

# $\chi^2$ of Tracks in GlueX

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# Motivation

- understand origin of  $\chi^2$  of each track
- will be used in kinematic fitting of events
- currently, strong hints that the numbers are unreliable...
  - ▶ efficiency of reconstruction
  - ▶ large number of events with  $\chi^2$  too low/high

# Line of Attack

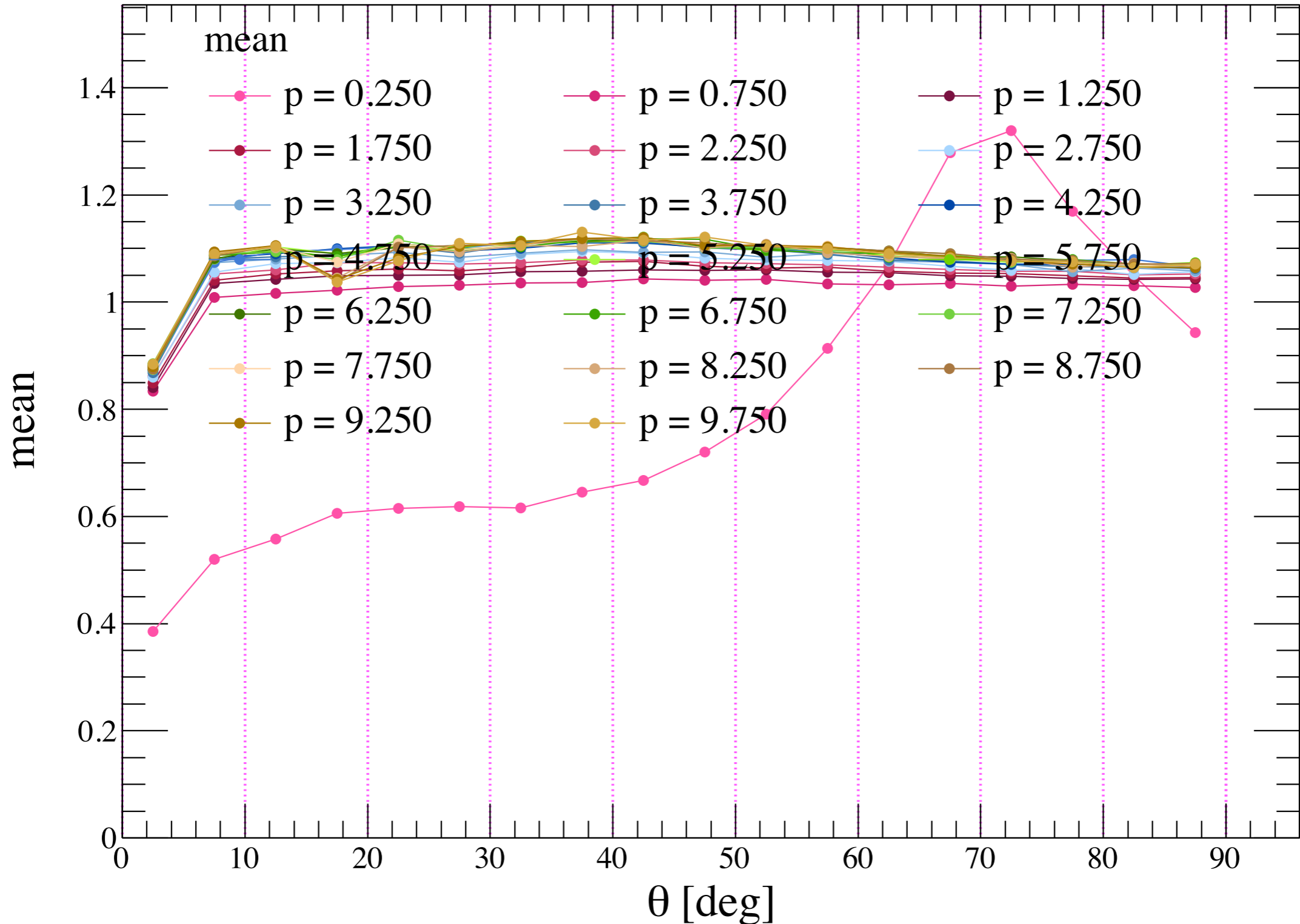
- previously used multi-particle events
- complicated to look at individual tracks due to multiplicity
- now using particle gun
- now possible to match up generated vs. reconstructed momentum

