# Coordinate Systems in GlueX

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**Abstract**

This document describes the different coordinate systems in GlueX and how to move between them. In particular, events are reconstructed in the GlueX lab frame, while results are generally presented in the center of mass frame. Studies of the decay of mesons produced in GlueX normally occur in the Gottfreid-Jackson and the Helicity frames.

# 1 Introduction

The goal of the GlueX Experiment [1] at Jefferson lab is to search for hybrid mesons using a linearly-polarized photon beam. Recent reviews on hybrid mesons [2, 3] provide an excellent description of the current experimental and theoretical situation. While not guaranteed to be true, it is in principle possible to produce all exotic hybridmesons in photoproduction [4].

In GlueX, we are looking at meson ($M$) photoproduction reactions of the form

$$\vec{\gamma} p \quad \rightarrow \quad pM \,, \tag{1}$$

where the mesonic system $M$ subsequently decays into two or more daughter mesons. The amplitude analyses that are to be carried out in GlueX will look at the decay angular distributions of $M$, and that of its daughter particles. In doing these analyses, there are a number of coordinate systems that are important in the analyses. These are discussed in this note.

# 2 The GlueX Lab Frame

The GlueX experiment is carried out in the so-call laboratory frame, or *Lab Frame* for short. An incident photon of energy $E_\gamma$ travels in the $+\hat{z}$ direction and interacts with a proton at rest in the GlueX liquid hydrogen target. In this frame, the $\hat{y}$ direction is taken as up and the $\hat{x}$ direction is taken such that we have a right-handed coordinate system:

$$\hat{x} \quad = \quad \hat{y} \times \hat{z} \,,$$

which, if looking in the $= \hat{z}$ direction, is to the left, as shown on Figure 1.

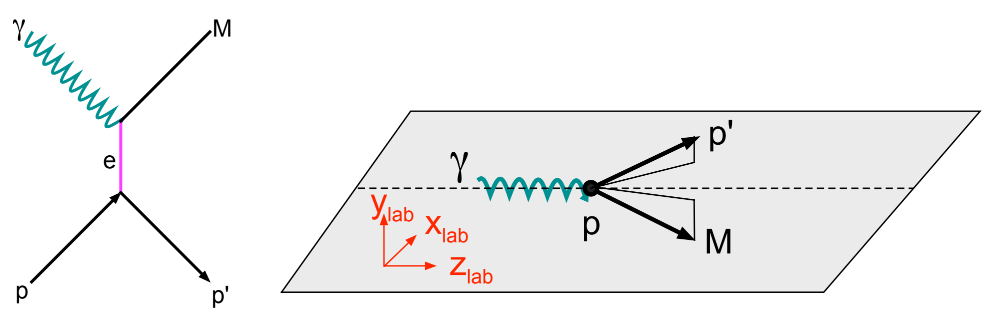



Figure 1: The Lab-Frame coordinates in Gluex. The $z$-axis is along the direction of the incident photon beam, the $y$-axis is normal to the hall floor, pointing up, and the $x$-axis is pointing to the left of the beam, forming a right-handed coordinate system.

In GlueX, the final-state particles are reconstructed in the lab frame, so their momentum and energy are reported in this frame. Given an initial photon of energy $E_\gamma$, we can compute the Mandelstam $s$ as

$$s \quad = \quad \left(E_\gamma + m_p c^2\right)^2 - \left(E_\gamma\right)^2$$

$$s \quad = \quad m_p c^2 \left(2E_\gamma + m_p c^2\right) \,. \tag{2}$$

# 3 The Center of Mass Frame

Most experimental results are reported in the center of mass frame. The frame is defined with the same axis orientations as the lab frame, but with the total momentum of the system being zero. We move from the lab frame to the center of mass frame by performing a Lorentz boost along the negative $z$ axis. The magnitude of the $\beta$ of the boost is given as

$$\beta = \frac{E_\gamma}{E_\gamma + m_p c^2},$$

and the $\gamma$ factor is

$$\begin{aligned} \gamma &= \frac{E_\gamma + m_p c^2}{\sqrt{m_p c^2 \left(m_p c^2 + 2E_\gamma\right)}} \\ \gamma &= \frac{1}{\sqrt{s}}\left(E_\gamma + m_p c^2\right). \end{aligned}$$

