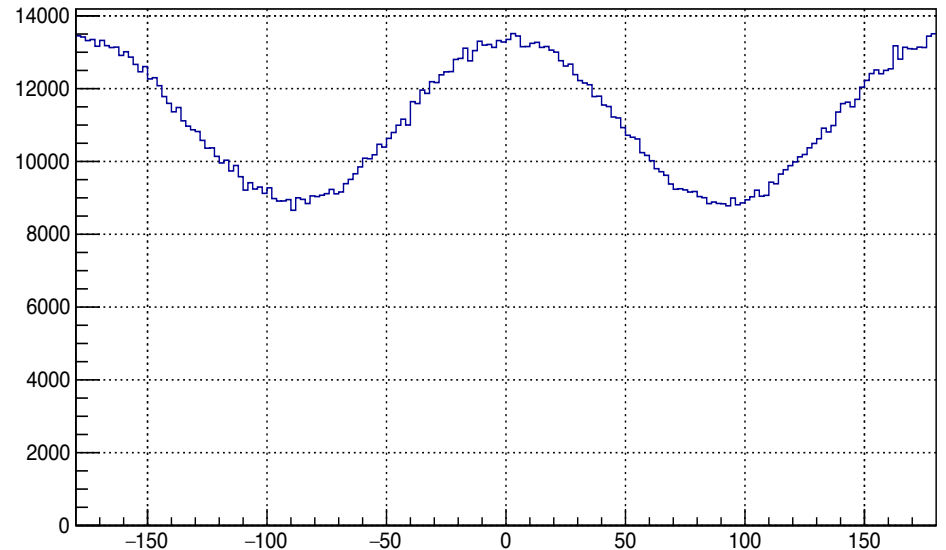
$M(\eta'\pi^0)$ [GeV/c²]



$\cos \theta_{GJ}$



φ



Φ

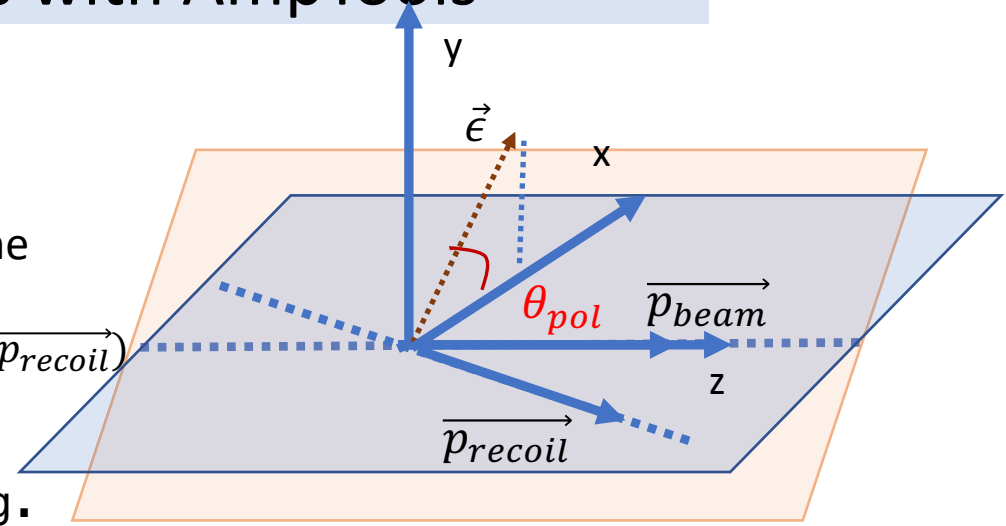
Lab frame

$$\begin{aligned}\vec{y} &= \vec{p}_{beam} \times (-\vec{p}_{recoil}) \\ \vec{x} &= \vec{y} \times \vec{p}_{beam} \\ \vec{z} &= \vec{x} \times \vec{y}\end{aligned}$$

$\theta_{pol} = 1.7$ Deg.

$$\vec{\epsilon} = (\cos(\theta_{pol}), \sin(\theta_{pol}), 0)$$

$$\Phi = \text{arctg}(\vec{y} \cdot \vec{\epsilon}, \vec{p}_{beam} \cdot (\vec{\epsilon} \times \vec{y}))$$



