The GDH Program at Jefferson Lab

S. Širca, U of Ljubljana, Slovenia

Tallahassee, February 21, 2024

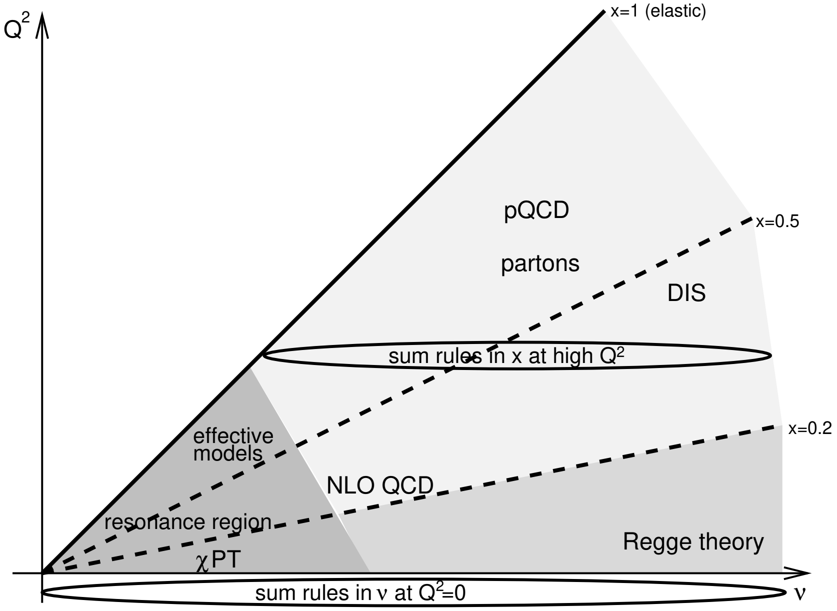
The ABC of GDH

- GDH = Gerasimov, Drell, Hearn (1966)
- Relates difference $\Delta\sigma \equiv \sigma_{3/2}-\sigma_{1/2} \equiv \sigma_{\rm P}-\sigma_{\rm A}$ of spin-dependent total photo-production XS to anomalous magnetic moment κ and mass M of arbitrary particle:

$$I_{\rm GDH} = \int_{\nu_0}^{\infty} \frac{\sigma_{3/2}(\nu)-\sigma_{1/2}(\nu)}{\nu}\,{\rm d}\nu = 4\pi^2\alpha\, S\,\frac{\kappa^2}{M^2}$$

- Fundamental QFT statement; valid for any spin S ... but:
  - RHS for proton/ neutron known to ∼ 8 digits: $I^p_{\rm GDH} \approx 205\ \mu$b, $I^n_{\rm GDH} \approx 232\ \mu$b
  - $\Delta\sigma$ for p (n) known at few % level, but only to $\nu = E_\gamma \approx 2.9$ GeV (2 GeV)
  - $\Delta\sigma$ at large ν unknown; domain of Regge theory
  - $1/\nu$ weight emphasizes threshold region, $\nu_0 \geq m_\pi(1+m_\pi/2M_N)$ for p/n, thus sum rule saturated by $\nu \approx 3$ GeV (?)

Drechsel, Tiator, Annu. Rev. Nucl. Part. Sci. **54**, 69 (2004)



GDH: why the question mark

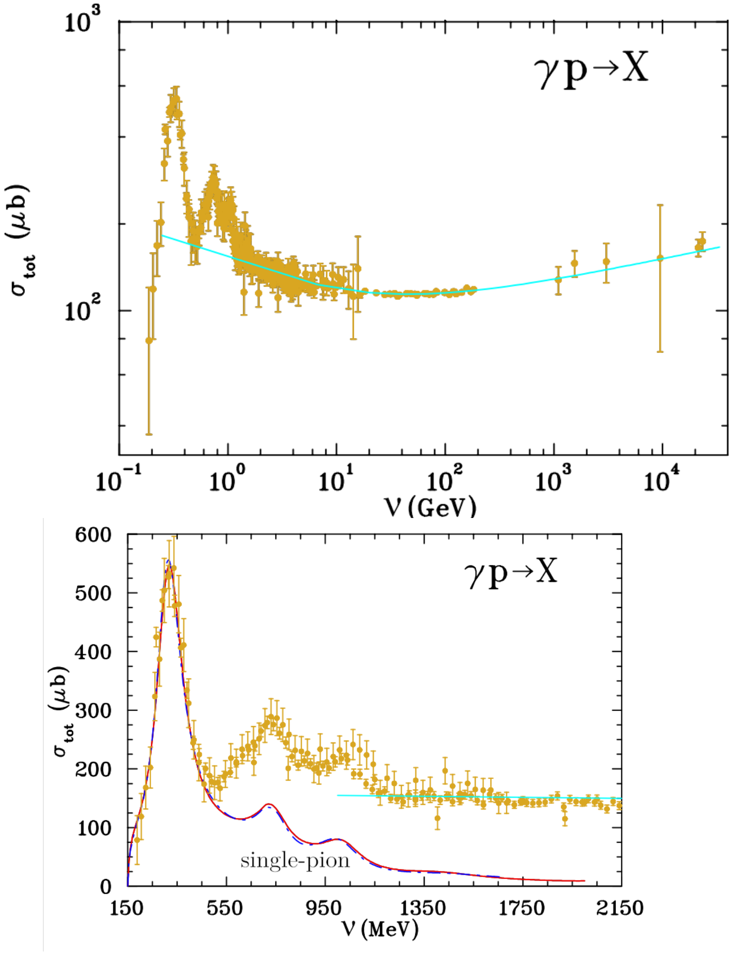
- Unpolarized “sum rule” (for $\sigma_{\text{tot}} \equiv \sigma_{3/2} + \sigma_{1/2} \equiv \sigma_{\text{P}} + \sigma_{\text{A}}$ on p/n):

$$\int_{\nu_0}^{\infty} \left(\sigma_{3/2}(\nu) + \sigma_{1/2}(\nu)\right)\, \mathrm{d}\nu = -\frac{\pi\alpha}{M_N}$$

  - ▷ LHS > 0, RHS < 0 (?)
  - ▷ Divergent integrand (?)
  - ▷ Pomeron exchange (1961)

    Regge parameterization of the XS,
    good up to $s = M_N(M_N + 2\nu) \approx (250\,\text{GeV})^2$:
    $\sigma_{\text{tot}} = (129\, s^{-0.4545} + 67.7\, s^{0.08})\,\mu\text{b}$
  - ▷ If $\int \sigma_{\text{tot}}(\nu)\, \mathrm{d}\nu$ is divergent, what are the implications for the convergence of $\int (\Delta\sigma(\nu)/\nu)\, \mathrm{d}\nu$ and asymptotic behaviour of $\Delta\sigma(\nu)$?
  - ▷ ∃ several considerations why the sum rule may need to be modified

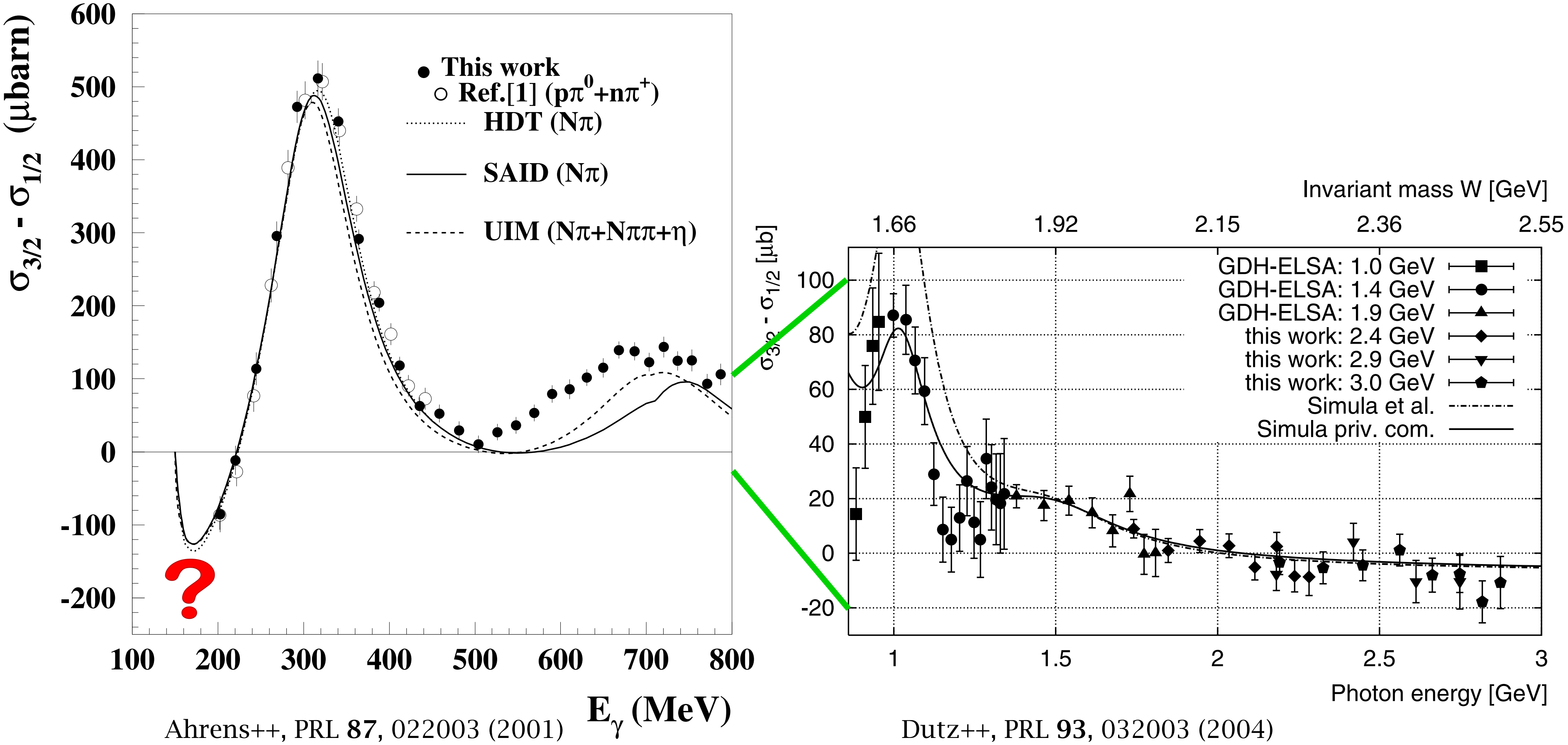
    Pantförder, arXiv:hep-ph/9805434



Strakovsky++, PRC **105**, 045202 (2022)

Measurements of $\Delta\sigma \neq$ evaluations of the GDH integral

- Threshold region important due to $1/\nu$ weight ⇒ extrapolation
  (Use models like MAID/SAID: both give $I^{p}_{\mathrm{GDH}}(\nu \leq 0.2\,\mathrm{GeV}) \approx -28\,\mu\mathrm{b}$)
- Phenomenological input for high-ν
- MAMI, ELSA: $0.2\,\mathrm{GeV} \leq \nu \leq 2.9\,\mathrm{GeV}$ (p), $0.2\,\mathrm{GeV} \leq \nu \leq 1.8\,\mathrm{GeV}$ (n)
  “Typical results” (proton) + standard problem near threshold:



Ahrens++, PRL **87**, 022003 (2001)

Dutz++, PRL **93**, 032003 (2004)

Measurements of $\Delta\sigma(\nu)$, the grand total

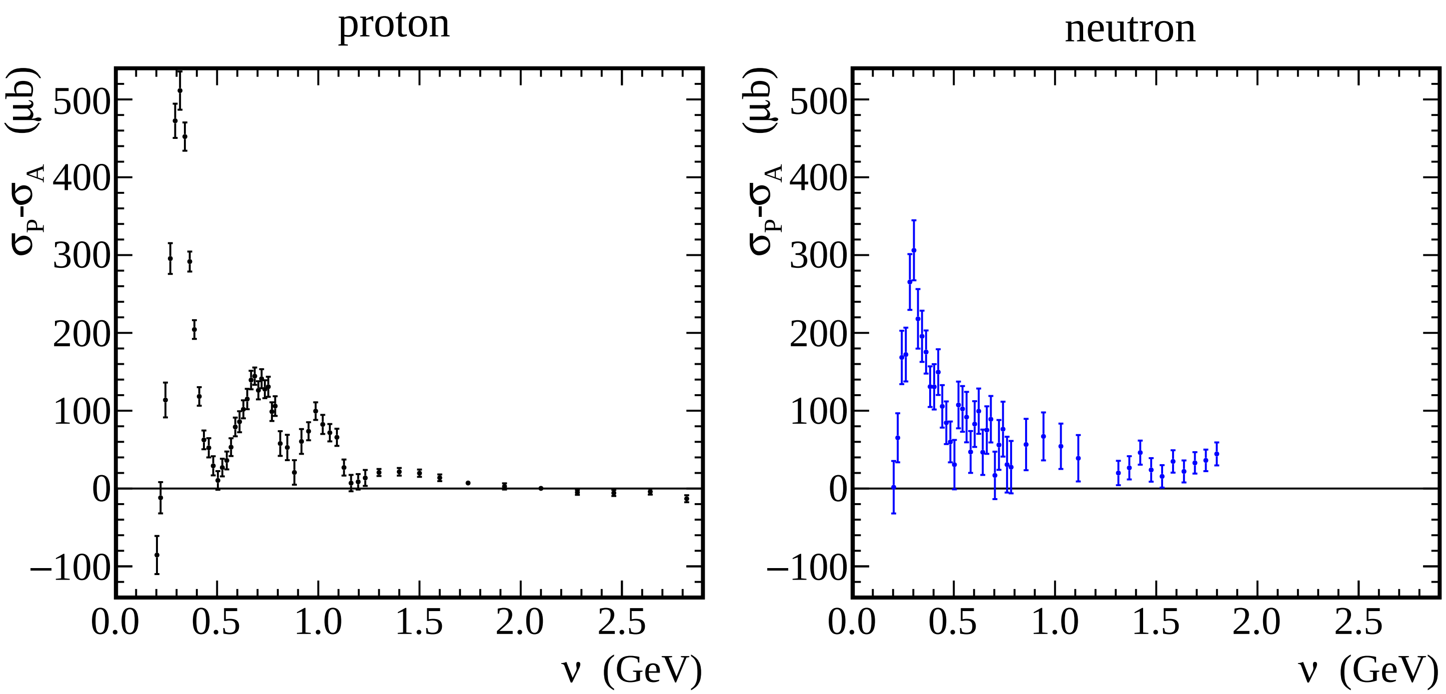
Other measurements exist, e. g. CLAS g9 (JLab @ 6 GeV): 1–π contrib up to 2 GeV, 2–π contrib up to 3 GeV, under analysis, etc. etc.