# Events

- generate single particle events in bins of  $p$ ,  $\theta$
- 20k events in
  - ▶ 0.5 GeV/c bins for  $0 < p < 10$
  - ▶ 5° bins for  $0^\circ < \theta < 90^\circ$
- total: 360 bins
- generated for  $p$ ,  $\pi^+$ ,  $\pi^-$

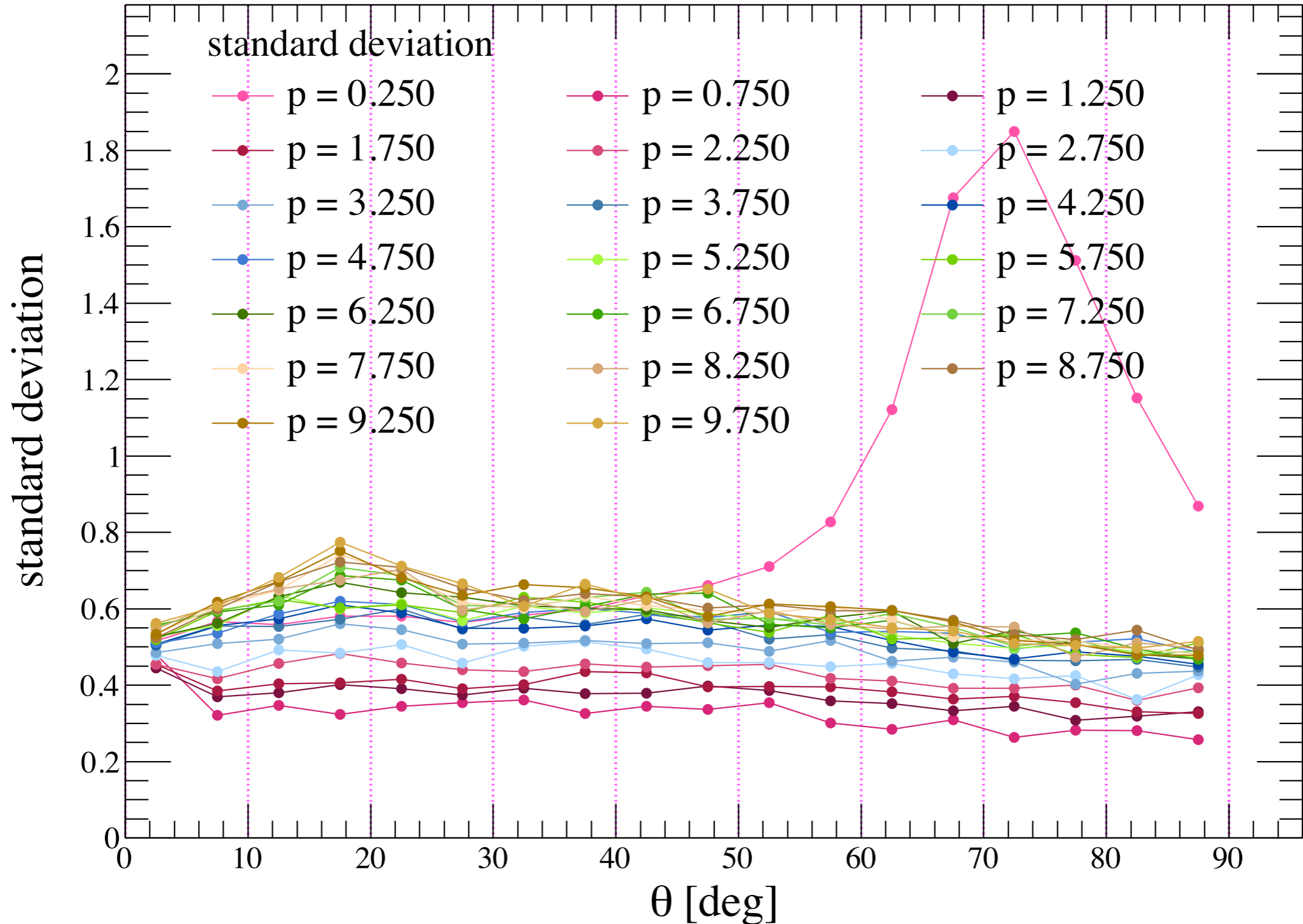
# Distributions

- mean number of tracks in each bin  
(20k generated)