From this, the $\vec{\beta}$ of the boost is given as

$$\vec{\beta} = \left(0, 0, -\frac{E_\gamma}{E_\gamma + m_p c^2}\right),$$

and the Lorentz boost can be expressed as a matrix equation as in 3.

$$\begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix}_{cm} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix}_{lab} \tag{3}$$

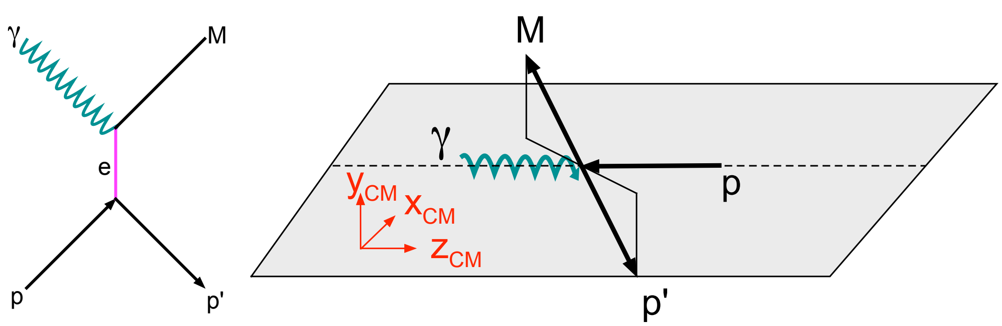



Figure 2: The GlueX center of mass frame has the same coordinate directions as the lab frame. The $z$-axis is along the incident photon direction, the $y$ axis is normal to the hall floor, pointing up and the $x$-axis is to beam left.

# 4 The Gottfried-Jackson and Helicity Frames for Mesons

For reactions of the type in equation 1, the produced meson $M$ typically decays into two (or more) daughter particles. There are two additional reference frames that we use to analyze these decays.

They are the Gottfried-Jackson ($GJ$) frame and the helicity ($\mathcal{H}$) frame. Both are measured in the rest frame of $M$, where we move to the $M$ rest frame with a Lorentz boost from the center-of-mass frame. The frames both have the same $y$ axis, but their $z$ axes differ. For the $GJ$ frame, the $z$-axis is taken as the direction of the incident photon in the $GJ$ frame. Forr the $\mathcal{H}$ frame, the $z$-axis is taken opposite the direction of the Lorentz boost to get to the frame (along the direction of $M$ in the center of mass).

It is important to note that one cannot just perform a Lorentz boost from the lab frame to the rest frame of $M$, one has to first move to the center of mass, and then to $M$. Unless the two Lorentz boosts are colinear, the combination of the two is the product of a pure Lorentz boost and a rotation. Thus, the direct Lorentz boost from the lab frame to the $M$ frame will differ by the product of the two boosts by a rotation. This is known as the *Wigner rotation*, and while it is possible to compute this rotation, it is often easier just to start from the center of mass frame.

To get the Lorentz Boost to the two frames, we need the four-momentum of $M$ ($p_M^\mu$ in the center-of-mass frame). It is of the form

$$p_M^\mu = \begin{pmatrix} E_M & p_{x_M}\, c & p_{y_M}\, c & p_{z_M}\, c \end{pmatrix} ,$$

where the magnitude of the three-momentum $p_A$ is

$$p_M = \sqrt{p_{x_M}^2\, c^2 + p_{y_M}^2\, c^2 + p_{z_M}^2\, c^2} \, .$$

The magnitude of $\beta$and the relativistic factor $\gamma$ are given as

$$\beta = \frac{p_M\, c}{E_M} \tag{4}$$

$$\gamma = \frac{E_M}{m_M} , \tag{5}$$

where $m_A$ is the mass of particle $A$. We also define the $\beta$ along each axis as

$$\beta_x = \frac{p_{x_M}\, c}{E_M}$$
$$\beta_y = \frac{p_{y_M}\, c}{E_M}$$
$$\beta_z = \frac{p_{z_M}\, c}{E_M} \, .$$

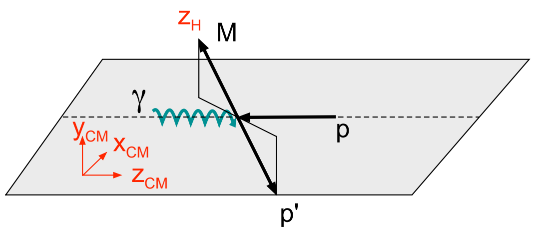
The Lorentz boost is given by the matrix equation 6.