Extractions of the neutron $\Delta\sigma$ from d, ^{3}He etc. require subtractions depending on the target, e.g. LiD: $\Delta\sigma^{d,n}(\nu) = \text{corr}(\nu)\Delta Y^{\text{LiD}} - g^{d,n}\Delta\sigma^{p}(\nu)$ and involve theoretical assumptions

The (unmeasured) **high-ν region** is interesting in its own way

in spite of the $1/\nu$ weight ... more on this later

All experiments combined and rebinned



(Some) JLab experiments on spin SRs, spin polarizabilities etc.

- Hall B: EG1a
  g_1^p down to $Q^2 = 0.15$
- Hall A: E94–010 (Cates, Chen, Meziani)
  $g_1^{^3\mathrm{He}}(x, Q^2)$, $g_2^{^3\mathrm{He}}(x, Q^2)$, $\Gamma_1^{^3\mathrm{He}}(Q^2)$, ...
  $\Longrightarrow n$
- Hall A: E97–110 (Chen, Deur, Garibaldi) — "small-angle GDH/n"
  $\Gamma_1^{^3\mathrm{He}}(Q^2)$, $I_{TT}^{^3\mathrm{He}}(Q^2)$, $\gamma_0^{^3\mathrm{He}}(Q^2)$, ...
  $\Longrightarrow n$
- Hall B: EG4 / E03–006 (Ripani, Battaglieri, Deur, de Vita) — "small-angle GDH/p"
  $\Gamma_1^p(Q^2)$, $I_{TT}^p(Q^2)$, $\gamma_0^p(Q^2)$, ... at low Q^2
- Hall B: EG4 / E05–111 (Deur, Dodge, Ripani, Slifer)
  $\Gamma_1^d(Q^2)$, $I_{TT}^d(Q^2)$, $\gamma_0^d(Q^2)$, ... at low Q^2
  $\Longrightarrow n$
- Hall A: E08–027 (Camsonne, Chen, Crabb, Slifer)
  $g_1^p(x, Q^2)$, $g_2^p(x, Q^2)$, $I_{TT}^p(Q^2)$, ... at low Q^2 (only one Q^2 point for I_{TT}^p)

All these observables can be related to the GDH integral in one way or another ...

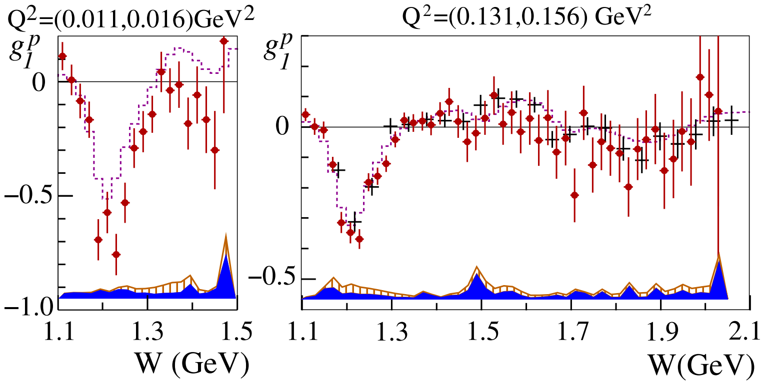
... here is just one example ⟹

Generalized GDH sum rule ($Q^2 \neq 0$)

CLAS EG4 (2006)

$$\Gamma_1^p(Q^2) = \int_0^{1^-} g_1^p(x,Q^2)\,\mathrm{d}x \to -\frac{Q^2\kappa_p^2}{8M^2} \quad \text{as} \quad Q^2 \to 0$$

- GDH sum given by the slope of $\Gamma_1^p(Q^2)$ at $Q^2 = 0$
- Proton target, very low Q^2:



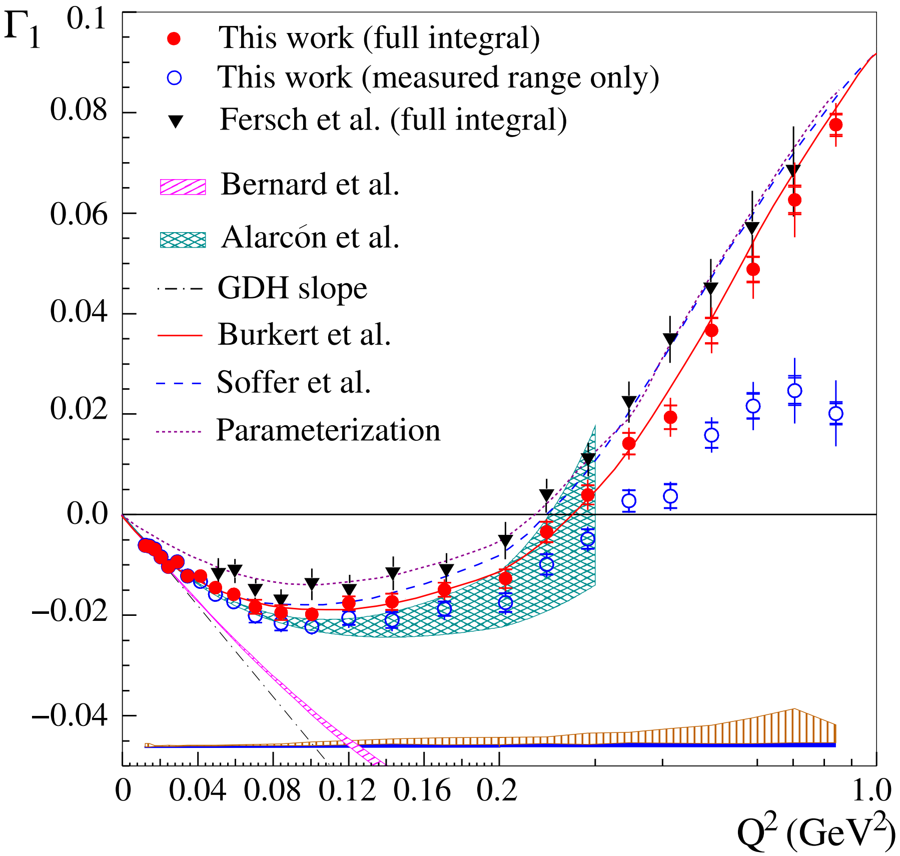
▷ $W \geq 1.15$ GeV (avoid elastic tail)

▷ Used parameterization of previous data to evaluate contributions from the low-x region (down to $x \approx 10^{-3}$) and the high-x region (from W_{thr} up to 1.15 GeV)

▷ Offers unique test of χEFT

Zheng++, Nat. Phys. **17**, 736 (2021)

proton



The slope of $\Gamma_1^p(Q^2) \longrightarrow$ the value of $I_{TT}^p(Q^2)$ at $Q^2 = 0$

Recall:

$$\frac{8\pi^2\alpha}{M^2} I_{TT}(0) = -\frac{2\pi^2\alpha\kappa^2}{M^2} = -I_{\text{GDH}}$$

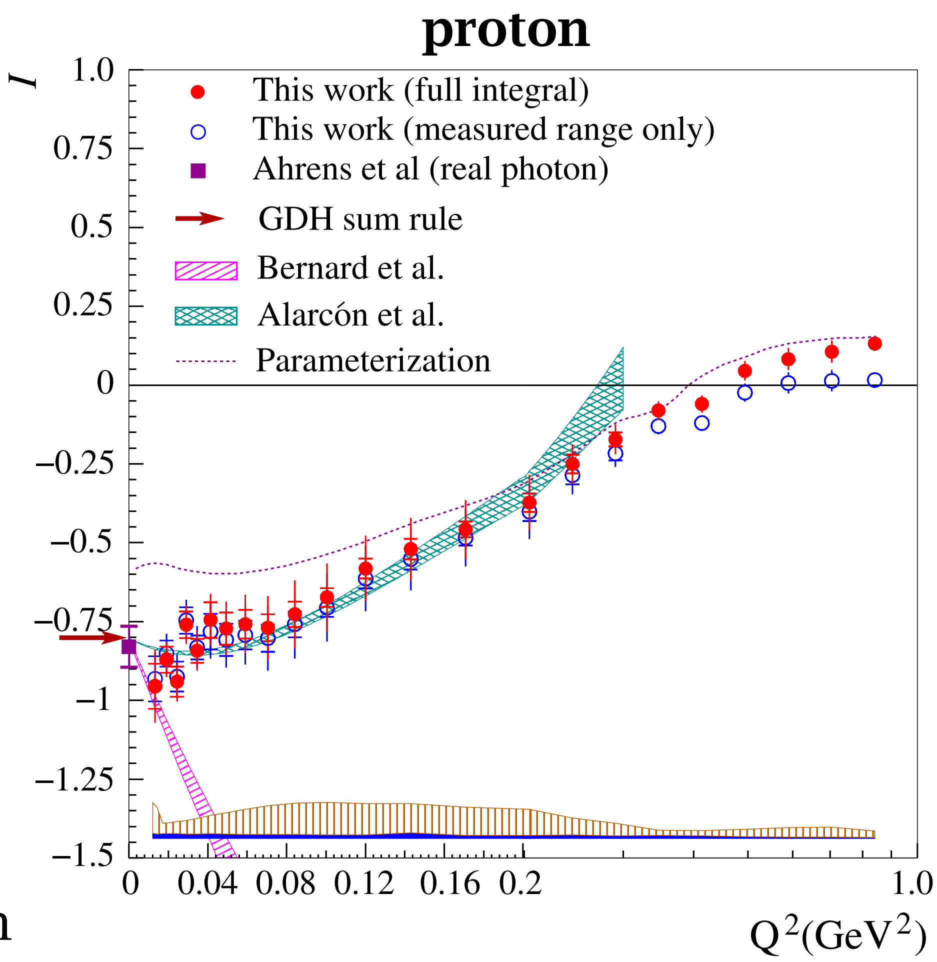
- EG4 result on proton:

$$I_{TT}^{p,\text{EG4}}(0) = -0.798 \pm 0.042$$

$$I_{TT}^{p}(\text{GDH}) = -\tfrac{1}{4}\kappa_p^2 = -0.804\ldots$$

$$I_{TT}^{p}(\text{MAMI} = -0.832 \pm 0.023 \pm 0.063$$

(from photo-production)



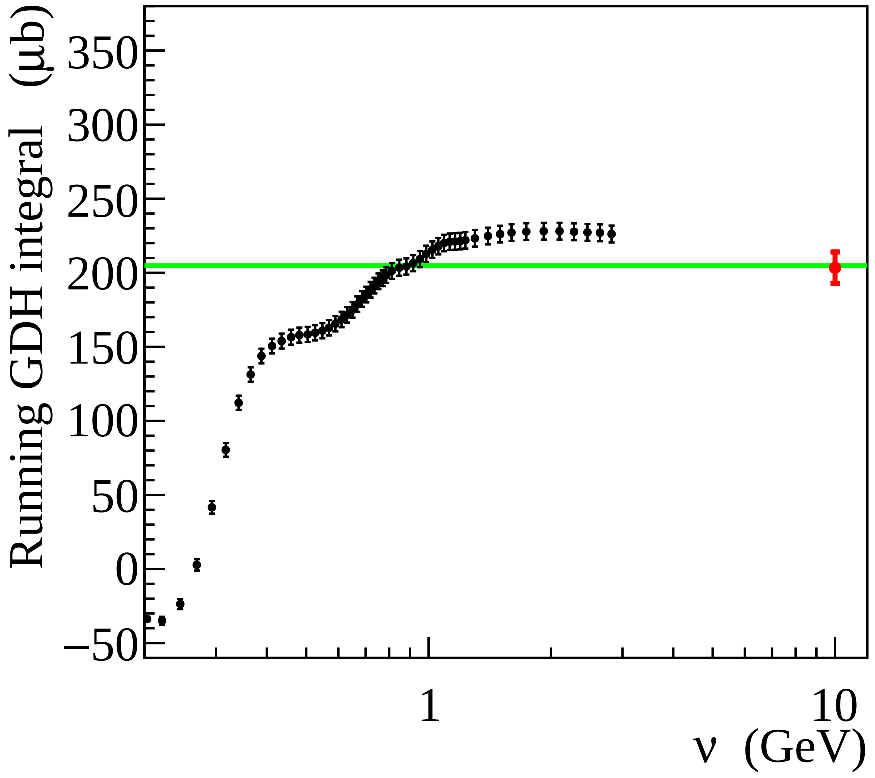
- Issue of $Q^2 \to 0$ extrapolation: Manifestly Lorentz-invariant BχPT vs. heavy-baryon frameworks
- Even more pertinent (and drastic) in the case of generalized longitudinal spin polarizability