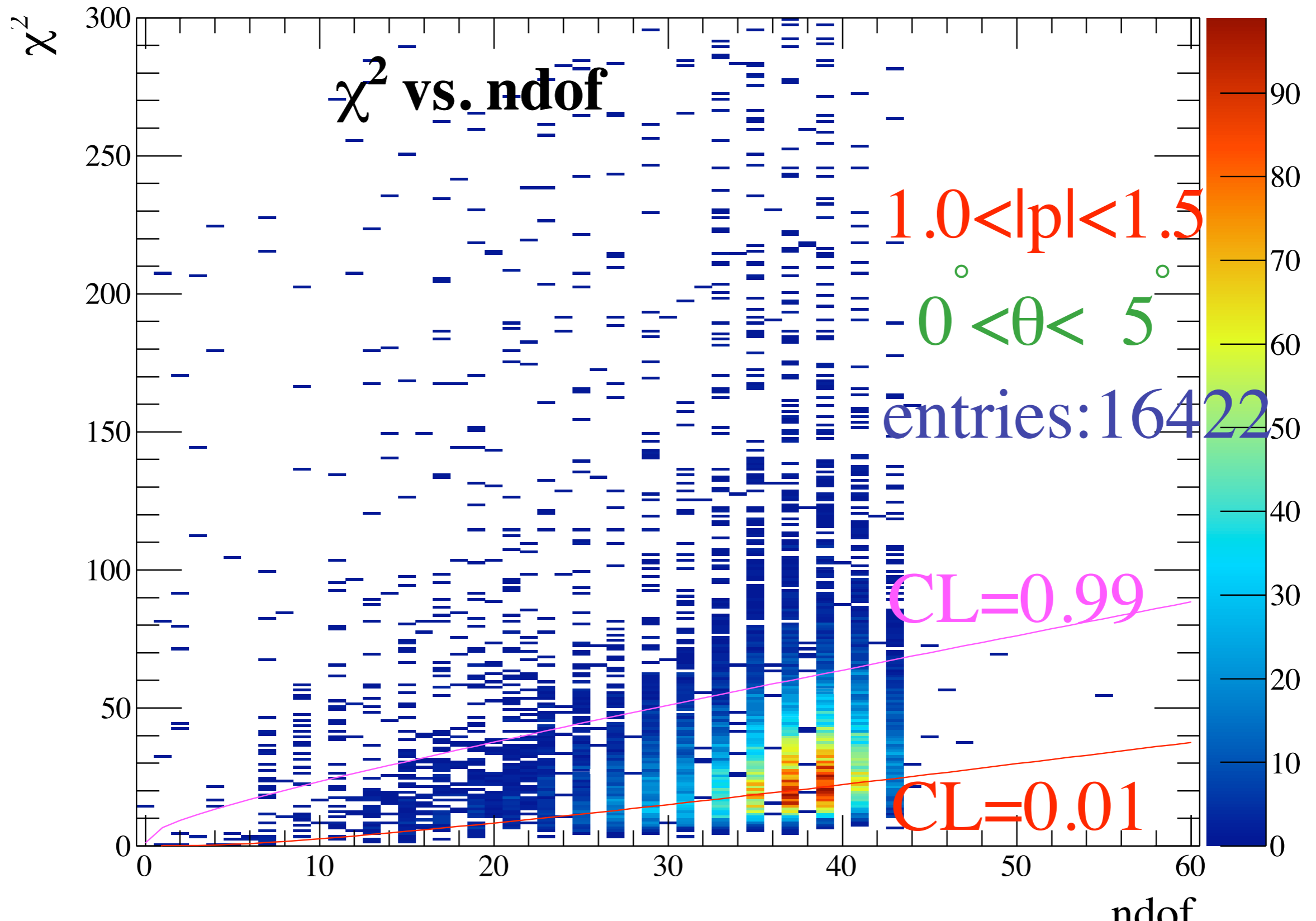
# Distributions

- standard deviation of number of tracks



