Quarks and Mesons : Lecture 1

Fermion - Antifermion Systems

fermions spin $1/2$ $|s, m_s\rangle = |1/2, \pm\frac{1}{2}\rangle$ $\uparrow \downarrow$

S_z

wave function are antisymmetric.

positronium $e^- e^+$

muonium $\mu^- \mu^+$

quarkonium $q\,\bar{q}$

} particle + antiparticle.

SPIN

S

orbital angular momentum L

total angular momentum J

$$J = L \oplus S = |L-S|, |L-S+1|, \dots |L+S|$$

Spin: $S_1 = 1/2$, $S_2 = 1/2$ $S = S_1 \oplus S_2 = \frac{1}{2} \oplus \frac{1}{2} = 0$ or 1

$\uparrow_1 \uparrow_2$, $\uparrow_1 \downarrow_2$, $\downarrow_1 \uparrow_2$, $\downarrow_1 \downarrow_2$ } 4 possible combination.

$S = 0$ $m_s = 0$ 1

$S = 1$ $m_s = +1, 0, -1$ 3

symmetric combination $1/\sqrt{2}\left[\,|\uparrow_1 \downarrow_2\rangle + |\downarrow_1 \uparrow_2\rangle\,\right]$ ← spin 1

antisymmetric combination $1/\sqrt{2}\left[\,|\uparrow_1 \downarrow_2\rangle - |\downarrow_1 \uparrow_2\rangle\,\right]$ ← spin 0

Clebsch Gordon Coefficients.

	J M	J M
m_1 m_2		

$1/2 \times 1/2$

	1 0	0 0
$+\frac{1}{2}$ $-\frac{1}{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$
$-\frac{1}{2}$ $+\frac{1}{2}$	$1/\sqrt{2}$	$-1/\sqrt{2}$

$$|\uparrow_1 \downarrow_2\rangle = 1/\sqrt{2}\,|1\,0\rangle + 1/\sqrt{2}\,|0\,0\rangle$$

$$|\downarrow_1 \uparrow_2\rangle = 1/\sqrt{2}\,|1\,0\rangle - 1/\sqrt{2}\,|0\,0\rangle$$

$$|1\,0\rangle = 1/\sqrt{2}\left[\,|\uparrow_1 \downarrow_2\rangle + |\downarrow_1 \uparrow_2\rangle\,\right]$$

$$|0\,0\rangle = 1/\sqrt{2}\left[\,|\uparrow_1 \downarrow_2\rangle - |\downarrow_1 \uparrow_2\rangle\,\right]$$

$\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad \downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ related to $SU(2)$

$$\underset{}{2 \otimes 2} = \underset{\chi_s}{3} \oplus \underset{\chi_a}{1}$$

$$\chi' = U \chi$$

$$U = e^{\frac{1}{2} i \theta \hat{n} \times \vec{\tau}}$$

$\vec{\tau}_i = \sigma_i$ $\quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

$$U = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \qquad \hat{n} = \hat{y}$$

$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Include orbital angular momentum $L = 0, 1, 2, 3, 4, \ldots$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ S P D F G

$$^{2S+1}L_J$$

$L=0$	$S=0$	$J=0$	$^1S_0$
$L=0$	$S=1$	$J=1$	$^3S_1$
$L=1$	$S=0$	$J=1$	$^1P_1$
$L=1$	$S=1$	$J=0,1,2$	$^3P_0$, $^3P_1$, $^3P_2$

$L=2 \quad S=0 \qquad J=2 \qquad {}^1D_2$

$L=2 \quad S=1 \qquad J=1,2,3 \qquad {}^3D_1,\ {}^3D_2,\ {}^3D_3$

Conserved Quantum numbers under strong interactn

J, Q, P - parity, C Charge-Conjugn

particl $\longleftrightarrow$ anti partl

Parity $\vec{r} \to -\vec{r}$ $\qquad q\bar{q} \leftarrow (-1)$

$$P\,\psi(r) = \psi(-\vec{r}) = \eta_p\,\psi(\vec{r})$$

$$\psi(\vec{r}) = R(r)\,Y_{lm}(\theta,\phi)$$

$$\psi(-\vec{r}) = R(r)\,Y_{lm}(\pi-\theta,\pi+\phi) = R(r)(-1)^L\,Y_{lm}(\theta,\phi)$$

$$\boxed{P = -(-1)^L}$$

Charge Conjugation C

$$\psi(\vec{r},\vec{s}) = R(r)\,Y_{lm}(\theta,\phi)\,\chi(\vec{s})$$

$$\left.\begin{matrix} q \to \bar{q} \\ \bar{q} \to q \end{matrix}\right\} \quad \vec{r} = -\vec{r} \longrightarrow (-1)^L$$

for spin: treat bosons + fermions diff enth.

fermion anti fermn (fact -1)

$$C = (-1)^{L+S}$$

$^{2S+1}L_J \longrightarrow J^{PC}$

	L	S	J	P	C	
$^1S_0$	0	0	0	-1	+1	$J^{PC} = 0^{-+}$
$^3S_1$	0	1	1	-1	-1	$J^{PC} = 1^{--}$
$^1P_1$	1	0	1	+1	-1	$J^{PC} = 1^{+-}$
$^3P_{0,1,2}$	1	1	0,1,2	+1	+1	$J^{PC} = 0^{++}, 1^{++}, 2^{++}$
$^1D_2$						$J^{PC} = 2^{-+}$
$^3D_{1,2,3}$						$J^{PC} = 1^{--}, 2^{--}, 3^{--}$
$^1F_3$						$J^{PC} = 3^{+-}$
$^3F_{2,3,4}$						$J^{PC} = 2^{++}, 3^{++}, 4^{++}$

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