Zheng++, Nat. Phys. **17**, 736 (2021)

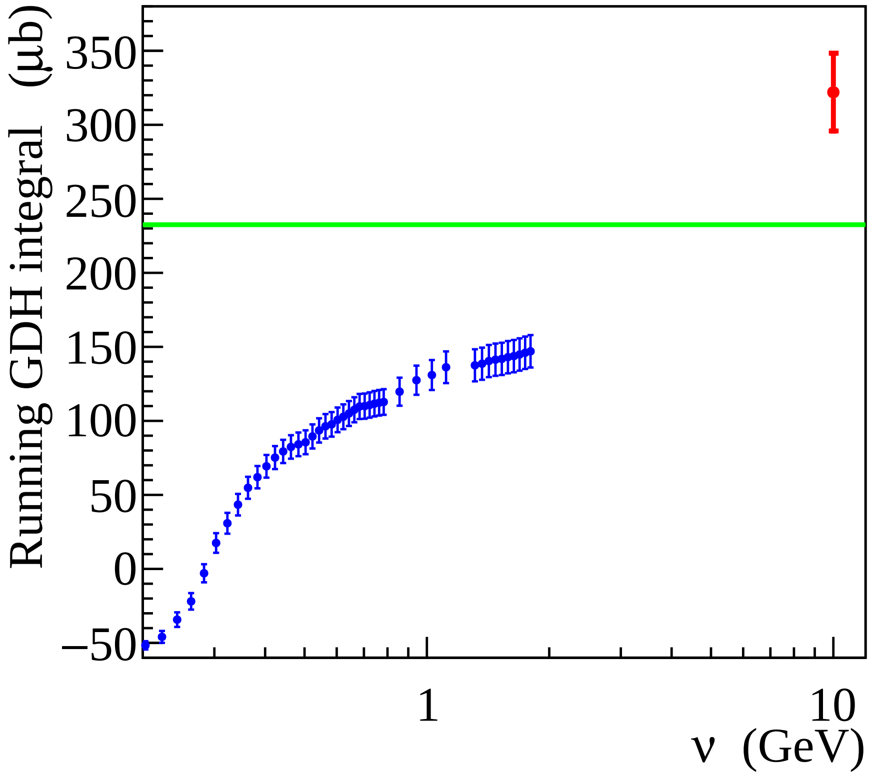
Alarcon++, PRD **102**, 114026 (2020)

Running GDH integral

$$\int_{\nu_0}^{\nu} \frac{\Delta\sigma(\nu')}{\nu'}\, d\nu'$$



proton



neutron

Contributions below 0.2 GeV: ≈ $-28\,\mu$b (proton), ≈ $-41\,\mu$b (neutron)

Red points (not really at $\nu = 10$!): from generalized GDH integral at $Q^2 \to 0$

The GDH integrand in the Regge framework

s-dependence of real/virtual polarized photo-absorption:

$$\Delta\sigma = \left[I c_1 s^{\alpha_{a_1}-1} + c_2 s^{\alpha_{f_1}-1} + c_3 \frac{\log s}{s} + \frac{c_4}{\log^2 s} \right] F(s, Q^2)$$

$I = \pm 1 =$ p/n isospin factor, α_{a_1}, α_{f_1} = intercepts of a_1 and f_1 Regge trajectories
For $Q^2 = 0$, log terms negligible, $F(s, Q^2)$ simplifies to a constant → absorb in c_1, c_2 $\Longrightarrow$

$$\Delta\sigma = I c_1 s^{\alpha_{a_1}-1} + c_2 s^{\alpha_{f_1}-1}$$

$c_1 = (-34.1 \pm 5.7)\,\mu\text{b}$, $\alpha_{a_1} = 0.42 \pm 0.23$, $c_2 = (209.4 \pm 29.0)\,\mu\text{b}$, $\alpha_{f_1} = -0.66 \pm 0.22$

Decompose κ_p, κ_n into iv/is components, $\kappa_p = (\kappa_\text{S} + \kappa_\text{V})/2$, $\kappa_n = (\kappa_\text{S} - \kappa_\text{V})/2$

$$\Longrightarrow \kappa_{p,n}^2 = \tfrac{1}{4}\kappa_\text{S}^2 \pm \tfrac{1}{2}\kappa_\text{V}\kappa_\text{S} + \tfrac{1}{4}\kappa_\text{V}^2$$

Split the GDH sum rule accordingly (I_GDH^SS, I_GDH^VV, and I_GDH^VS)

$$\Longrightarrow I_\text{GDH}^\text{VS} = \int_{E_\gamma^\text{thr}}^{\infty} \left(\sigma_{3/2}^\text{VS} - \sigma_{1/2}^\text{VS} \right) \frac{\text{d}E_\gamma}{E_\gamma} = \tfrac{1}{2}\kappa_\text{V}\kappa_\text{S}\frac{2\pi^2\alpha}{M^2}$$

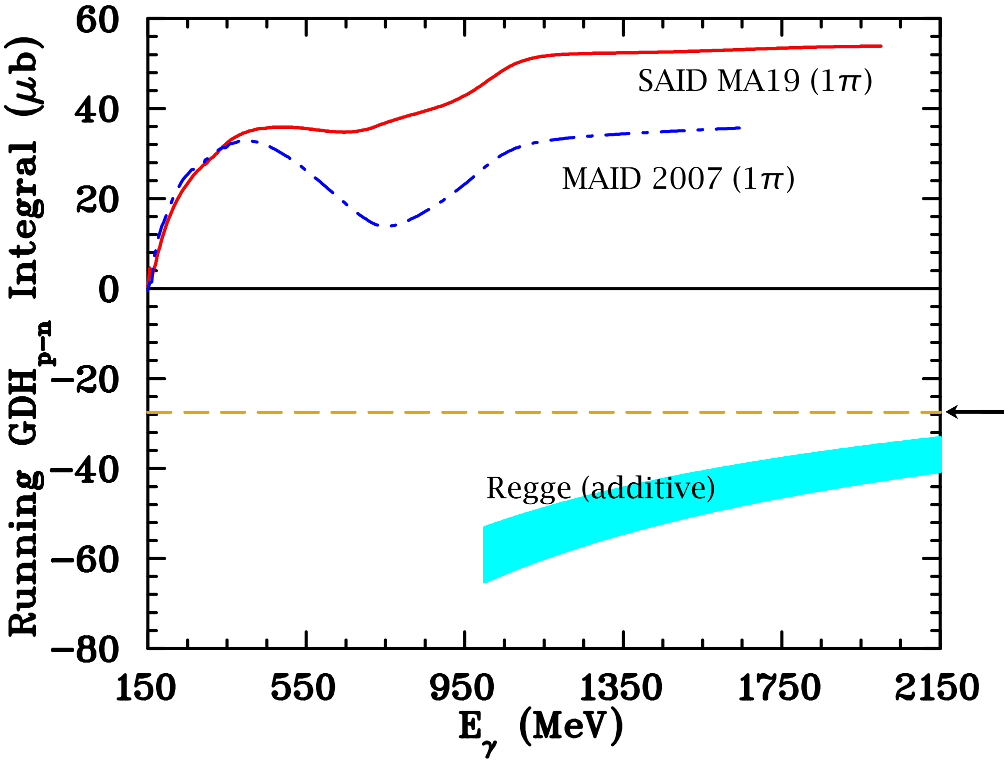
Since $\kappa_p^2 - \kappa_n^2 = \kappa_\text{V}\kappa_\text{S}$, the isovector GDH sum rule amounts to

$$\int_{E_\gamma^\text{thr}}^{\infty} \frac{\Delta\sigma_{p-n}}{E_\gamma}\,\text{d}E_\gamma = 2 I_\text{GDH}^\text{VS} \approx -27.5\,\mu\text{b}$$

Isovector GDH sum rule à la Regge

In Regge theory, $\Delta\sigma_{p-n}$ is driven by the a_1 trajectory alone:

$$\Delta\sigma_{p-n}^{\text{Regge}} = 2c_1 s^{\alpha_{a_1}-1}$$



← GDH prediction (−27.5 μb)

Strakovsky++, PRC **105**, 045202 (2022)

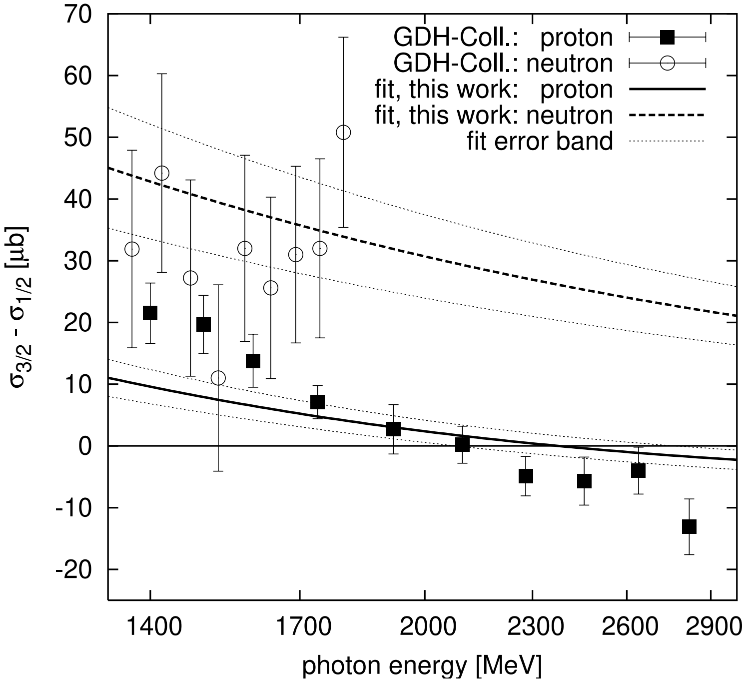
⟹ Understanding the magnitude and sign of α_{a1} is important

⟹ **“REGGE”** — JLab Experiment E12–20–011

New GDH effort: “REGGE” — JLab Experiment E12–20–011

(M. M. Dalton, A. Deur, SŠ, J. Stevens)

- $\Delta\sigma$ at high ν **unknown**
- High ν = domain of Regge theory
- If the GDH sum rule failed, it would happen at high ν (not in the low-ν region, even if it dominates in the sum)



Strategy:

▷ Measure on both proton and neutron (deuteron) to allow for isospin separation
Regge: is/iv contributions to $\Delta\sigma$ come from different meson families: $f_1(1285)/a_1(1260)$

▷ Extend energy coverage: $3 < \nu < 12\,\text{GeV}$

▷ Hall D @ JLab ideally suited for this study
cross-check with MAMI/ELSA at $\nu < 3\,\text{GeV}$ would be nice, but invasive to other Halls

▷ Measure yield difference $\Delta Y(\nu) = N^+ - N^-$
$\longrightarrow$ make sure $\Delta\sigma(\nu)/\nu$ decreases rapidly enough
$\longrightarrow$ investigate the power-law behavior of $\Delta\sigma(\nu)$, i.e. establish b in $\Delta\sigma(\nu) = a\nu^b$

(▷ Determine absolute $\Delta\sigma(\nu)$: later)

