The Flatte Parametrization of the $a_0(980)$

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In the Crystal Barrel paper on $\bar{p}p \to K_L K^\pm \pi^\mp$ [1], a Flatte parametrization [2] is used for the $a_0(980)$ to accurately account for both the $\eta\pi$ and the $K\bar{K}$ decay. That parametrization is given as

$$F_0(m) \;=\; \beta_0' \frac{\begin{pmatrix} g_1 \\ g_2 \end{pmatrix}}{m_0^2 - m^2 - i\left(\rho_1(m) g_1^2 + \rho_2(m) g_2^2\right)}\,, \qquad (1)$$

where m_0 is the nominal mass of the $a_0(980)$, m is the running mass of the $a_0(980)$, g_1 and g_2 are the couplings to the $\eta\pi$ and $K\bar{K}$ channels. The paper and an earlier Crystal Barrel paper on the analysis of $\eta\pi^0\pi^0$ [3] final states finds

$$\begin{aligned} g_{\eta\pi} &= 353\,\mathrm{MeV} \\ g_{K\bar{K}} &= 311\,\mathrm{MeV}\,. \end{aligned}$$

That paper also finds a pole mass of the a_0 of $m_{a_0} = 984.45$ MeV. The parameter β_0' describes the production of the resonance from the initial $p\bar{p}$ and the $\rho_i(m)$s are the phase space available for the decay with an a_0 of mass m, and is given as

$$\rho_i(m) \;=\; \frac{2q(m)}{m}\,, \qquad (2)$$

where the decay momentum of the a_0 to daughters of mass m_1 and m_2 is given as

$$q(m) \;=\; \frac{\sqrt{\left[m^2 - (m_1 - m_2)^2\right]\,\left[m^2 - (m_1 + m_2)^2\right]}}{2m}\,. \qquad (3)$$

What will ultimately replace the Breit-Wigner for the $a_0(980)$ is $\rho \mid F \mid^2$.

For the case of $a_0 \to K_S K_S$, we would have the decay momentum being

$$\begin{aligned} q_{K\bar{K}}(m) &= \frac{1}{2}\sqrt{m^2 - 4\,m_{K_S}^2} && m > 2m_{K_S} \\ q_{K\bar{K}}(m) &= 0 && m \le 2m_{K_S}\,. \end{aligned}$$

and for the decay to $\eta\pi$, we have

$$q_{\eta\pi}(m) \;=\; \frac{\sqrt{\left[m^2 - (m_\eta - m_\pi)^2\right]\,\left[m^2 - (m_\eta + m_\pi)^2\right]}}{2m}\,.$$

From these, the phase space factors are

$$\begin{aligned} \rho_{K\bar{K}}(m) &= \sqrt{1-\left(\frac{2m_{K_S}}{m}\right)^2} \quad m > 2m_{K_S} \\ \rho_{K\bar{K}}(m) &= 0 \quad\quad\quad\quad\quad\quad m \leq 2m_{K_S}\,. \end{aligned}$$

and

$$\rho_{\eta\pi}(m) = \frac{\sqrt{[m^2-(m_\eta-m_\pi)^2]\,[m^2-(m_\eta+m_\pi)^2]}}{m^2}\,.$$

So, the Flatte formulation for the $K\bar{K}$ decay will be

$$F_{K\bar{K}}(m) = \frac{g_{K\bar{K}}}{m_{a_0}^2 - m^2 - i\left(\rho_{\eta\pi}(m)g_{\eta\pi}^2 + \rho_{K\bar{K}}(m)g_{K\bar{K}}^2\right)}\,,$$

and the function to replace the Breit-Wigner form is

$$BW(m) = c\,\rho_{K\bar{K}}(m)\mid F_{K\bar{K}}(m)\mid^2\,, \tag{4}$$

where c is a normalization constant. This function will be 0 for $m \leq 2m_{K_S}$, and then turn on after that, yielding a bump above 1 GeV.

References

[1] A. Abele *et al.* [Crystal Barrel], **$\bar{\mathbf{p}}\mathbf{p}$ annihilation at rest into $\mathbf{K_L K^\pm}\pi^\mp$**, Phys. Rev. D**57**, 3860 (1998).

[2] S. M. Flatté, **Coupled-channel analysis of the $\eta\pi$ and $\mathbf{K\overline{K}}$ systems near $\mathbf{K\overline{K}}$ threshold**, Phys. Lett. B**63**, 224 (1976).

[3] C. Amsler *et al.* [Crystal Barrel], **Observation of a new $\mathbf{I^G(J^{PC}} = \mathbf{1^-(0^{++})}$ resonance at 1450 MeV**, Phys. Lett. B**333**, 277 (1994).