Quarks and Mesons : Lecture 2

Quarks and Isospin

Isospin $|I \; I_z\rangle$

Nuclear physics p & n had same properties

$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$p = |\tfrac{1}{2} \, +\tfrac{1}{2}\rangle$ $\quad n = |\tfrac{1}{2} \, -\tfrac{1}{2}\rangle$ $\quad |I, I_z\rangle$

isospin is a property of up & down quark

$u = |\tfrac{1}{2} \, +\tfrac{1}{2}\rangle$ $\quad d = |\tfrac{1}{2} \, -\tfrac{1}{2}\rangle$

$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

SU(2)

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

$2 \otimes 2 = 3 \oplus 1$ $\quad I=1, \; I_z = +1, 0, -1$

$I=0 \quad I_z=0$

$|0\,0\rangle = 1/\sqrt{2}\left[|ud\rangle - |du\rangle\right]$ antisym.

$|1\,1\rangle = |uu\rangle$ $\quad |1\,-1\rangle = |dd\rangle$ $\quad |1\,0\rangle = \frac{1}{\sqrt{2}}\left[|ud\rangle + |du\rangle\right]$ sym

$2 \otimes 2 \otimes 2 = 2 \otimes [3 \oplus 1]$

$= 2 \otimes 3 + 2 \otimes 1$ ← doublet

$4 \oplus 2$

$2 \times 1: \quad 1/\sqrt{2}\left[(ud - du)\,u\right] \quad 1/\sqrt{2}\left[(ud - du)\,d\right]$

$I = 1 \,,\; I = 1/2$

$1 \oplus 1/2 \rightarrow 3/2 \text{ or } 1/2$

$$|3/2 \; +1/2\rangle = \frac{1}{\sqrt{3}} |1 1\rangle |\tfrac{1}{2} \, -\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}} |1 0\rangle |\tfrac{1}{2} \, \tfrac{1}{2}\rangle$$

$$|1/2 \; +1/2\rangle = -\frac{1}{\sqrt{3}} |1 0\rangle |\tfrac{1}{2} \, \tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}} |1 1\rangle |\tfrac{1}{2} \, -\tfrac{1}{2}\rangle$$

$$-\frac{1}{\sqrt{3}} \left[\frac{ud + du}{\sqrt{2}} u - \sqrt{2} \; uud \right] = |\tfrac{1}{2} \; +\tfrac{1}{2}\rangle$$

$$\frac{1}{\sqrt{3}} \left[\frac{ud + du}{\sqrt{2}} d - \sqrt{2} \; ddu \right] \qquad |\tfrac{1}{2} \; -\tfrac{1}{2}\rangle$$

$$|\tfrac{3}{2} \; +\tfrac{3}{2}\rangle = uuu \qquad |\tfrac{3}{2} \; -\tfrac{3}{2}\rangle = ddd$$

$$|\tfrac{3}{2} \; +\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}} (uud + udu + duu)$$

$$|\tfrac{3}{2} \; -\tfrac{1}{2}\rangle = \frac{1}{\sqrt{3}} (udd + dud + ddu)$$

Quarks + Anti quarks

$2 \qquad \bar{2}$

$$\begin{pmatrix} u \\ d \end{pmatrix} \longrightarrow \begin{pmatrix} \bar{d} \\ \bar{u} \end{pmatrix}$$ simple assumption.

$$\begin{pmatrix} u' \\ d' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

$$\begin{pmatrix} \bar{u}' \\ \bar{d}' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix}$$

$$\bar{2} = \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$$

$$\begin{pmatrix} \bar{d}' \\ -\bar{u}' \end{pmatrix} = \begin{pmatrix} \cos\theta & +\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$$

$\bar{2} \otimes 2 = 1 + 3$ $\quad d \to -\bar{u} \quad u \to \bar{d}$

$I = 0 \quad \frac{1}{\sqrt{2}}\left(|ud\rangle - |du\rangle\right) \to \frac{1}{\sqrt{2}}\left[|\bar{d}d\rangle + |\bar{u}u\rangle\right]$ symmetric

$$\left.\begin{array}{ll} I = 1 & |uu\rangle \to \bar{d}u \\ & |dd\rangle \to -\bar{u}d \\ & \frac{1}{\sqrt{2}}\left(|ud\rangle + |du\rangle\right) \to \frac{1}{\sqrt{2}}\left(\bar{d}d - \bar{u}u\right) \end{array}\right\} \text{anti sym.}$$

Isospin is conserved: G-parity, related to C-parity

$\pi^+ \quad \pi^0 \quad \pi^-$

$\quad J^{PC} = 0^{-+}$ $\qquad C|\pi^+\rangle \approx |\pi^-\rangle$

$$G = C(-1)^I = (-1)^{L+S+I}$$

$(I^G)\, J^{PC}$

	J^{PC}	I	I^G
${}^1S_0$	0^{-+}	0	0^+
		1	1^-
${}^3S_1$	1^{--}	0	0^-
		1	1^+