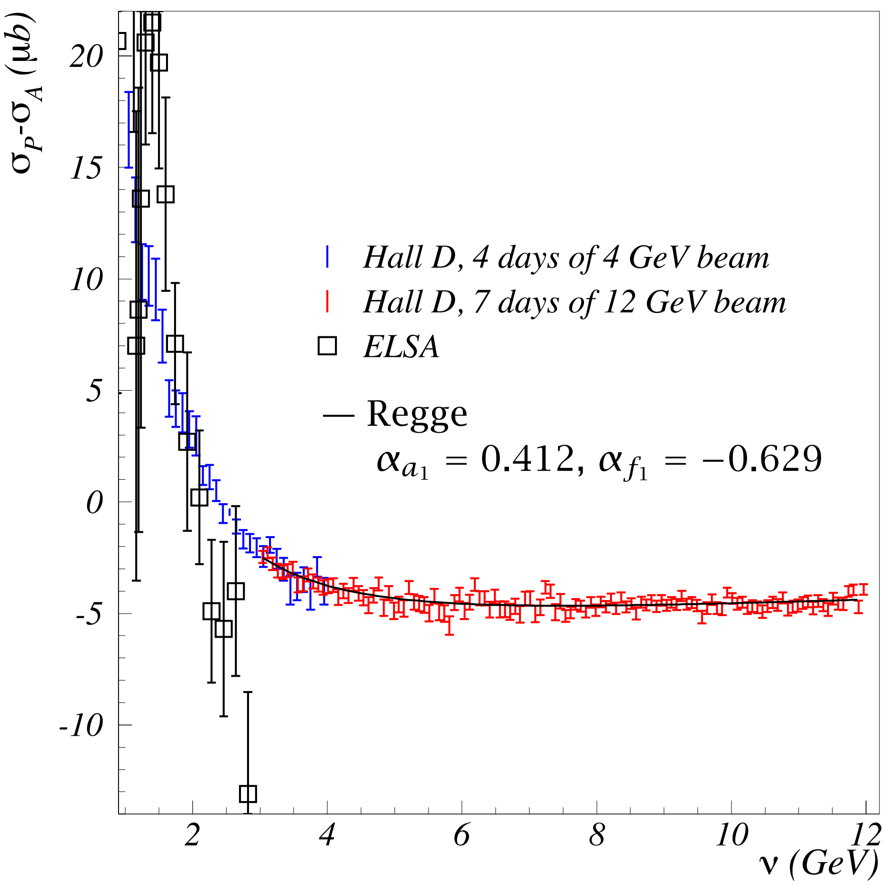
“REGGE” — JLab Experiment E12–20–011

Setup:

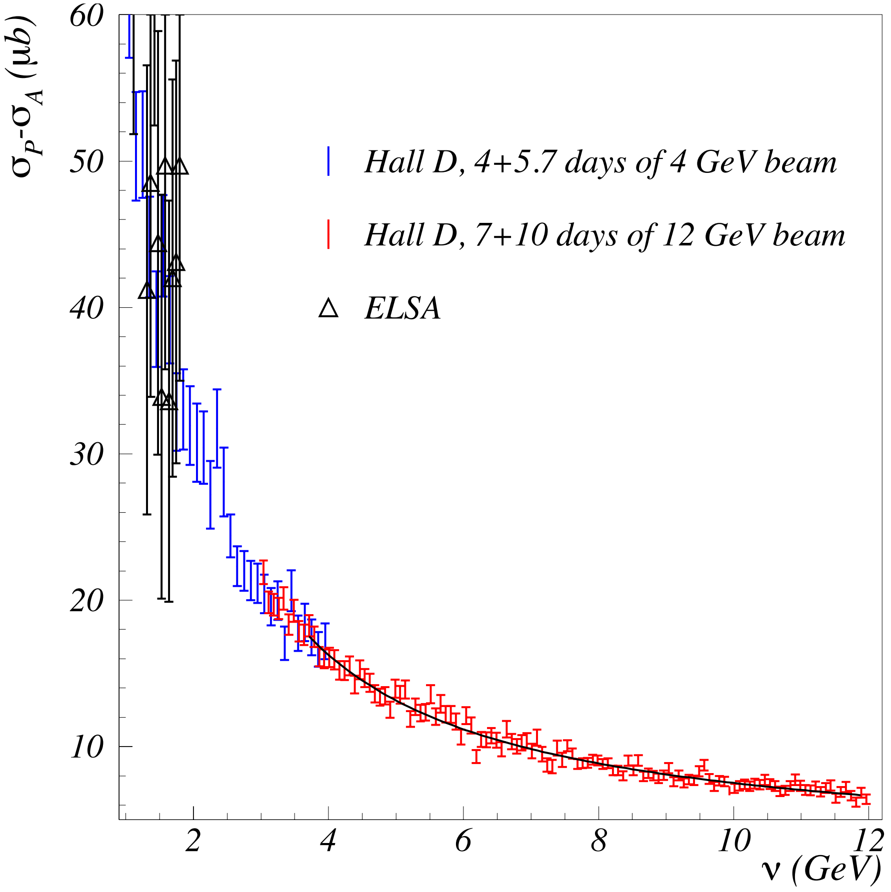
- Circularly polarized tagged photon beam ✓
  Generated by electrons from CEBAF with $P_e \approx 80\,\%$ on amorphous radiator
  Increasing P_e at large ν compensates the decrease in bremsstrahlung flux and XS
- Longitudinally polarized target: (a new) FROST
  Chosen against HDice (= does not allow extension to polarization of heavier nuclei)
  Dynamical nuclear polarization on butanol (C_4H_9OH), p and d polarizations up to 90 %
  Desired sustainable flux: $\approx 10^8/\mathrm{s}$ or more
  Dilution (and other unpolarized backgrounds) cancel:
  $(N^+ + N^0) - (N^- + N^0) = N^+ - N^-$
- Large solid angle detector ✓
  FCal (with $PbWO_4$ upgrade), BCal: 0.4° to 145° polar, 2π azimuthal coverage
  Unpolarized XS $\approx 120\,\mu$b $\Longrightarrow$ DAQ rate $\approx$ 33 kHz on H-butanol, $\approx$ 40 kHz on D-butanol
  $+$ target window $+$ EM backgrounds

- Note: solely to establish the fall-off of $\Delta\sigma(\nu)/\nu$, the ν-independent normalization factors (flux, ρ_t, P_e, P_t, $\int\Delta\Omega$) are irrelevant

“REGGE” — Expected results



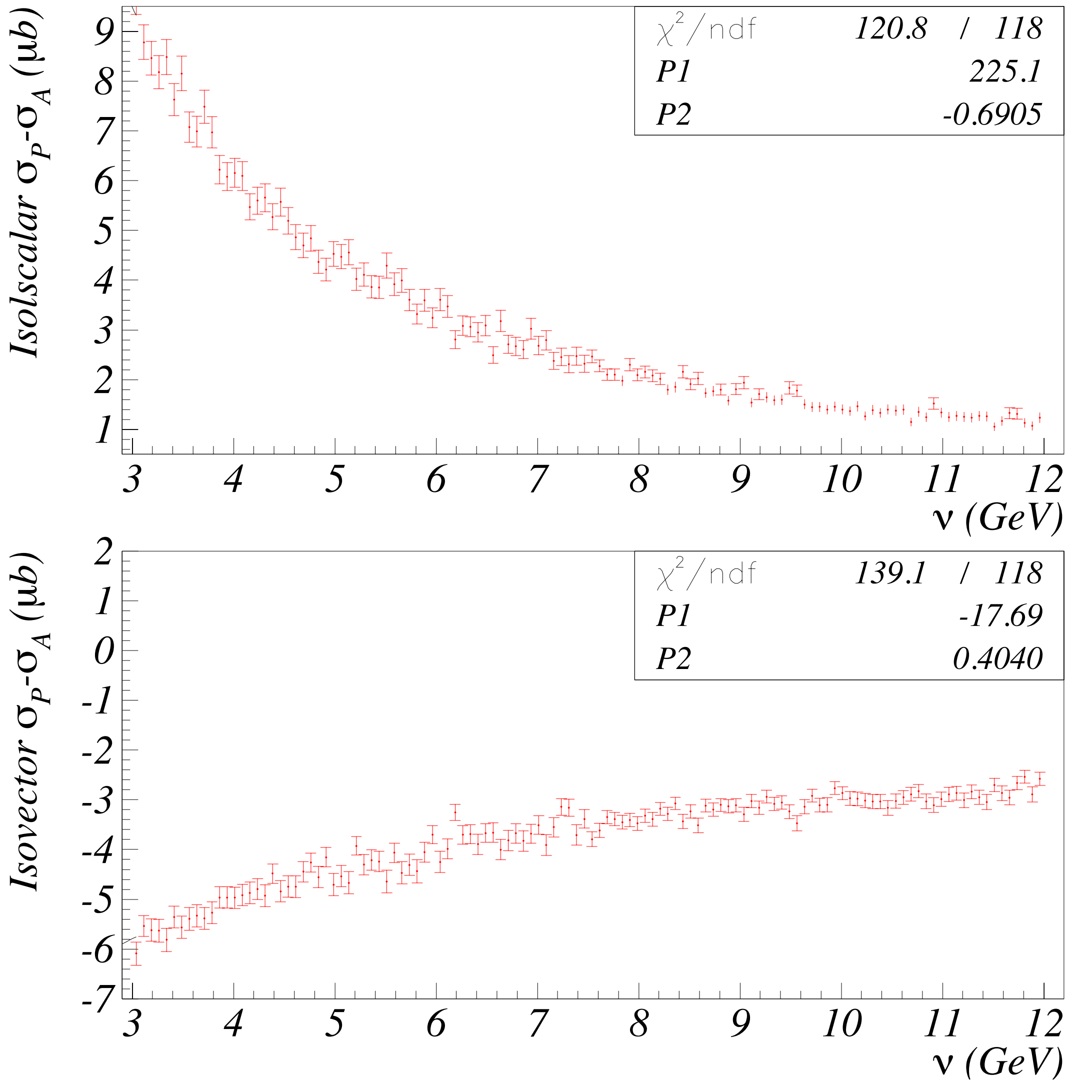
proton



neutron (from deuteron)

“REGGE” — Isospin decomposition

Based on the substraction $\Delta\sigma_{\mathrm{n}} = \Delta\sigma_{\mathrm{d}}/(1 - 1.5\omega_D) - \Delta\sigma_{\mathrm{p}}$



(Select) motivation for “REGGE”

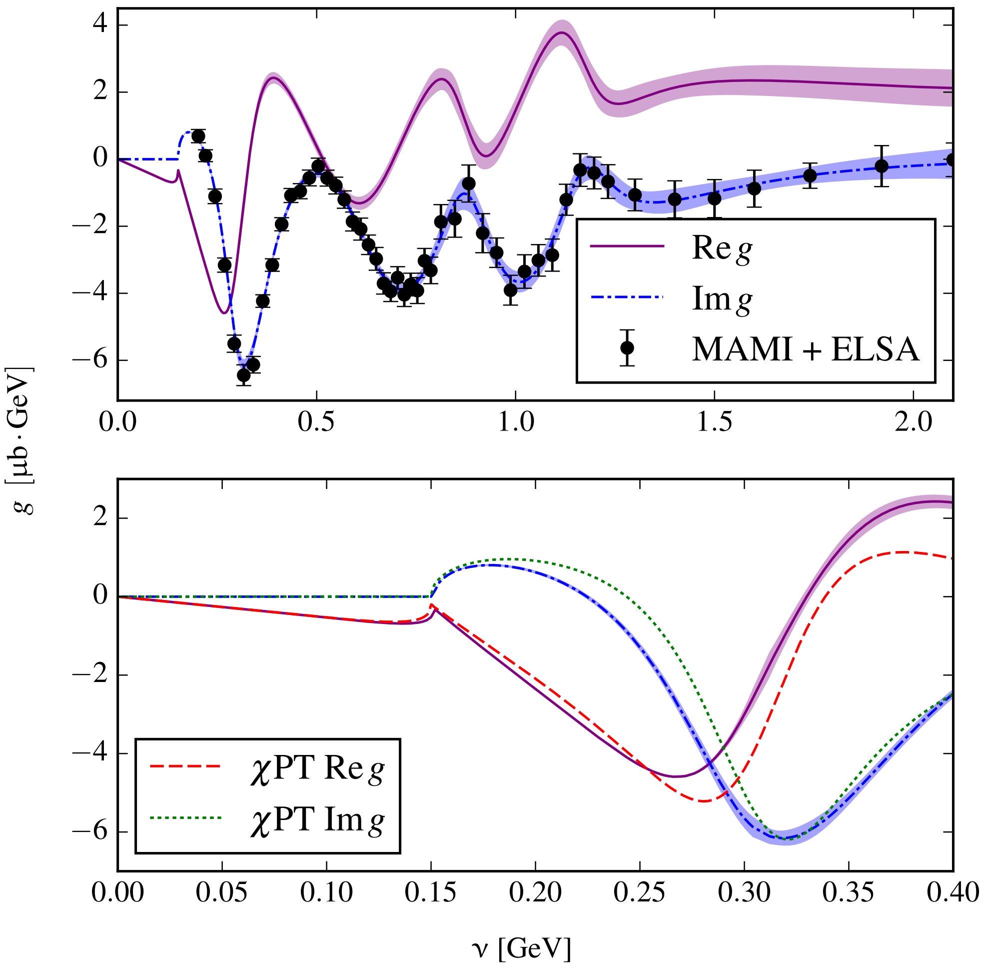
- Access Compton physics without resorting to dedicated Compton setup
- Relation of $\Delta\sigma$ to spin-dependent Compton amplitude g:

$$\mathrm{Im}\, g(\varepsilon) = \frac{\varepsilon}{8\pi}\left(\sigma_{3/2} - \sigma_{1/2}\right)$$

- Access to the real part via DR:

$$\mathrm{Re}\, g(\nu) = \frac{2\nu}{\pi} P \int_0^\infty \frac{\mathrm{Im}\, g(\varepsilon)}{\varepsilon^2 - \nu^2}\, \mathrm{d}\varepsilon$$

$\Longrightarrow$ Extend Re g—Im g “symbiosis” cross-check to beyond 10 GeV (sixfold energy range)



Hagelstein++, Prog. Part. Nucl. Phys. **88**, 29 (2016)