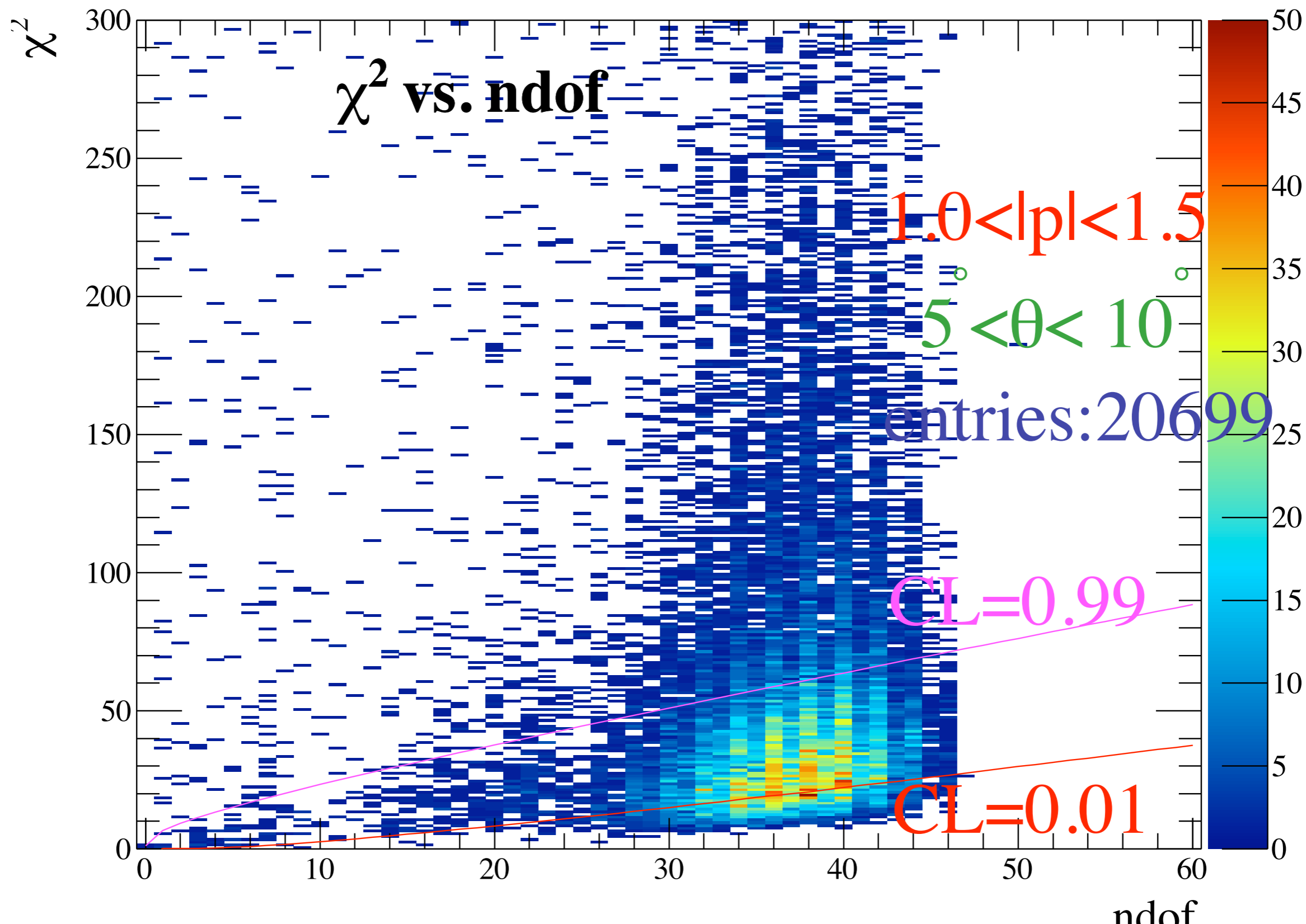
# Distributions

- $\chi^2$  vs. ndof for each track



