

## Quarks and Mesons : Lecture 4

### Mesons in $SU(3)$

3 lightest quarks.

anti quarks

$u, d, s$

$\bar{u}, \bar{d}, \bar{s}$

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

$$B(q) = \frac{1}{3}$$

$$B(\bar{q}) = -\frac{1}{3}$$

$u, d$  have isospin  $\frac{1}{2}$

$s$  has  $I=0$

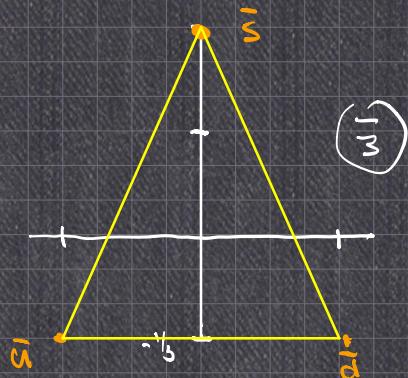
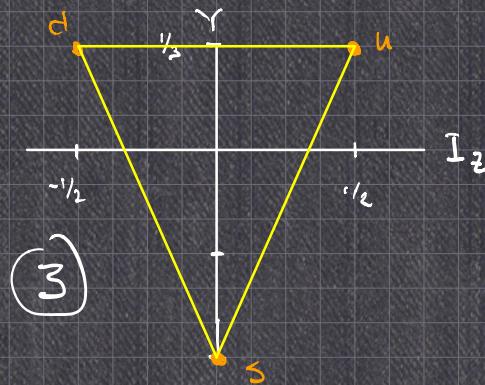
$s$  has strangeness  $S=-1$

$u, d$   $S=0$

Hypercharge  $Y = B + S$

$I$	$I_z$	$S$	$Y$
$u$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{3}$
$d$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$
$s$	$0$	$0$	$-1$

$I$	$I_z$	$S$	$Y$
$\bar{u}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{3}$
$\bar{d}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{3}$
$\bar{s}$	$0$	$0$	$+1$



$$3 \otimes \bar{3} = 8 \oplus 1$$

(these are on top of the  $SU(2)$ )

isospin combination

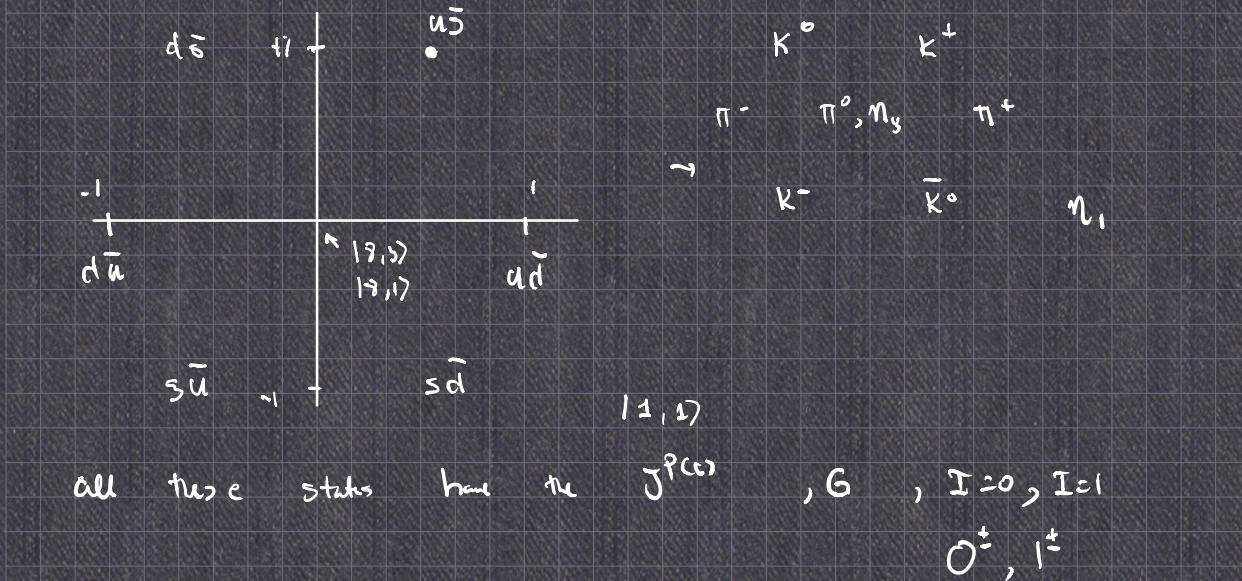
$$\pi^+, \pi^-, \pi^0$$

$s\bar{s}$  mix into  $u\bar{u}, d\bar{d}$

$$|3,3\rangle \rightarrow \pi^+, \pi^-, \pi^0 \quad \gamma_2 (\bar{d}\bar{d} - \bar{u}\bar{u}) \pi^0$$

$$|8,1\rangle \rightarrow \frac{1}{\sqrt{6}} (\bar{u}\bar{u} + \bar{d}\bar{d} - 2\bar{s}\bar{s})$$

$$|1,1\rangle \rightarrow \frac{1}{\sqrt{3}} (\bar{u}\bar{u} + \bar{d}\bar{d} + \bar{s}\bar{s})$$



Naming of Mesons:

L	S	J <sup>PC</sup>	I <sup>G</sup>	.	I <sup>G</sup>	.	.	I <sup>G</sup>
0	0	0 <sup>-+</sup>	1 <sup>-</sup>	π	0 <sup>+</sup>	n	n'	γ <sub>2</sub> K
0	1	1 <sup>--</sup>	1 <sup>+</sup>	ρ	0 <sup>-</sup>	ω	φ	γ <sub>2</sub> K <sup>*</sup>
1	0	1 <sup>+-</sup>	1 <sup>+</sup>	b <sub>1</sub>	0 <sup>-</sup>	h <sub>1</sub>	h <sub>1'</sub>	γ <sub>2</sub> K <sub>1</sub>
1	1	0 <sup>++</sup>	1 <sup>-</sup>	a <sub>0</sub>	0 <sup>+</sup>	f <sub>0</sub>	f <sub>0'</sub>	γ <sub>2</sub> K <sub>0</sub> <sup>*</sup>
1	1	1 <sup>++</sup>	1 <sup>-</sup>	a <sub>1</sub>	0 <sup>+</sup>	f <sub>1</sub>	f <sub>1'</sub>	γ <sub>2</sub> K <sub>1</sub>
1	1	2 <sup>++</sup>	1 <sup>-</sup>	a <sub>2</sub>	0 <sup>+</sup>	f <sub>2</sub>	f <sub>2'</sub>	γ <sub>2</sub> K <sub>2</sub> <sup>*</sup>
2	0	2 <sup>-+</sup>	1 <sup>-</sup>	π <sub>2</sub>	0 <sup>+</sup>	n <sub>2</sub>	n <sub>2'</sub>	γ <sub>2</sub> K <sub>2</sub>

2	1	1 <sup>-</sup>	1+	$\rho_1$	0 <sup>-</sup>	$\omega_1$	$\phi_1$	$\nu_2$	$K^*_2$
2	1	2 <sup>--</sup>	1+	$\rho_2$	0 <sup>-</sup>	$\omega_2$	$\phi_2$	$\nu_2$	$K^*_2$
2	1	3 <sup>--</sup>	1+	$\rho_3$	0 <sup>-</sup>	$\omega_3$	$\phi_3$	$\nu_3$	$K^*_3$

$|81\rangle$      $|11\rangle$     can mix

$$\eta_8, \omega_3 \quad \eta_1, \omega_1$$

Vector mesons    1<sup>--</sup>     $\omega_8, \omega_1 \rightarrow \omega \equiv \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$

$$\phi \approx 55^\circ$$

ideal mixing:

PDG:

$$\begin{pmatrix} f \\ f' \end{pmatrix} = \begin{pmatrix} \cos \Theta_n & \sin \Theta_n \\ -\sin \Theta_n & \cos \Theta_n \end{pmatrix} \begin{pmatrix} |11\rangle \\ |18\rangle \end{pmatrix}$$

$$f = \cos \Theta_n \left[ \underbrace{\bar{u}\bar{u} + \bar{d}\bar{d} - \bar{s}\bar{s}}_{\sqrt{3}} \right] + \sin \Theta_n \left[ \underbrace{\bar{u}\bar{u} + \bar{d}\bar{d} - 2\bar{s}\bar{s}}_{\sqrt{6}} \right]$$

$$f = (\bar{u}\bar{u} + \bar{d}\bar{d}) \left[ \frac{\cos \Theta_n}{\sqrt{3}} + \frac{\sin \Theta_n}{\sqrt{6}} \right] + \bar{s}\bar{s} \underbrace{\left[ \frac{\cos \Theta_n}{\sqrt{3}} - \sqrt{\frac{2}{3}} \sin \Theta_n \right]}_{}$$

$$\sin \Theta_n = \frac{1}{\sqrt{2}}, \quad \cos \Theta_n = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\cos \Theta_n = \sqrt{2} \sin \Theta_n$$

$$\tan \Theta_n = \frac{1}{\sqrt{2}}$$

$$\boxed{\Theta_I = 35.26^\circ}$$

$$f' = -\bar{s}\bar{s}$$

Pseudo scalar → Mesons:  $\eta \rightarrow f', \quad \eta' \rightarrow f$

$$\begin{pmatrix} \eta' \\ \eta \end{pmatrix} = \begin{pmatrix} \cos \Theta_p & \sin \Theta_p \\ -\sin \Theta_p & \cos \Theta_p \end{pmatrix} \begin{pmatrix} |1\rangle \\ |2\rangle \end{pmatrix}$$

$$\Theta_p \approx -7^\circ \quad \cos \Theta_p \approx 1 \quad \sin \Theta_p \approx 0$$

$$\eta' \approx |1\rangle \quad \eta \approx |2\rangle$$

$$s\bar{s} \approx |1\rangle \quad s\bar{s} \approx |2\rangle$$

$$\eta' = \sin \psi_p |N\rangle + \cos \psi_p |S\rangle$$

$$\eta = \cos \psi_p |N\rangle - \sin \psi_p |S\rangle$$

$$\psi_p = \Theta_I - \Theta_S$$

Excited QNS:

$J^{PC}$	$I^G$	$I^G$	$I$
$0^{--}$	$ +$	$\rho_0$	$0^-$
$0^{+-}$	$ +$	$b_0$	$0^-$
$1^{-+}$	$ ^-$	$\pi_1$	$0^+$
$2^{+-}$	$ ^+$	$b_2$	$0^-$
$3^{-+}$	$ ^-$	$\pi_3$	$0^+$
			$w_0$
			$\phi_0$
			$\gamma_2$
			$K_S^{\pm}$
			$h_2$
			$h_2'$
			$\gamma_2$
			$K_0^{\pm}$
			$\eta_1$
			$\eta_1'$
			$\eta_2$
			$K_2^{\pm}$
			$\eta_2'$
			$\eta_3$
			$\eta_3'$
			$\gamma_2$
			$K_3^{\pm}$