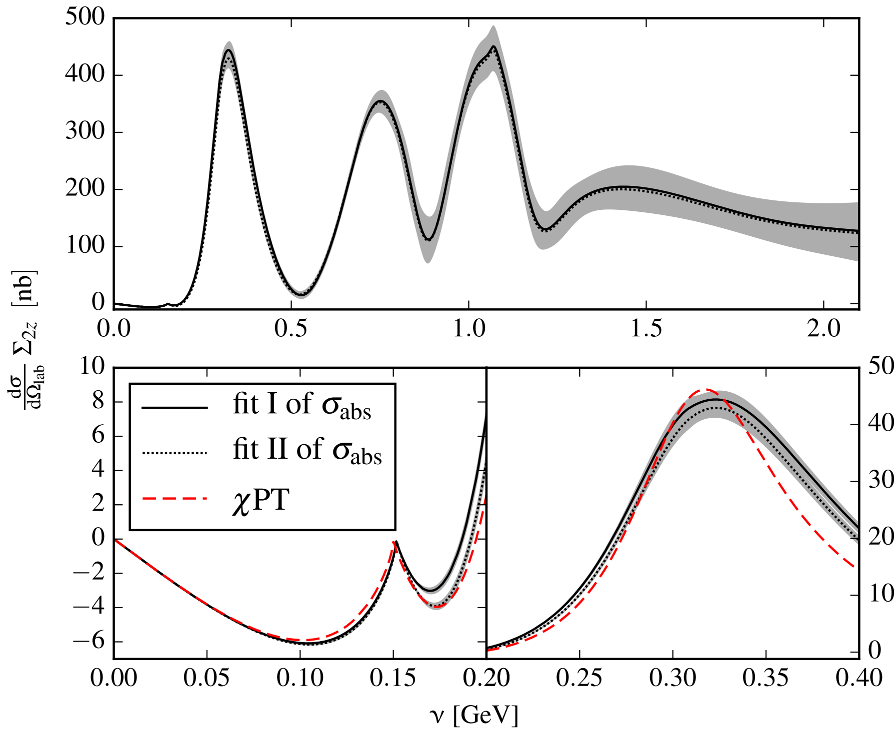
(Select) motivation for "REGGE"

- If both Re g and Im g are known precisely enough (and given f, the unpolarized amplitude, which is well measured), one can determine the differential XS and the beam-target asymmetry in the fwd direction:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=0} = |f|^2 + |g|^2, \qquad \Sigma_{2z}|_{\theta=0} = -\frac{2\mathrm{Re}\,(f g^*)}{|f|^2 + |g|^2}$$

- $\Sigma_{2z} = \Delta\sigma/\sigma_{\text{tot}}$ provides information on (all four) spin polarizabilities; very sensitive to chiral loops

$\Longrightarrow$ Reduce uncertainties of Σ_{2z} by precise measurements of $\Delta\sigma(\nu)$ at high ν



Hagelstein++, Prog. Part. Nucl. Phys. **88**, 29 (2016)

(Select) motivation for "REGGE"

- Regge: $\Delta\sigma_{\mathrm{p-n}}$ driven by the a_1 trajectory:

$$\Delta\sigma_{\mathrm{p-n}}^{\mathrm{Regge}} = 2c_1 s^{\alpha_{a_1}-1}$$

- Conflicting determinations:

	α_{a_1}
DIS fit (approx. values)	0.45
Photo/electro-production fit	0.31 ± 0.04
Regge expectation	-0.34

- Problem: $a_1(1260)$ is the only $I^G(J^{PC}) = 1^-(1^{++})$ meson to form a "trajectory", while the second candidate, the $a_1(1640)$, has been omitted from the PDG Summary Tables (needs confirmation)

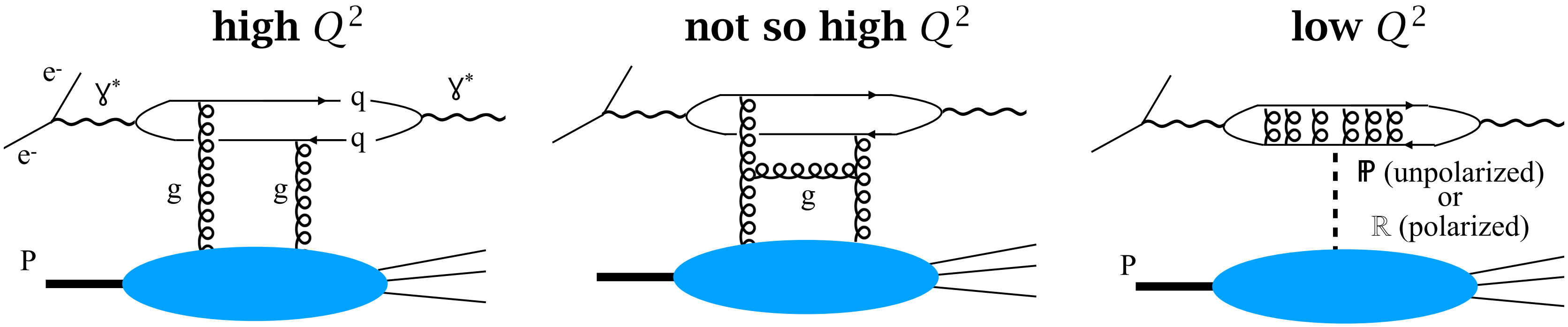
$\Longrightarrow$ A precise measurement of $\Delta\sigma$ at high ν for both proton and neutron targets would help to remove this uncertainty. $\longrightarrow$ Note: the intercept is given by

$$\alpha_{a_1} = 1 - \alpha' m_{a_1}^2$$

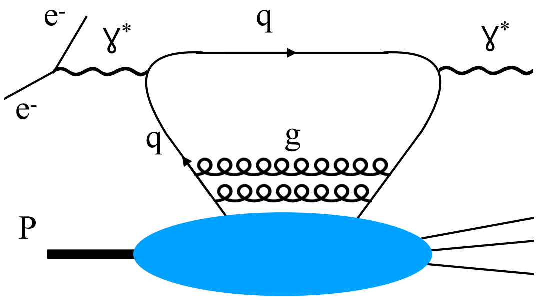
where $\alpha' = 1/(2\pi\sigma) \approx 0.88\,\mathrm{GeV}^{-2}$ and σ is the string tension

(Select) motivation for “REGGE”

- Explore transition between polarized DIS and diffraction regimes
- Diffractive scattering ⇔ diquark picture


high Q^2 **not so high Q^2** **low Q^2**

- Other mechanisms exist, connecting to DIS parton model, e. g.



- Doubly-polarized $\vec{e}$–$\vec{p}$ scattering filters out $\mathbb{P}$ exchanges to reveal non-singlet $\mathbb{R}$ exchange ⟹ relevant to EIC

(Select) motivation for “REGGE”

- Polarizability correction to hyperfine splitting in hydrogen

$$\begin{aligned} E_{\mathrm{HFS}}(nS) &= \left[1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{structure}}\right] E_{\mathrm{Fermi}}(nS) \\ \Delta_{\mathrm{structure}} &= \Delta_Z + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}} \end{aligned}$$

Relative uncertainties of the three terms: 140 ppm, 0.8 ppm, 86 ppm, respectively vs. precision of forthcoming PSI measurement of E_{HFS}: 1 ppm

“REGGE” can contribute to the uncertainty reduction of Δ_{pol}:

$$\Delta_{\mathrm{pol}} = \frac{\alpha m_{\mathrm{e}/\mu}}{2\pi(1+\kappa)M}[\delta_1 + \delta_2]$$

$$\delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left(\{\cdots\} + \frac{8M^2}{Q^2}\int_0^{x_0} \mathrm{d}x\, g_1(x,Q^2)\{\cdots\}\right)$$

The GDH integrand at general values of ν and Q^2:

$$\Delta\sigma(\nu) = -\frac{8\pi^2\alpha}{MK_{\gamma^*}}\left(g_1(\nu,Q^2) - \frac{Q^2}{\nu^2}g_2(\nu,Q^2)\right)$$

Work in progress: “REGGEoN” — JLab LOI12–23–004

(M. M. Dalton, A. Deur, SŠ, J. Stevens)

REGGEon == REGGE on Nuclei

Magnetic moment of particle with charge $e_0 Q$, mass M and spin $\vec{S}$:

$$\vec{\mu} = \frac{e_0}{M}(\underset{\text{Dirac}}{\underset{\uparrow}{Q}} + \underset{\text{anomalous}}{\underset{\uparrow}{\kappa}})\vec{S}$$

For a nucleus of mass $M \approx AM_p$ and charge Ze_0:

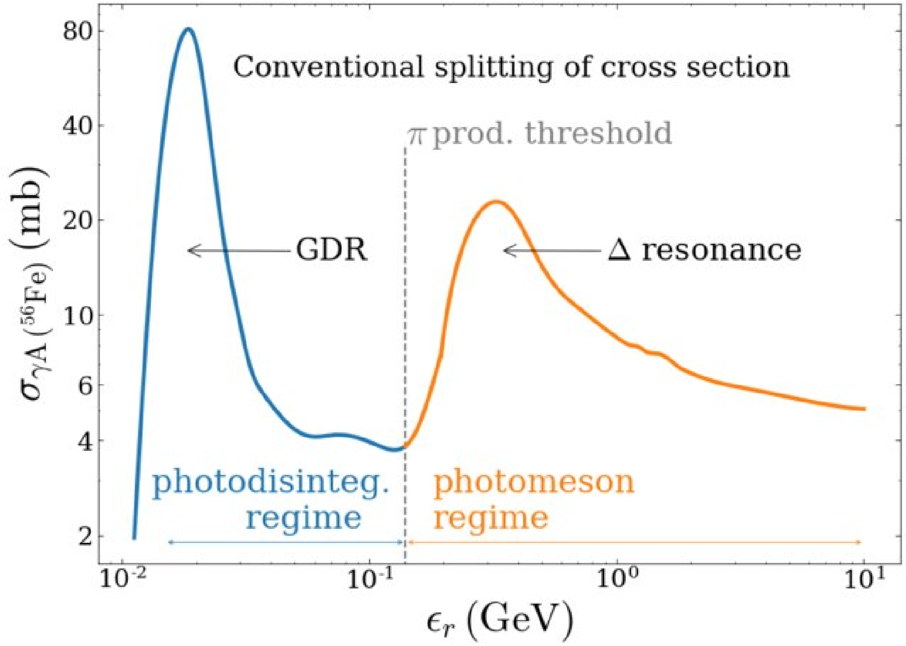
$$\vec{\mu} \approx \frac{e_0}{AM_p}(Z + \kappa)\vec{S} \quad \Longrightarrow \quad \underline{\kappa = \frac{A}{2|\vec{S}|}\frac{\mu}{\mu_N} - Z}$$

$\Longrightarrow$ Compute κ for all stable nuclei with non-zero spin

$\Longrightarrow$ Compute the static part of the GDH sum

“REGGEoN” — Photo-disintegration vs. photo-production

The GDH integral for a **nucleus** has contributions from the whole photo-absorption spectrum:



(Note: no data on the polarized XS, $\Delta\sigma$, exist for $A > 3$!)

Region below π threshold: dominated by properties of *nucleus*
Region above it: dominated by properties of *nucleons*
(coherent photo-production: small)

Example: ^{7}Li $\left(J^P = \frac{3}{2}^-\right)$: polarization carried by single $1p_{3/2}$ nucleon

$$I_{\text{GDH}}^{p^*} \approx 270\,\mu\text{b},\ I_{\text{GDH}}^{p} = 204.78\,\mu\text{b},\ I_{\text{GDH}}^{^7\text{Li}} = 83.4\,\mu\text{b}$$

“REGGEoN” — Modification of properties of bound nucleons

A nucleon in the nuclear medium will be modified
$\Longrightarrow$ modification of **both sides** of the *nucleon* sum rule

Bass, Acta Phys. Pol. B **52**, 42 (2021)
Bass++, Eur. Phys. J. A **59**, 238 (2023)

Static side: guidance for κ^*, M^* from QMC model:

$$\frac{M_N^*}{M_N} \approx \frac{M_\Delta^*}{M_\Delta} \approx \left(1 - 0.2\frac{\rho}{\rho_0}\right), \quad \frac{\kappa_N^*}{\kappa_N} \approx \left(1 + 0.1\frac{\rho}{\rho_0}\right), \quad \rho \ll \rho_0$$

Typical QMC predictions (depending on bag radius):

$$\frac{M_N^*(\rho_0)}{M_N} \approx 0.9, \quad \frac{\kappa_N^*(\rho_0)}{\kappa_N} \approx 1.05 \quad \Longrightarrow \quad \underline{\left(\frac{\kappa^*(\rho_0)}{M_N^*(\rho_0)}\right)^2 \bigg/ \left(\frac{\kappa}{M_N}\right)^2 \approx 1.3}$$

Saito++, Prog. Part. Nucl. Phys. **58**, 1 (2007)

Dynamic (integral) side: modification of the integral due to in-medium shifts of resonance masses “probed” by the $1/\nu$ factor in the integrand

- ▷ “large” effect for $\Delta(1232)$, $1/\nu \leftrightarrow 1/M_\Delta^*$
- ▷ small effect for $D_{13}(1520)$, $S_{11}(1535)$, ... (?)
- ▷ 3rd resonance region + Regge domain: situation unclear:
  $+18\,\mu\text{b} - 15\,\mu\text{b}$ for proton vs. $+16\,\mu\text{b} - 89\,\mu\text{b}$ for neutron