Lab frame

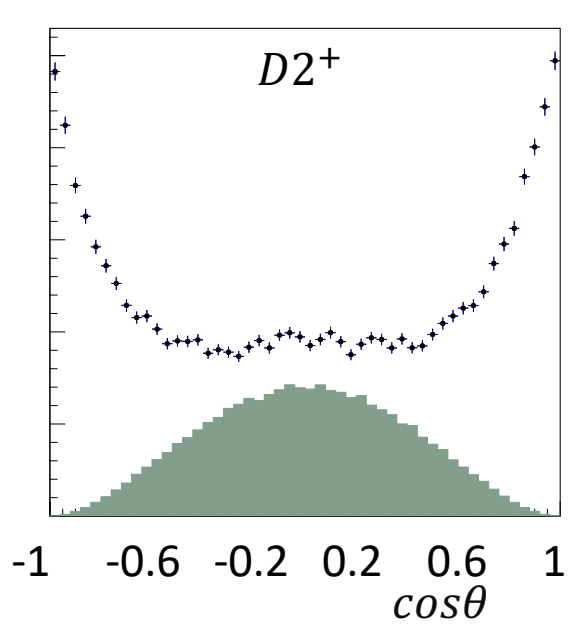
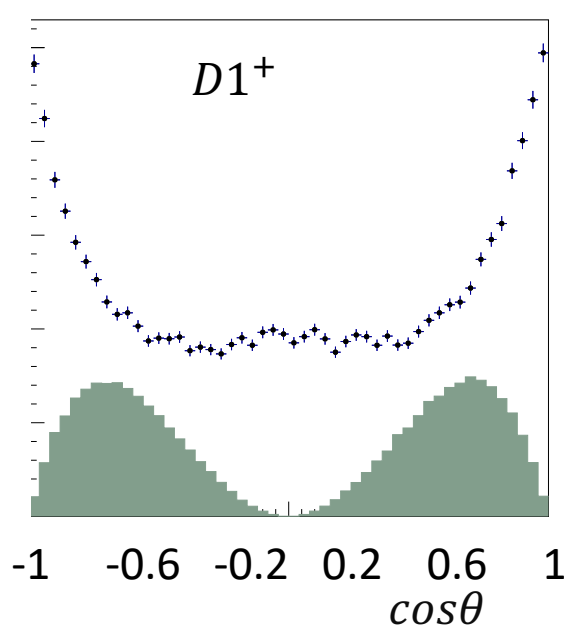
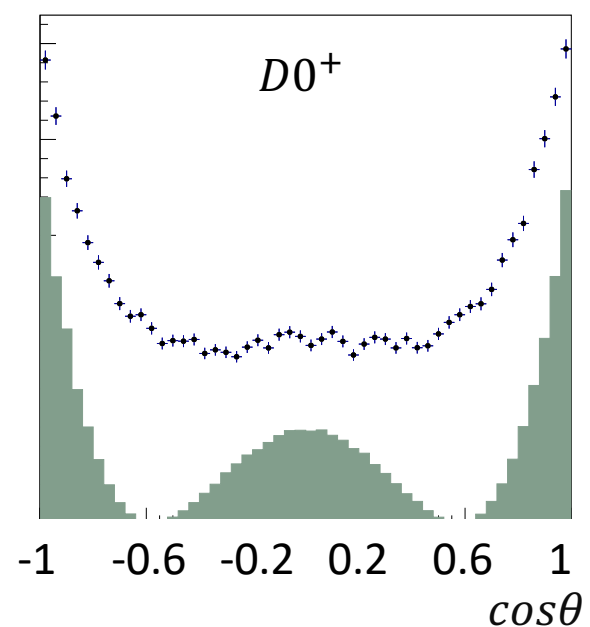
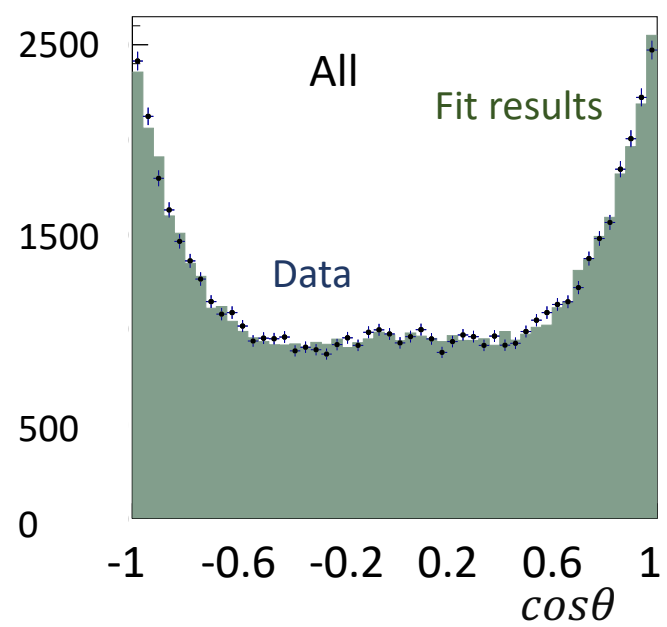