# Distributions

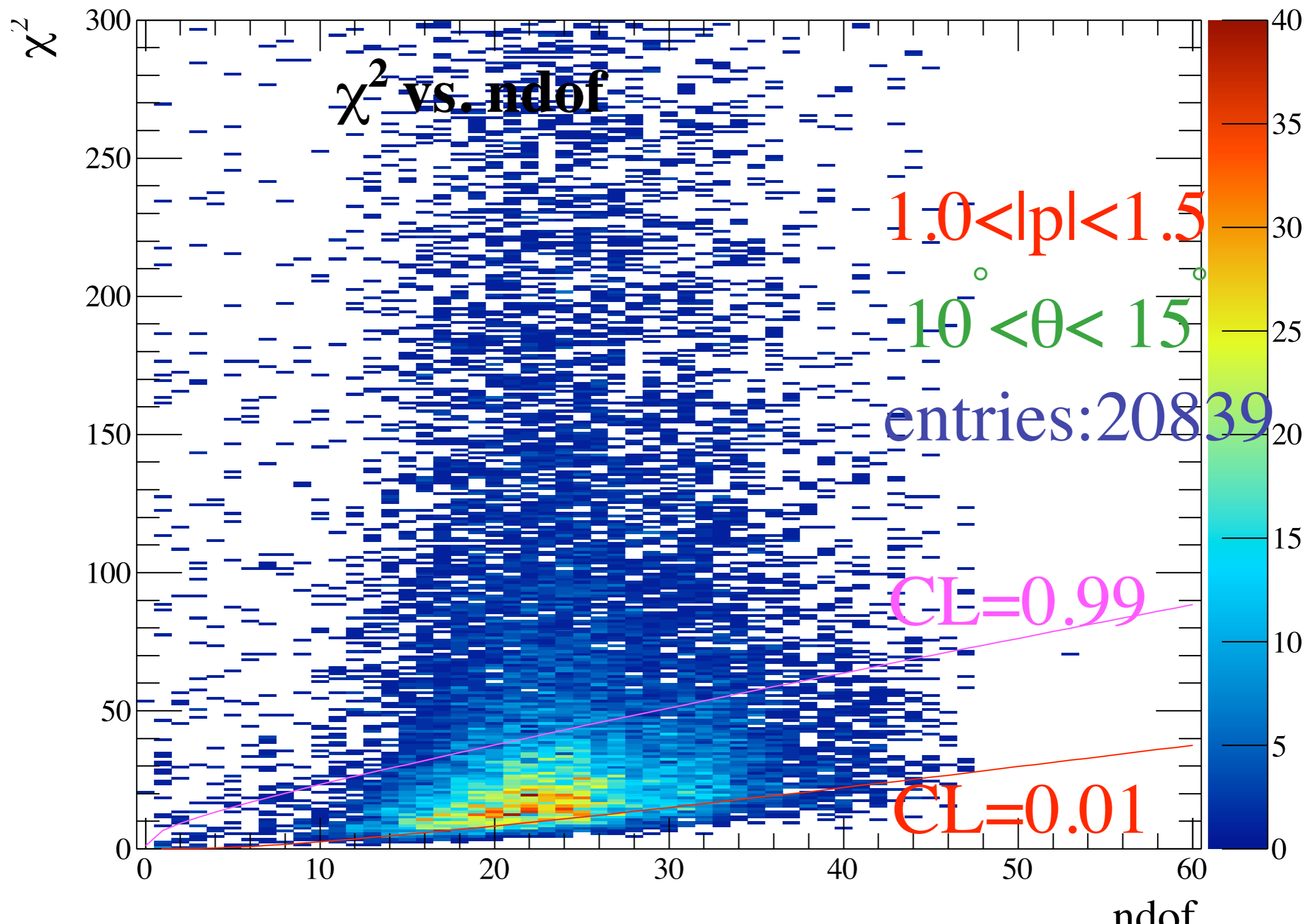
- $\chi^2$  vs. ndof for each track