“REGGEoN” — Candidate nuclei

	J^π	μ	κ	M	I_{GDH}
^{1}H	$\frac{1}{2}^+$	2.793	1.793	0.9383	204.8
^{2}H	1^+	0.857	−0.1426	1.875	0.6484
^{3}He	$\frac{1}{2}^+$	−2.128	−8.383	2.808	499.9
^{7}Li	$\frac{3}{2}^-$	3.256	4.598	6.532	83.39
^{13}C	$\frac{1}{2}^-$	0.702	3.131	12.11	3.753
^{17}O	$\frac{5}{2}^+$	−1.894	−14.44	15.83	233.4
^{19}F	$\frac{1}{2}^+$	2.628	40.94	17.69	300.5

- Choice will depend on target feasibility / FOM / other considerations
- The strongest candidate is ^{7}Li:
  - ▷ Also the subject of unpolarized (E12–10–008) and polarized (E12–14–001: $Q^2 > 1\,\mathrm{GeV}^2$) EMC experiments at JLab
  - ▷ A GDH measurement will provide the $Q^2 \to 0$ limit ...
  - ▷ ... and help to establish which of the two competing explanations of the EMC effect (MF or SRC) is most likely
- **Low-ν part (up to $\approx$ 3 GeV) at ELSA?**

Conclusions

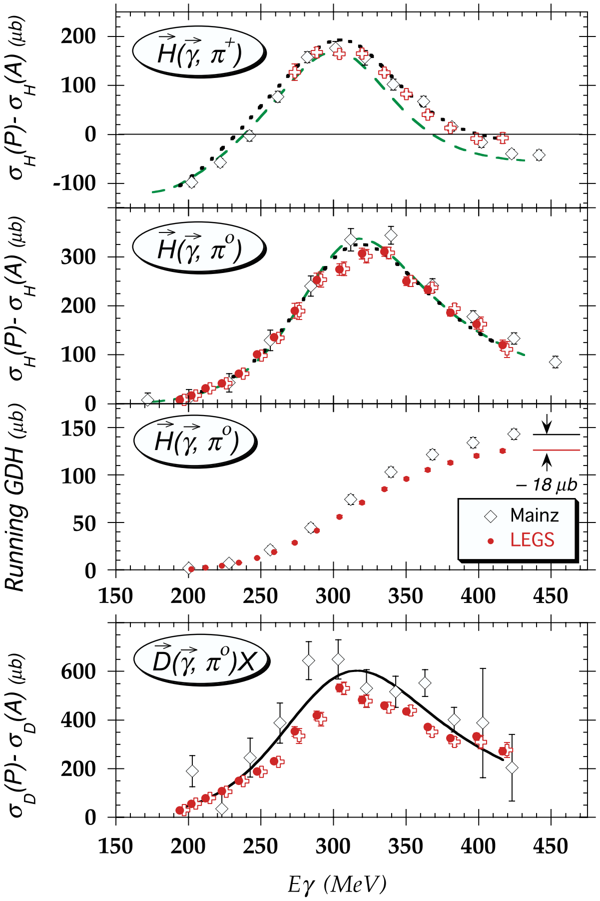
- Running GDH integral sort-of converges for proton …
  … but not at all convincingly for neutron
  - ▷ Threshold (low-ν) issues
  - ▷ High-ν (“Regge”) concerns
  - ▷ Imbalance of sum rule saturation in terms of single-π vs. all other channels (p vs. n, GDH vs. GGT)
- Reasonable agreement of real-γ results with extractions from e-scattering experiments extrapolated to $Q^2 = 0$
  - ▷ Understanding of $I_{\mathrm{GDH}}(Q^2 \to 0)$, $\gamma_0(Q^2 \to 0)$, $\Gamma_1(Q^2 \to 0)$ etc. not at the same level
- New approved experiment: REGGE in Hall D @ JLab to study the high-ν behavior of $\Delta\sigma$
- JLab Letter of Intent (June 2023): REGGEoN (= REGGE on Nuclei)

Spare slides

Measurements of $\Delta\sigma$

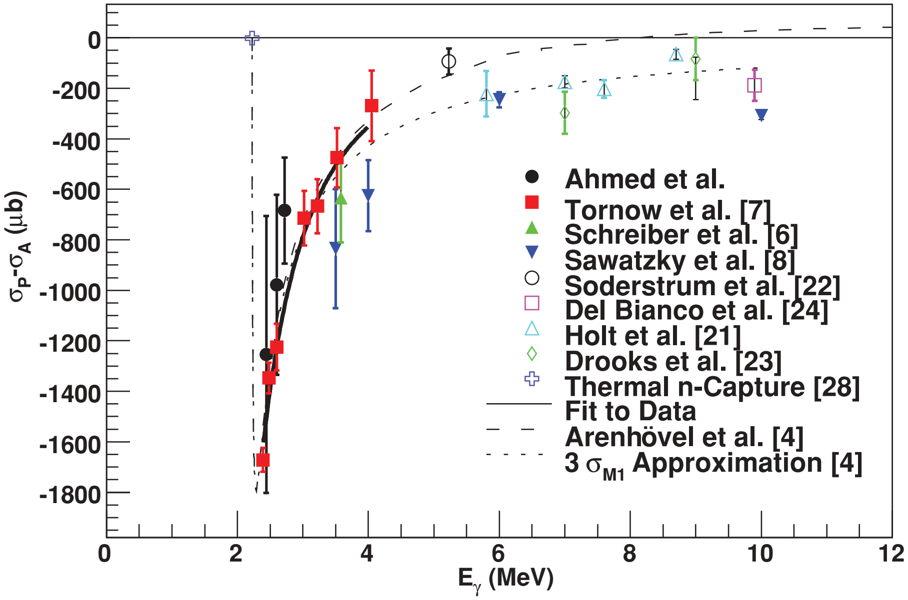
The **threshold region** is very important due to $1/\nu$ weight
$\Longrightarrow$ Use models like MAID/SAID: both give $I^p_{\mathrm{GDH}}(\nu \leq 0.2\,\mathrm{GeV}) \approx -28\,\mu\mathrm{b}$

- Low-ν accessible at facilities like LEGS (BNL): $0.2\,\mathrm{GeV} \leq \nu \leq 0.4\,\mathrm{GeV}$ ...



Hoblit++, PRL **102**, 172002 (2009)

- ... or TUNL (e. g. deuteron with huge $\Delta\sigma$ just above the photo-disintegration threshold):



Ahmed++, PRC **77**, 044005 (2008)

- (Precise) low-ν data is also crucial for extrapolations (guiding threshold models)

Generalization of the GDH sum rule to $Q^2 \neq 0$

Based on the ν-expansion of the VVCS amplitude in the dispersion relation

$$\mathrm{Re}\, A_{\mathrm{VVCS}}(\nu, Q^2) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\mathrm{Im}\, A_{\mathrm{VVCS}}(\nu', Q^2)}{\nu' - \nu}\, \mathrm{d}\nu'$$

- **LO: generalized GDH**

$\Delta\sigma \equiv -2\sigma_{TT}$

$$I_{TT}(Q^2) = \frac{M^2}{4\pi^2\alpha} \int_{\nu_0}^{\infty} \frac{K_{\gamma^*}}{\nu} \frac{\sigma_{TT}}{\nu}\, \mathrm{d}\nu = \frac{2M^2}{Q^2} \int_0^{x_0} \left[g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right] \mathrm{d}x$$

$$\frac{8\pi^2\alpha}{M^2} I_{TT}(0) = -\int_{\nu_0}^{\infty} \frac{\Delta\sigma(\nu)}{\nu}\, \mathrm{d}\nu = -\frac{2\pi^2\alpha\kappa^2}{M^2} = -I_{\mathrm{GDH}}$$

- **NLO: forward spin polarizability:** Gell-Mann–Goldberger-Thirring SR:

$$\gamma_0(Q^2) = \frac{1}{2\pi^2} \int_{\nu_0}^{\infty} \frac{K_{\gamma^*}}{\nu} \frac{\sigma_{TT}}{\nu^3}\, \mathrm{d}\nu = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[\cdots \quad \text{as above} \quad \cdots \right] \mathrm{d}x$$

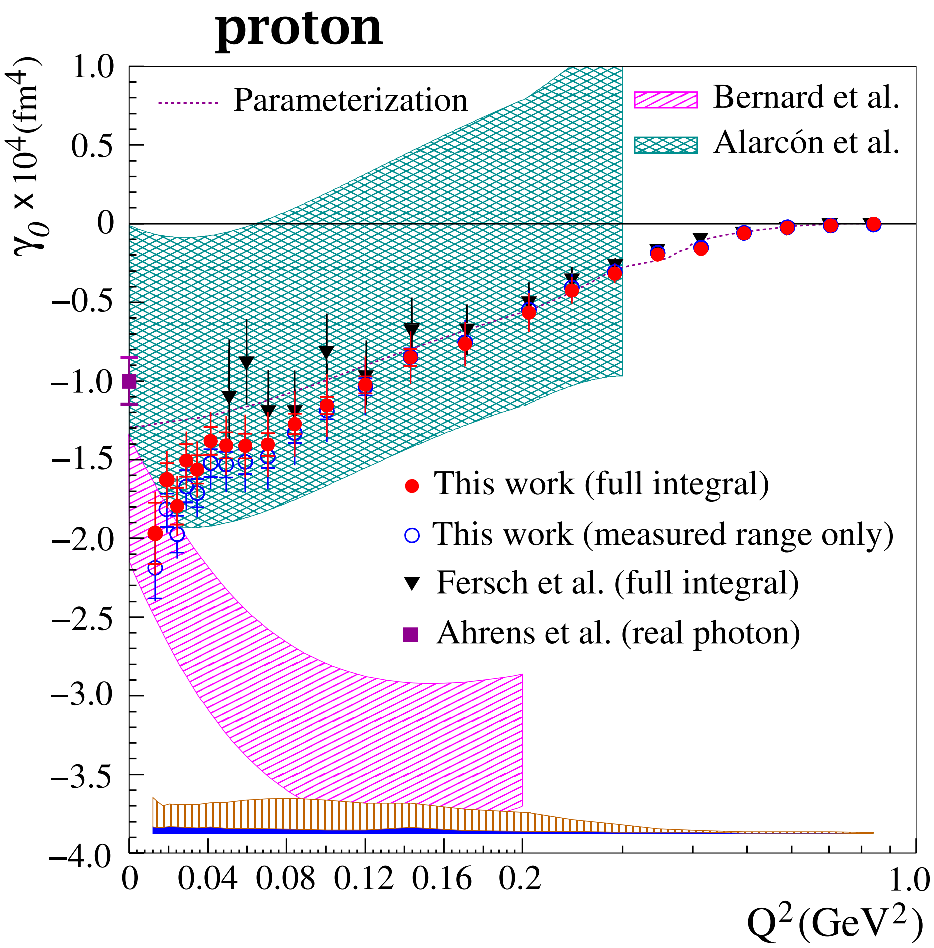
$$\gamma_0 = -\frac{1}{4\pi^2} \int_{\nu_0}^{\infty} \frac{\Delta\sigma(\nu)}{\nu^3}\, \mathrm{d}\nu \equiv I_{\mathrm{GGT}}$$

Details of the formalism, conventions etc.:
Deur++, Rep. Prog. Phys. **82**, 076201 (2019)

Spin polarizability γ_0, proton

$$\gamma_0(Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 g_1(x, Q^2)\,\mathrm{d}x$$

(if $x^2 g_2(x, Q^2)$ contribution neglected)

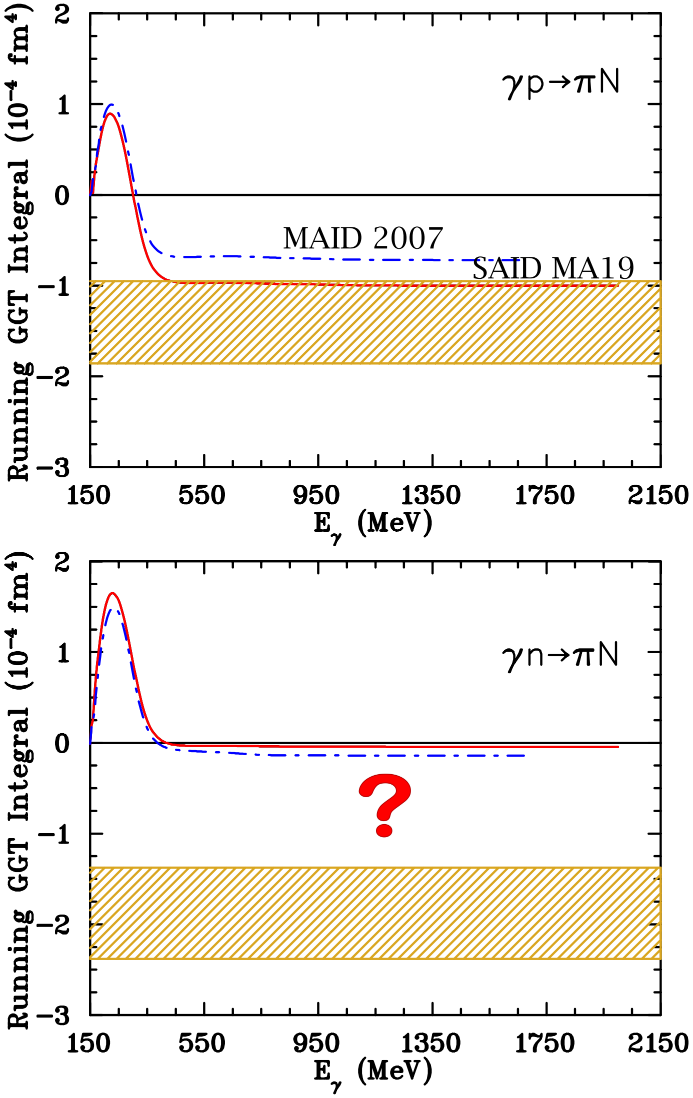


Zheng++, Nat. Phys. **17**, 736 (2021)