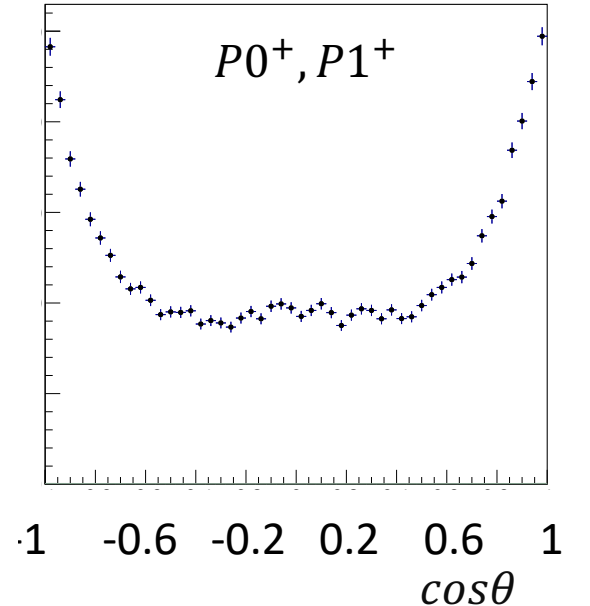
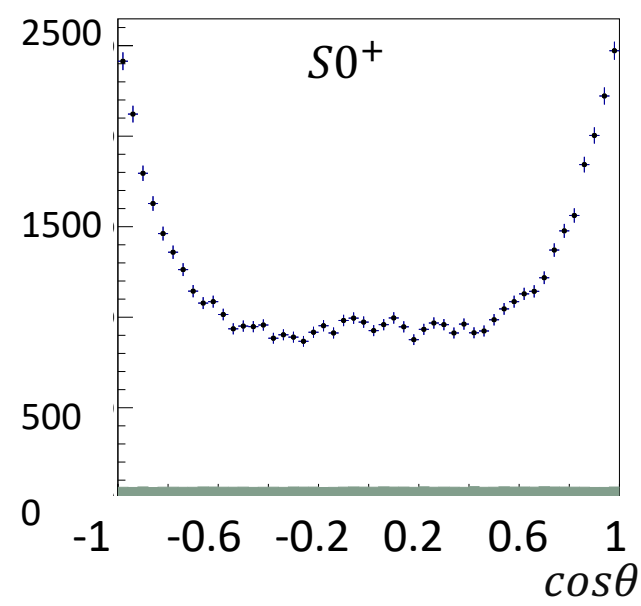
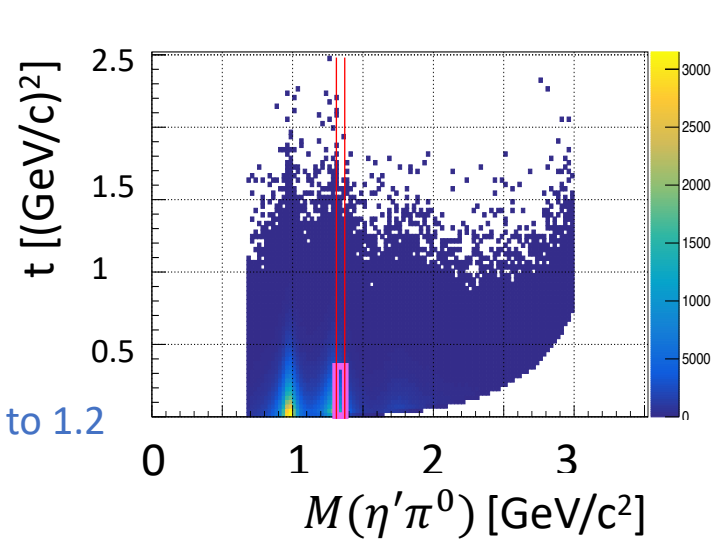
Results for bin $M=1.37$ and $t<0.3$

