Meson Decays to $K^*\overline{K}$ and $\overline{K}^*K$

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SU(3) Clebsch-Gordan Coefficients

The two-body decays of mesons A$\rightarrow B + C$ from one nonet to two other nonets are related by SU(3) Clebsch-Gordan Coefficients and constants:

$$\Gamma \sim \gamma^2$$

where γ is the product of a constant g_i and a clebsch, c_i.

In SU(3), we can have the following four types of decays:

Singlet to singlet + singlet: $|1\rangle \rightarrow |1\rangle\, |1\rangle$ g_{11}
Singlet to octet + octet: $|1\rangle \rightarrow |8\rangle\, |8\rangle$ g_1
Octet to octet + singlet: $|8\rangle \rightarrow |8\rangle\, |1\rangle$ g_{18}
Octet to octet + octet: $|8\rangle \rightarrow |8\rangle\, |8\rangle$ g_T

SU(3) Clebsch-Gordan Coefficients

(η_1) Singlet

$$\begin{pmatrix} K \\ \pi \\ \eta_8 \\ \overline{K} \end{pmatrix}$$ Octet

nonet mixing

$$f = \cos\theta\eta_1 + \sin\theta\eta_8$$
$$f' = \cos\theta\eta_8 - \sin\theta\eta_1$$

pseudoscalars

$$\eta = \cos\theta_P\eta_8 - \sin\theta_P\eta_1$$
$$\eta' = \cos\theta_P\eta_1 + \sin\theta_P\eta_8$$

Singlet to singlet + singlet: |1> → |1> |1> g_{11} c_i = 1

Octet to octet + singlet: |8> → |8> |1> g_{18}

$$\begin{pmatrix} K \\ \pi \\ \eta_8 \\ \overline{K} \end{pmatrix} \rightarrow \begin{pmatrix} K & \eta_8 \\ \pi & \eta_8 \\ \eta_8 & \eta_8 \\ \overline{K} & \eta_8 \end{pmatrix} \qquad c_i = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

SU(3) Clebsch-Gordan Coefficients

Singlet to octet + octet: |1> → |8> |8> g_1

$$(\eta_1) \to \left(\begin{array}{cccc} K\overline{K} & \pi\pi & \eta_8\eta_8 & \overline{K}K \end{array} \right) \qquad c_i = \frac{1}{\sqrt{8}} \left(\begin{array}{cccc} 2 & 3 & 1 & -2 \end{array} \right)^{\frac{1}{2}}$$

$$c_i = \left(\begin{array}{cccc} \frac{1}{2} & \sqrt{\frac{3}{8}} & \sqrt{\frac{1}{8}} & -\frac{1}{2} \end{array} \right)$$

This is used to argue that a pure glueball will decay pairs of pseudoscalars as

$$\Gamma(G \to K\overline{K}) : \Gamma(G \to \pi\pi) : (G \to \eta_8\eta_8) = 4 : 3 : 1$$

SU(3) Clebsch-Gordan Coefficients

Octet to octet + octet: $|8\rangle \rightarrow |8\rangle\,|8\rangle$ g_T

There are two of these, denoted $|8_1\rangle$ and $|8_2\rangle$, and the choice depends on the C-parities of the three nonets. Look at the decay involving the neutral particles so C-parity is defined.

When the decay is C-parity allowed, use $|8_1\rangle$:

$+\rightarrow+\,+$, $+\rightarrow-\,-$, $-\rightarrow+\,-$, $-\rightarrow-\,+$

When the decay is C-parity forbidden, use $|8_2\rangle$:

$+\rightarrow+\,-$, $+\rightarrow-\,+$, $-\rightarrow+\,+$, $-\rightarrow-\,-$

SU(3) Clebsch-Gordan Coefficients

Octet to octet + octet: |8> → |8> |8> g_T

$$\begin{pmatrix} K \\ \pi \\ \eta_8 \\ \overline{K} \end{pmatrix} \to \begin{pmatrix} K\pi & & K\eta_8 & \pi K & \eta_8 K \\ K\overline{K} & \pi\pi & \eta_8\pi & \pi\eta_8 & \overline{K}K \\ K\overline{K} & \pi\pi & \eta_8\eta_8 & & \overline{K}K \\ \pi\overline{K} & & \eta_8\overline{K} & \overline{K}\pi & \overline{K}\eta_8 \end{pmatrix}$$

$$|8_1>: c_i = \frac{1}{\sqrt{20}} \begin{pmatrix} 9 & & -1 & -9 & -1 \\ -6 & 0 & 4 & 4 & -6 \\ 2 & -12 & -4 & & -2 \\ 9 & & -1 & -9 & -1 \end{pmatrix}^{\frac{1}{2}}$$