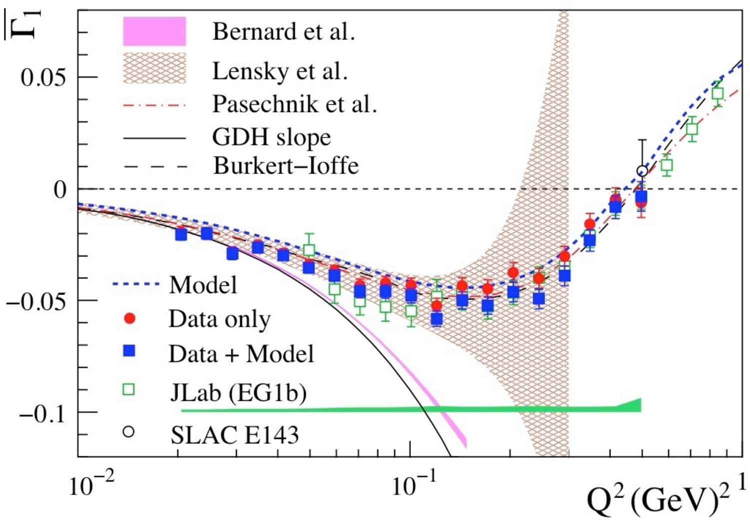
Strakovsky++, PRC **105**, 045202 (2022)

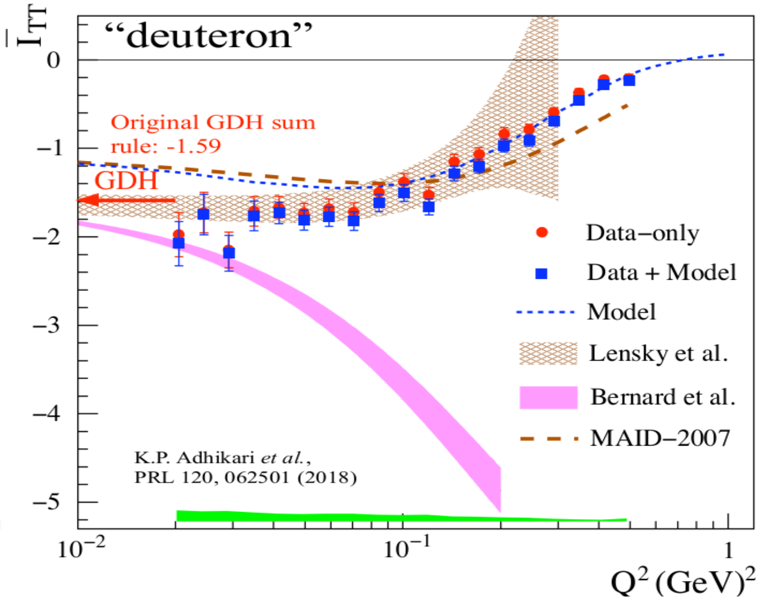
... compare to single-π contribution to the **"running" GGT integral**

$$I_{\mathrm{GGT}}(\nu) = -\frac{1}{4\pi^2} \int_{\nu_0}^{\nu} \frac{\Delta\sigma(\nu')}{\nu'^3}\,\mathrm{d}\nu'$$



Same exercise, deuteron ...





- Photo-disintegration part excluded

  $\Longrightarrow$ "deuteron" ≈ p+n ($I^{d}_{TT} = I^{p}_{TT} + I^{n}_{TT}$)

  $$I^{d,EG4}_{TT}(0) = -1.724 \pm 0.027 \pm 0.050$$

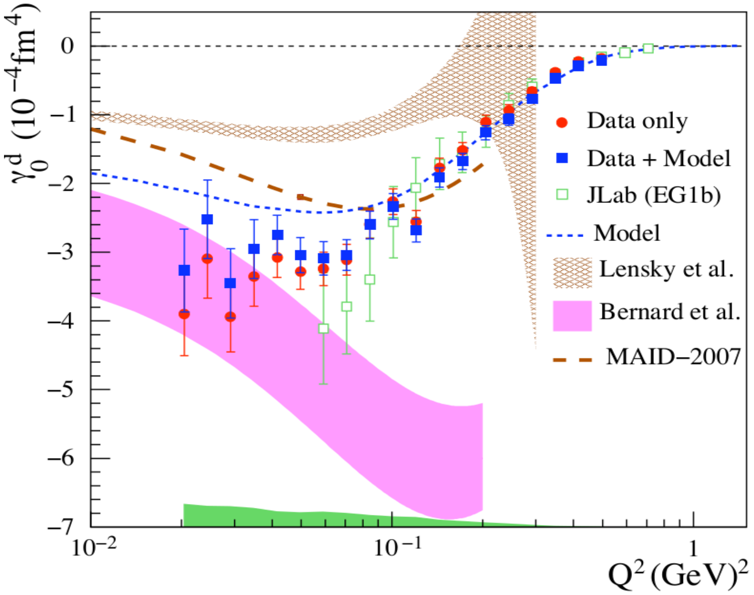
  $$I^{d}_{TT}(\mathrm{GDH}) = -1.59\ldots$$

  $\Longrightarrow$ extracted neutron information:

  $$I^{n,EG4}_{TT}(0) = -0.955 \pm 0.040 \pm 0.113$$

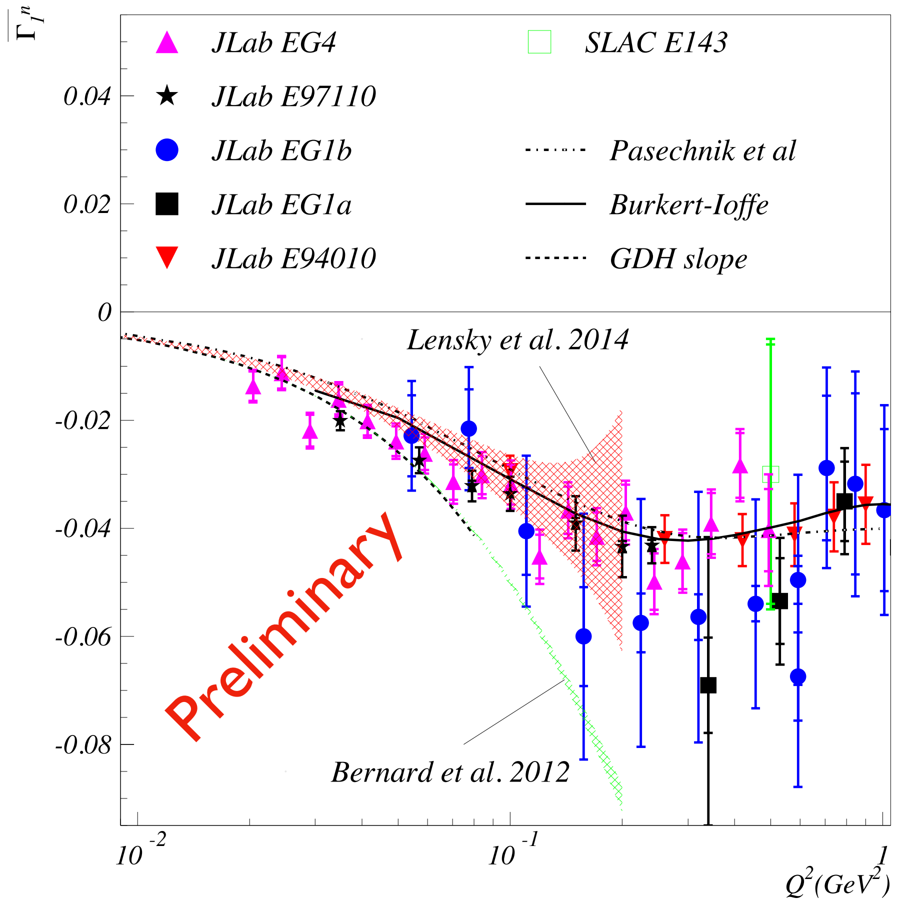
  $$I^{n}_{TT}(\mathrm{GDH}) = -0.803\ldots$$

  (agreement not so good ...)



Results on $\Gamma_1^n(Q^2)$ from E97–110 (^{3}He) and EG4 (d)

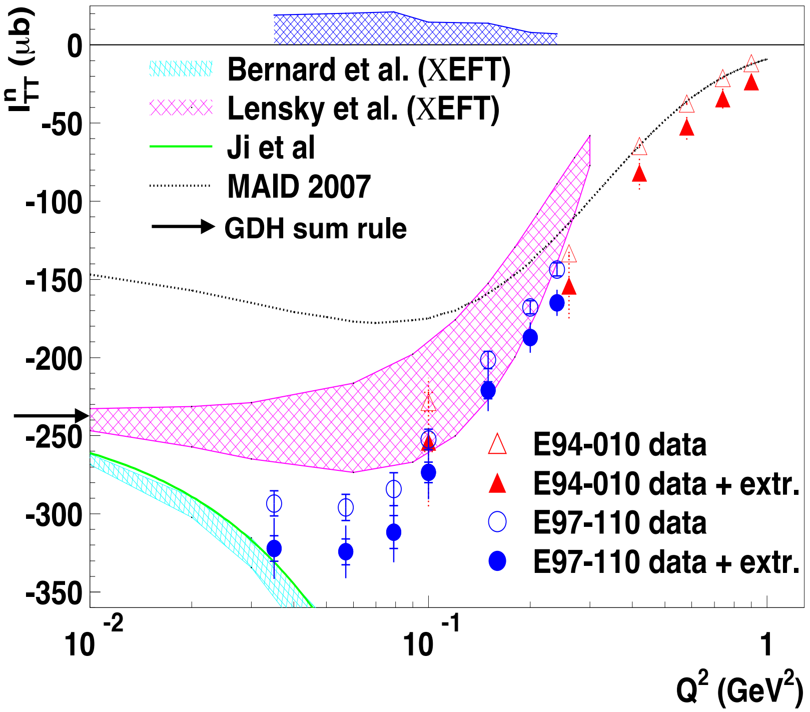
$$\Gamma_1^n = 2\Gamma_1^d/(1-1.5\omega_D) - \Gamma_1^p\,, \qquad \Gamma_1^p = \int_0^{1-} g_1^p(x,Q^2)\,\mathrm{d}x$$



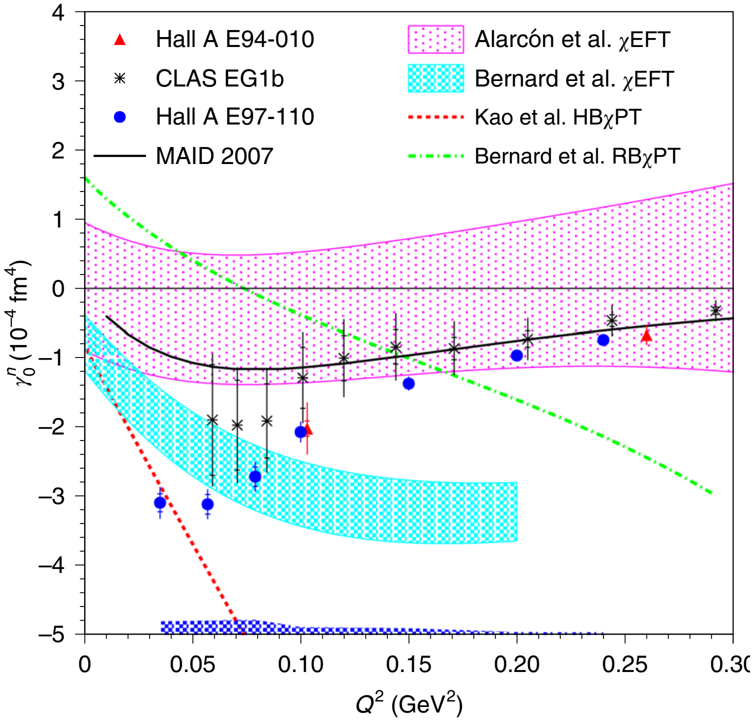
- Good mutual agreement E97–110 $\Longleftrightarrow$ EG4
- Good description in terms of NLO χPT at lowest Q^2

$I_{TT}^{n}(Q^2)$ and γ_0^n from E97–110 alone

GDH
Sum
Rule



Sulkosky++, PLB **805**, 135428 (2020)



Sulkosky++, Nat. Phys. **17**, 687 (2021)