Amplitudes used in fitting are $S0^+$, $P0^+$, $P1^+$, $D0^+$, $D1^+$, $D2^+$. Good starting values for fit parameters Fit results

Bin M, t

$M(\eta\pi^0)$ range from 0.7 to 3
 N bins=45
 Bin width ≈ 0.051 GeV/c^2

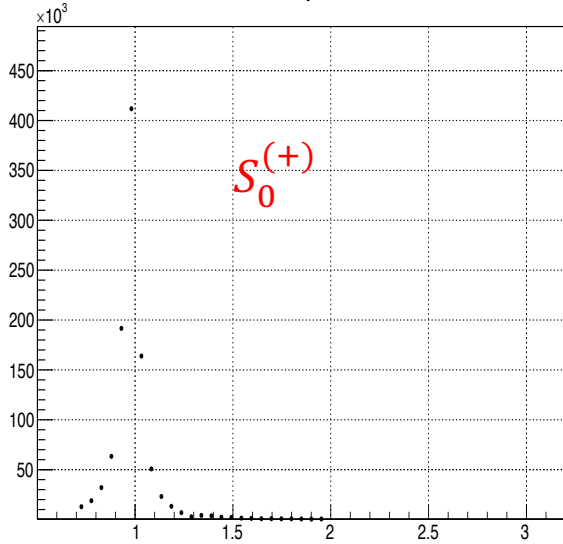
t range from 0 to 1.2
 N bins=4
 Bin width ≈ 0.3 $(\text{GeV}/c)^2$



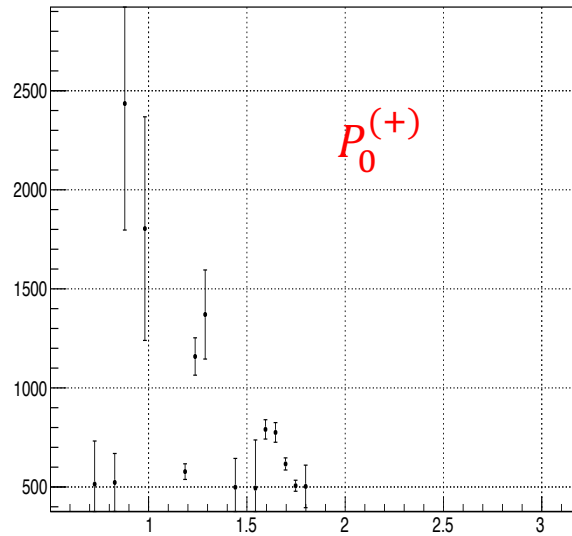
Fit 1 results (fitting in M and t bins)

Amplitudes used in fitting are amplitudes $S_0^{(+)}$, $P_0^{(+)}$, $P_1^{(+)}$, $D_0^{(+)}$, $D_1^{(+)}$, $D_2^{(+)}$. Good starting values for fit parameters