$$\begin{pmatrix} E \\ p_x\, c \\ p_y\, c \\ p_z\, c \end{pmatrix}_{decay} = \begin{pmatrix} \gamma & -\beta_x\gamma & -\beta_y\gamma & -\beta_z\gamma \\ -\beta_x\gamma & 1+(\gamma-1)\frac{\beta_x^2}{\beta^2} & (\gamma-1)\frac{\beta_x\beta_y}{\beta^2} & (\gamma-1)\frac{\beta_x\beta_z}{\beta^2} \\ -\beta_y\gamma & (\gamma-1)\frac{\beta_x\beta_y}{\beta^2} & 1+(\gamma-1)\frac{\beta_y^2}{\beta^2} & (\gamma-1)\frac{\beta_y\beta_z}{\beta^2} \\ -\beta_z\gamma & (\gamma-1)\frac{\beta_x\beta_z}{\beta^2} & (\gamma-1)\frac{\beta_y\beta_z}{\beta^2} & 1+(\gamma-1)\frac{\beta_z^2}{\beta^2} \end{pmatrix} \begin{pmatrix} E \\ p_x\, c \\ p_y\, c \\ p_z\, c \end{pmatrix}_{cm} \tag{6}$$

This reduces to equation 3 in the case where $\beta_x = \beta_y = 0$ and $\beta = \beta_z$.

For both the $GJ$ and $\mathcal{H}$ frames, the $\hat{y}$-axis is defined to be the normal to the production plane. This can be expressed in terms of the center of mass momentum $\vec{p}_\gamma$ and $\vec{p}_M$ as

$$\hat{y}_{GJ,\mathcal{H}} = \frac{\vec{p}_\gamma \times \vec{p}_M}{\mid \vec{p}_\gamma \times \vec{p}_M \mid} \, . \tag{7}$$



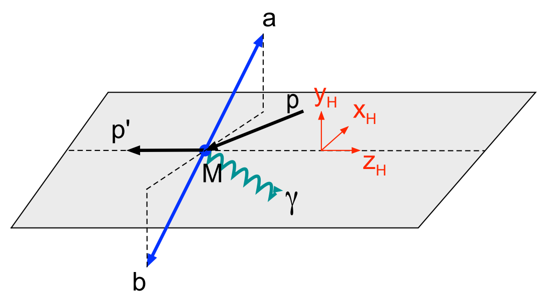




Figure 3: The Helicity Frame of of particle $M$ .

In the $\mathcal{H}$ frame, the $\hat{z}_{\mathcal{H}}$-axis is taken opposite to the direction of the boost, so

$$\hat{z}_{\mathcal{H}} = \frac{\vec{p}_M}{|\,\vec{p}_M\,|}\,, \tag{8}$$

and then $\hat{x}_{\mathcal{H}}$ is chosen to have a right-hand coordinate system,

$$\hat{x}_{\mathcal{H}} = \hat{y}_{\mathcal{H}} \times \hat{z}_{\mathcal{H}}\,. \tag{9}$$

For the case $M$ decaying into two or more daughter particles, it is often convenient to rotate the center of mass momentum of the daughter particles into the $\mathcal{H}$ coordinate system. For a daughter particle of momentum $\vec{p}_a$ in the center of mass frame, we can rotate it to the $\mathcal{H}$ coordinates using the unit vectors.

$$\vec{p}_a(\mathcal{H}) = (\vec{p}_a \cdot \hat{x}_{\mathcal{H}}\,,\, \vec{p}_a \cdot \hat{y}_{\mathcal{H}}\,,\, \vec{p}_a \cdot \hat{z}_{\mathcal{H}})$$