- Agreement with older data (E94–010, EG1b) at larger Q^2
- Poor match to either of the competing NLO χPT calculations
- Disagreement with MAID

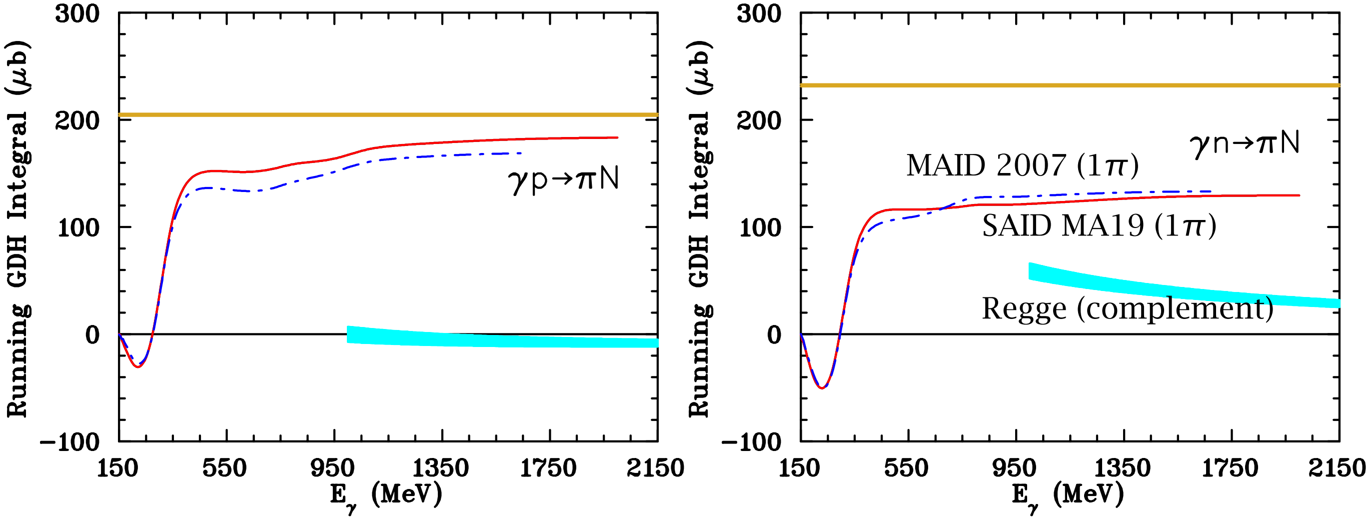
Individual contributions to the running GDH integral

TABLE 2 The contribution of various decay channels to the GDH integral I and the forward spin polarizability γ_0. The integration extends to $\nu_{\max} = 1.67$ GeV ($W_{\max} = 2$ GeV) except that the two-pion contribution is integrated only up to $\nu_{\max} = 800$ MeV

Reference	**Proton**	I_p	γ_0^p	**Neutron**	I_n	γ_0^n
(34)/(49)	$\pi^0 p$	157/142	−1.46/−1.40	$\pi^0 n$	145/147	−1.44/−1.44
(34)/(49)	$\pi^+ n$	7.5/44	0.82/0.55	$\pi^- p$	−21/−13	1.53/1.36
(54)	ηp	−9.0	0.01	ηn	−5.9	0.01
(55)	$\pi\pi N$	28	−0.07	$\pi\pi N$	19	−0.05
(53)	$K\Lambda,\ K\Sigma$	−4.0	<0.01	$K\Lambda,\ K\Sigma$	2.0	<0.01
(53)	$\omega p,\ \rho N$	−3.0	<0.01	$\omega n,\ \rho N$	2.1	<0.01
(44)/(45)*	Regge	−25/−9	<0.01	Regge	31/16	<0.01

p, contribution below ν_0 is of EM origin & suppressed by $\kappa_{\mathrm{QED}}/\kappa_p \approx 10^{-3}$ ($\kappa_{\mathrm{QED}} = \alpha/2\pi$ = Schwinger correction)

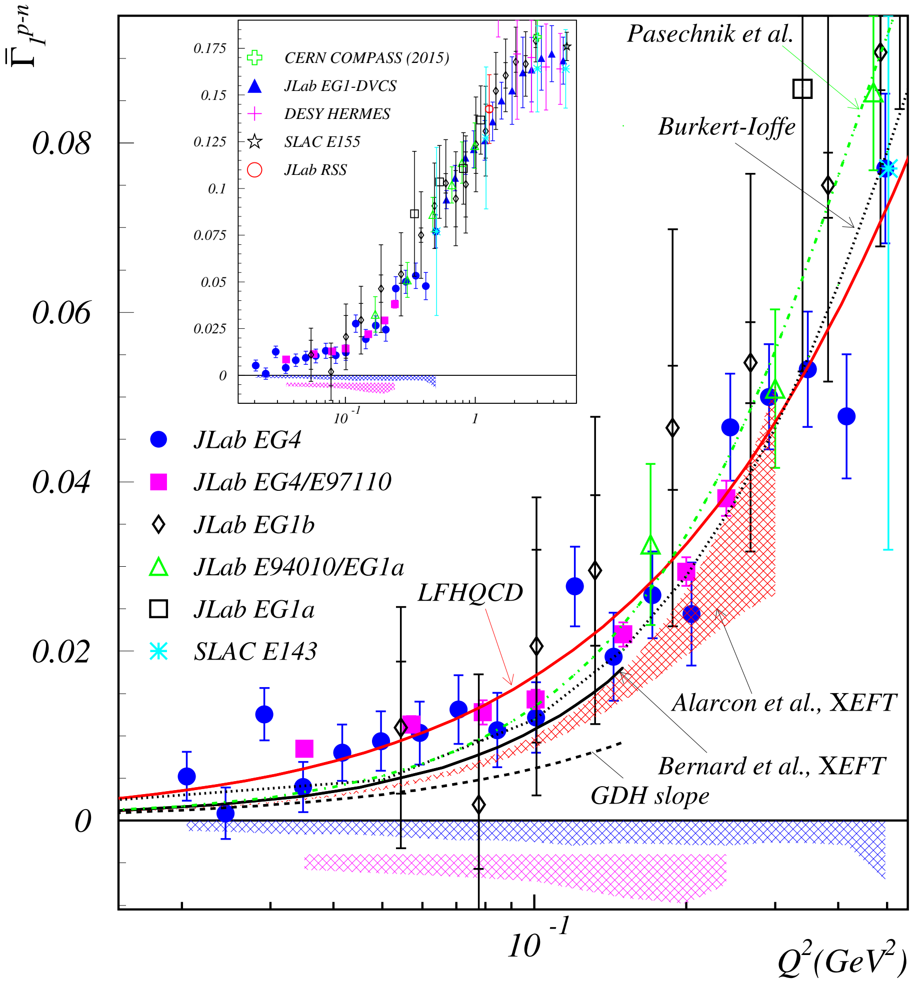
Drechsel, Walcher, Annu. Rev. Nucl. Part. Sci. **54**, 69 (2004)



Strakovsky++, PRC **105**, 045202 (2022)

Isovector GDH sum rule vs. Bjorken sum at very low Q^2

$$\overline{\Gamma}_1^{p-n}(Q^2)\Big|_{Q^2\to 0} = \frac{Q^2}{8}\left(\frac{\kappa_n^2}{M_n^2} - \frac{\kappa_p^2}{M_p^2}\right)$$



Best fit of the world data on $\overline{\Gamma}_1^{p-n}(Q^2)$ (full integral, with low-x contribution) using a fit function $bQ^2 + cQ^4$. The fit is performed up to $Q^2 = 0.244$ GeV^2. The "*uncor*" uncertainty designates the point-to-point uncorrelated uncertainty. It is the quadratic sum of the statistical uncertainty and a fraction of the systematic uncertainty determined so that $\chi^2/n.d.f = 1$ for the best fit, see Appendix. The "*cor*" uncertainty is the correlated uncertainty estimated from the remaining fraction of the systematic uncertainty. Also listed are results of fits applied to the predictions from χEFT and models.

Data set	$(b \pm uncor \pm cor)$ [GeV^{-2}]	$c \pm uncor \pm cor$ [GeV^{-4}]
World data	$0.182 \pm 0.016 \pm 0.034$	$-0.117 \pm 0.091 \pm 0.095$
GDH Sum Rule [17]	0.0618	-
χEFT Bernard et al. [13]	0.07	0.3
χEFT Alarcón et al. [15]	0.066(4)	0.25(12)
Burkert-Ioffe [29]	0.09	0.3
Pasechnik et al. [30]	0.09	0.4
LFHQCD [35]	0.177	-0.067

$\Longrightarrow$ only marginal agreement with χEFT

(somewhat surprising as contribution of $\Delta(1232)$ suppressed in this observable)

EG4 (p, d), E97-110 (^{3}He)

Deur++, PLB **825**, 136878 (2022)

Recall the decades of FF-medium-modification efforts!

An example: p recoil polarization components in $^{12}\mathrm{C}(\vec{e}, e'\vec{p})$:

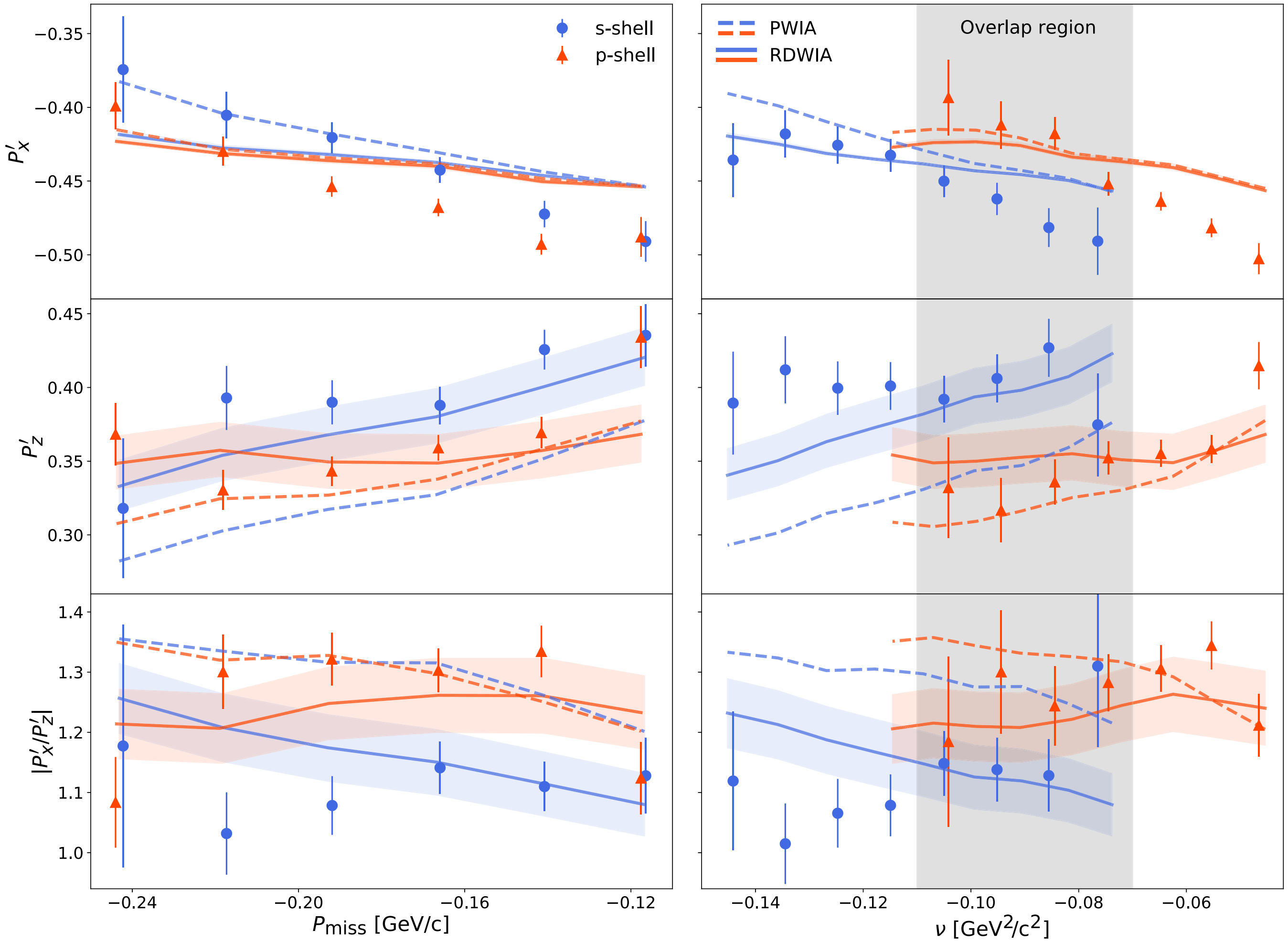


Fig. 3. The measured polarization components P'_x (top), P'_z (middle), and their ratio P'_x/P'_z (bottom) as a function of missing momentum (left) and virtuality (right). Shown are statistical uncertainties only. The lines represent RDWIA and PWIA calculations for the corresponding shell obtained using a slightly modified program from [2] (see text). The shaded colored regions correspond to RDWIA calculations with the form-factor ratio, G_E/G_M, modified by ±5%.

Kolar++, PLB **811**, 135903 (2020)