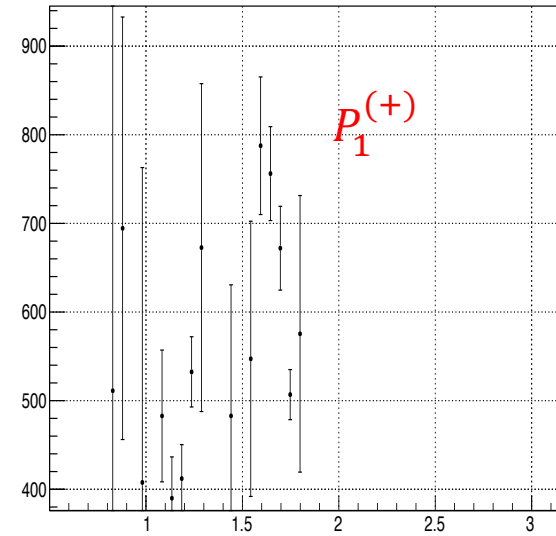
S0pl



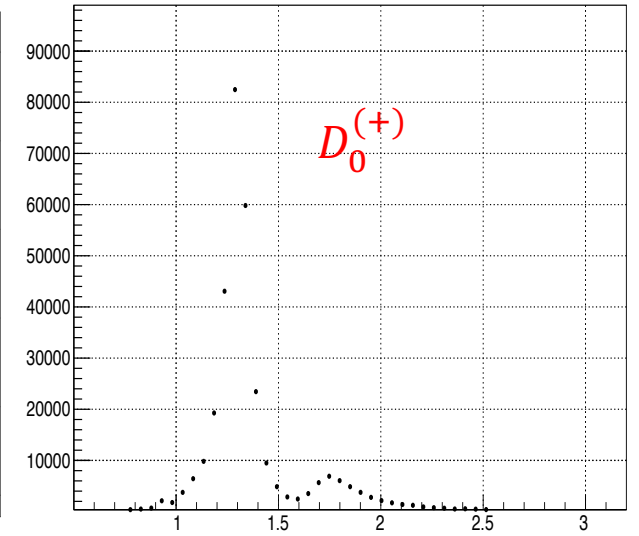
P0pl



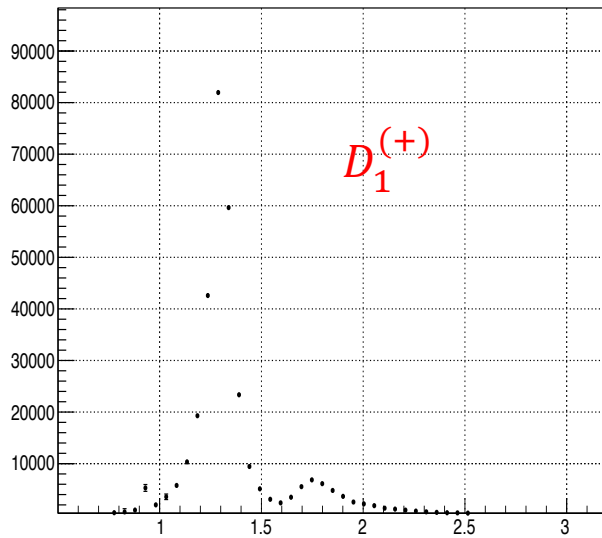
P1pl



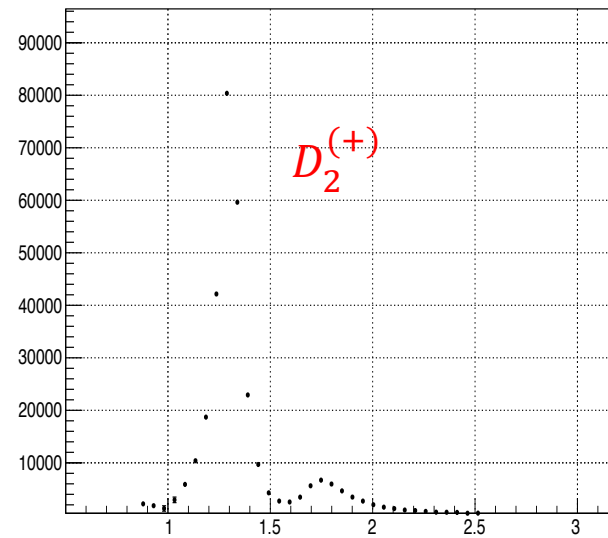
D0pl



D1pl



D2pl



All waves