These can then be boosted to the $\mathcal{H}$-frames using the simplified Lorentz boost, where we are bosting along the $\hat{z}_{\mathcal{H}}$ axis. Using $\beta$ and $\gamma$ from equations 4 and 5, the matrix form of the boost is

$$\begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix}.$$

For the case of the $GJ$ frame, there are two ways that we can proceed. Using equation 6, we can boost the center of mass momentum of the photon into the rest frame of $A$ yielding $\vec{p}_\gamma(M)$. We then define the $\hat{z}_{GJ}$-axis as

$$\hat{z}_{GJ} = \frac{\vec{p}_\gamma(M)}{|\,\vec{p}_\gamma(M)\,|}\,, \tag{10}$$

and then $\hat{x}_{GJ}$ is chosen to have a right-hand coordinate system,

$$\hat{x}_{GJ} = \hat{y}_{GJ} \times \hat{z}_{GJ}\,. \tag{11}$$

We then boost the individual momentum $\vec{p}_a$ into the rest frame of $M$, $\vec{p}_a(M)$, and then use the unit vectors to project out the components of momentum,

$$\vec{p}_a(GJ) = (\vec{p}_a(M) \cdot \hat{x}_{GJ}\,,\, \vec{p}_a(M) \cdot \hat{y}_{GJ}\,,\, \vec{p}_a(M) \cdot \hat{z}_{GJ})\,.$$

Alternatively, one can go first to the helicity frame, and then perform a rotation about the $\hat{y}_{\mathcal{H}}$ axis by an angle $\alpha$, where

$$\cos\alpha \quad = \quad \frac{\cos\theta_{cm}(M) - \beta}{1 - \beta\cos\theta_{cm}(M)} \, .$$

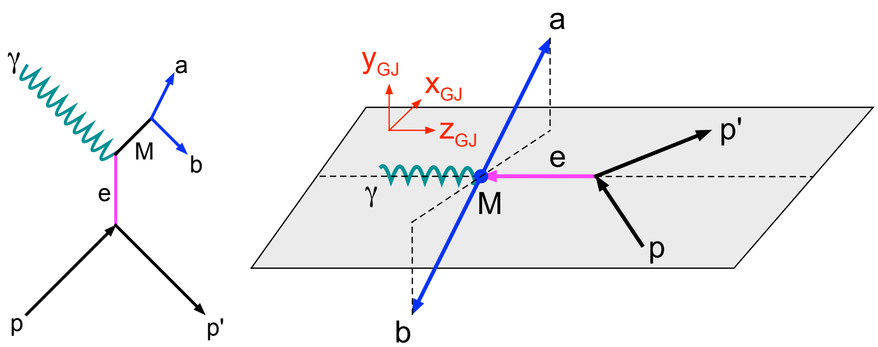



Figure 4: The Gottfried-Jackson frame for $M$.

## 5 Decays of Daughter Particles

For the photoproduction of $M$ through reaction 1, we have considered the subsequent decay of $M$ to daughter particles,

$$M \quad \rightarrow \quad ab$$

where we can look at the decay of $M$ in either the $GJ$ or $\mathcal{H}$ frames (of $M$). It often happens that one or both of the daughter particles ($a$ and/or $b$) subsequently decays to grand daughter particles, for example

$$a \quad \rightarrow \quad cd \, .$$

In this case, we usually look at the subsequent decay (of $a$) in the $\mathcal{H}$ frame for $a$. The procedure is the same, in that we start from the rest frame of $M$ and perform a Lorentz boost into the rest frame of $a$. The $\hat{y}$ axis is taken as the normal to the decay plane

$$\hat{y}_{GJ,\mathcal{H}} \quad = \quad \frac{\hat{z}_{\mathcal{H}} \times \vec{p}_a}{\mid \hat{z}_{\mathcal{H}} \times \vec{p}_a \mid} \, , \tag{12}$$

where $\vec{p}_a$ is measured in the rest frame of $M$. This formulation breaks down when the grand-daughter particles travel along the $\hat{z}_{\mathcal{H}}(M)$ axes, so care needs to be exercised in that limit. The $z$-axis is then defined as before, with

$$\hat{z}_{\mathcal{H}}(a) \quad = \quad \frac{\vec{p}_a}{\mid \vec{p}_a \mid} \, , \tag{13}$$

for the helicity frame. This procedure can continue to be extended for subsequent decays as well.

# References


[1] S. Adhikari et. al. [GlueX Collaboration], **The GlueX Beamline and Detector**, Nucl. Instrum. & Meth. A**987**, 164807 (2021). DOI: 10.1016/j.nima.2020.164807.

[2] C. A. Meyer and Y. van Haarlem, **The Status of Exotic-quantum-number Mesons** Phys. Rev. C**82**, 025208 (2010). DOI: 10.1103/PhysRevC.82.025208.

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[4] A. A. Meyer, **The Production and Decay of Normal and Exotic-Hybrid Mesons in GlueX**, GlueX-Doc 4788 (2020).