$\pi \to \eta_8\pi\,,\ K\overline{K}$ $\quad \pi \not\to \pi\pi$

$\eta_8 \to \pi\pi\,,\ \eta_8\eta_8\,,\ K\overline{K}$

$$|8_2>: c_i = \frac{1}{\sqrt{12}} \begin{pmatrix} 3 & & 3 & 3 & -3 \\ 2 & 8 & 0 & 0 & -2 \\ 6 & 0 & 0 & & 6 \\ 3 & & 3 & 3 & -3 \end{pmatrix}^{\frac{1}{2}}$$

$\pi \to \pi\pi\,,\ K\overline{K}$ $\quad \pi \not\to \eta_8\pi$

$\eta_8 \to K\overline{K}$ $\quad \eta_8 \not\to \pi\pi\,,\ \eta_8\eta_8$

Decays of f and f' in the |8$_2$> Case

A generic f and f'will be admixtures of singlet and octet states.

For the singlet component, $\eta_1 \to K\bar{K}\,,\ c_i = \frac{1}{2}\,,\ \eta_1 \to \bar{K}K\,,\ c_i = -\frac{1}{2}$

For the octet component, $\eta_8 \to K\bar{K}\,,\ c_i = \frac{1}{\sqrt{2}}\,,\ \eta_8 \to \bar{K}K\,,\ c_i = \frac{1}{\sqrt{2}}$

This sets us up for some interesting interference effects.

$$\begin{aligned}
\gamma\left(\eta_1 \to K\bar{K}\right) &= \frac{g_1}{2} \\
\gamma\left(\eta_1 \to \bar{K}K\right) &= -\frac{g_1}{2} \\
\gamma\left(\eta_8 \to K\bar{K}\right) &= \frac{g_T}{\sqrt{2}} \\
\gamma\left(\eta_8 \to \bar{K}K\right) &= \frac{g_T}{\sqrt{2}}
\end{aligned}$$

Decays of f and f' in the |8$_2$> Case

We will now specialize the daughter nonets to be vector and pseudoscalar, so the daughters will be $K^*\bar{K}$ and $\bar{K}^*K$.

$$\begin{aligned}
\gamma\left(f \to K^*\bar{K}\right) &= \sin\theta\frac{g_T}{\sqrt{2}} + \cos\theta\frac{g_1}{2} \\
\gamma\left(f \to \bar{K}^*K\right) &= \sin\theta\frac{g_T}{\sqrt{2}} - \cos\theta\frac{g_1}{2} \\
\gamma\left(f' \to K^*\bar{K}\right) &= \cos\theta\frac{g_T}{\sqrt{2}} - \sin\theta\frac{g_1}{2} \\
\gamma\left(f' \to \bar{K}^*K\right) &= \cos\theta\frac{g_T}{\sqrt{2}} + \sin\theta\frac{g_1}{2}
\end{aligned}$$

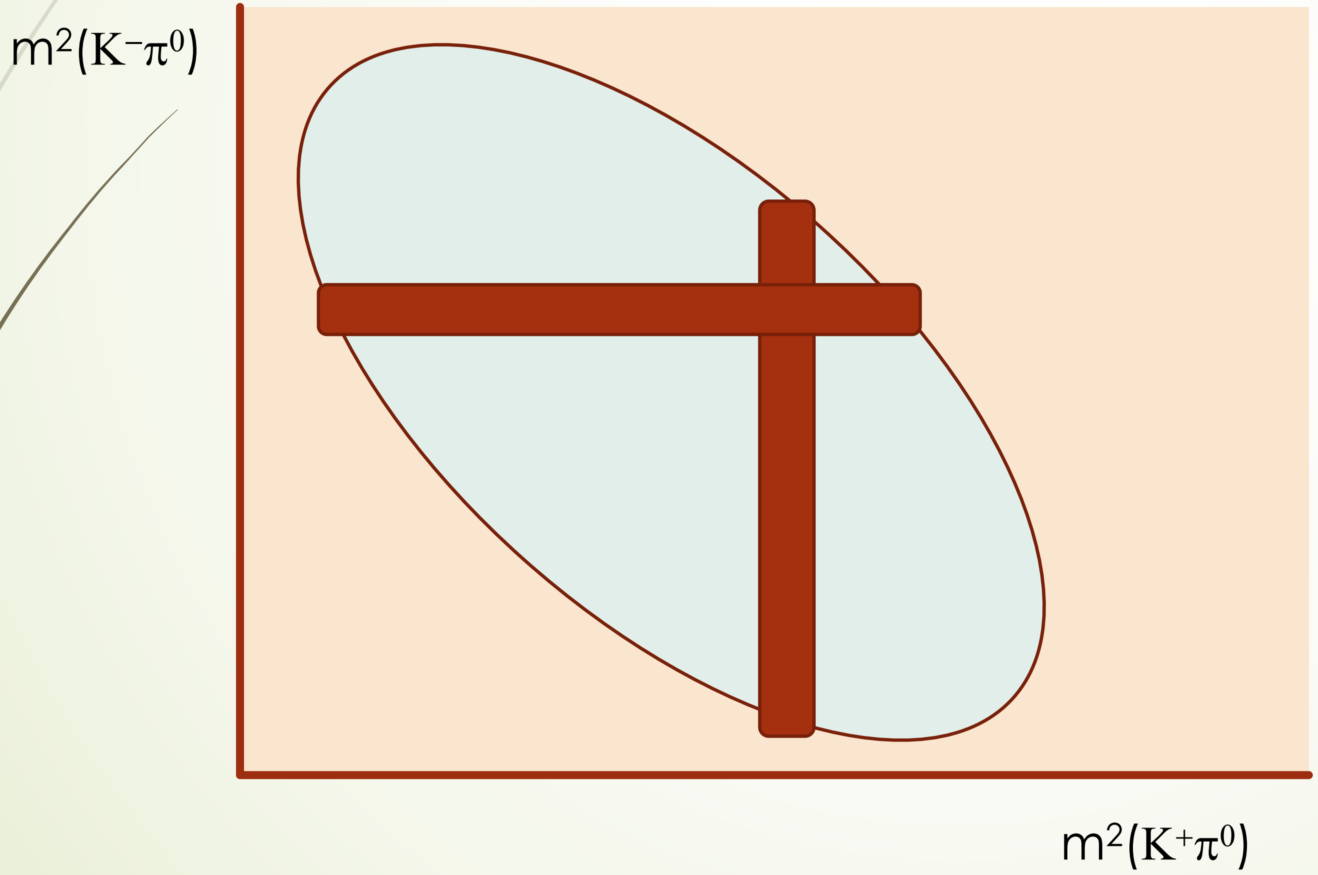
These could come from f_1/f_1', η/η' , η_2/η_2' or the exotic η_1/η_1' .

$$\begin{aligned}
\Gamma\left(f \to K^*\bar{K}\right) &\sim \sin^2\theta\frac{g_T^2}{2} + \cos^2\theta\frac{g_1^2}{4} + \sin\theta\cos\theta\frac{g_1 g_T}{2\sqrt{2}} \\
\Gamma\left(f \to \bar{K}^*K\right) &\sim \sin^2\theta\frac{g_T^2}{2} + \cos^2\theta\frac{g_1^2}{4} - \sin\theta\cos\theta\frac{g_1 g_T}{2\sqrt{2}}
\end{aligned}$$

These rates are not the same!

Dalitz Plot

Consider the $K^+ K^- \pi^0$ final state, so we have $K^{+*}K^-$ and $K^{-*}K^+$.
(I think including K_S will overly complicate this).



Choose finite regions in $KK\pi$ invariant mass and form Dalitz plots.

One K* band will be be stronger than the other one.

How useful is this?

- Many things can couple to K^*K, so any partial wave analysis is probably going to be very complicated.
- For an ideally-mixed nonet,

$$\begin{aligned}\Gamma\left(f \to K^* \bar{K}\right) &\sim \tfrac{1}{6}\left(g_T^2 + g_1 g_T + g_1^2\right)\\ \Gamma\left(f \to \bar{K}^* K\right) &\sim \tfrac{1}{6}\left(g_T^2 - g_1 g_T + g_1^2\right)\end{aligned}$$

- This could help disentangle η_1 and η_1' if we can see them.
- The ground state and hybrid 2^{-+} nonets may be close. This could help disentangle the four η_2 states.