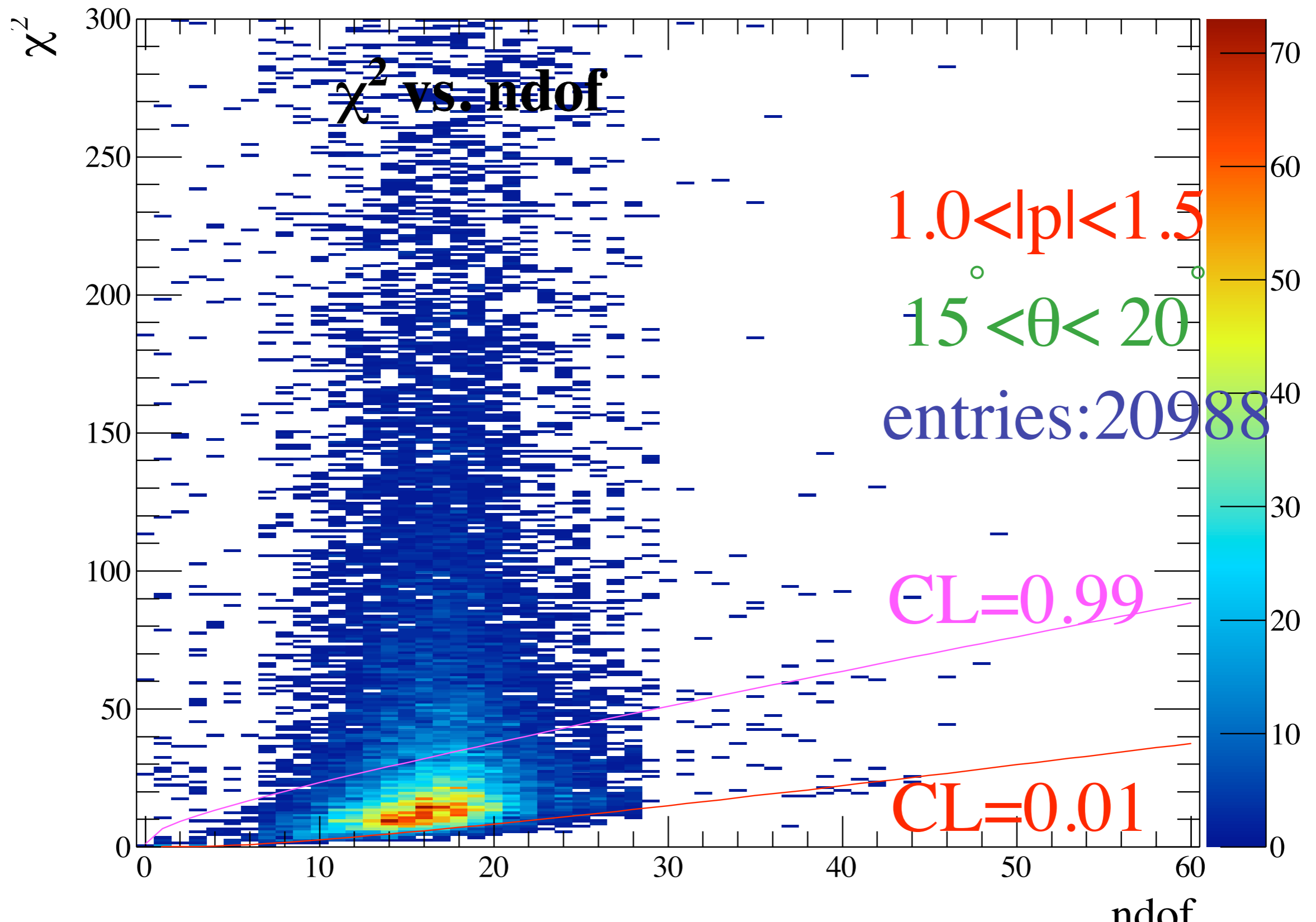
# Distributions

- $\chi^2$  vs. ndof for each track



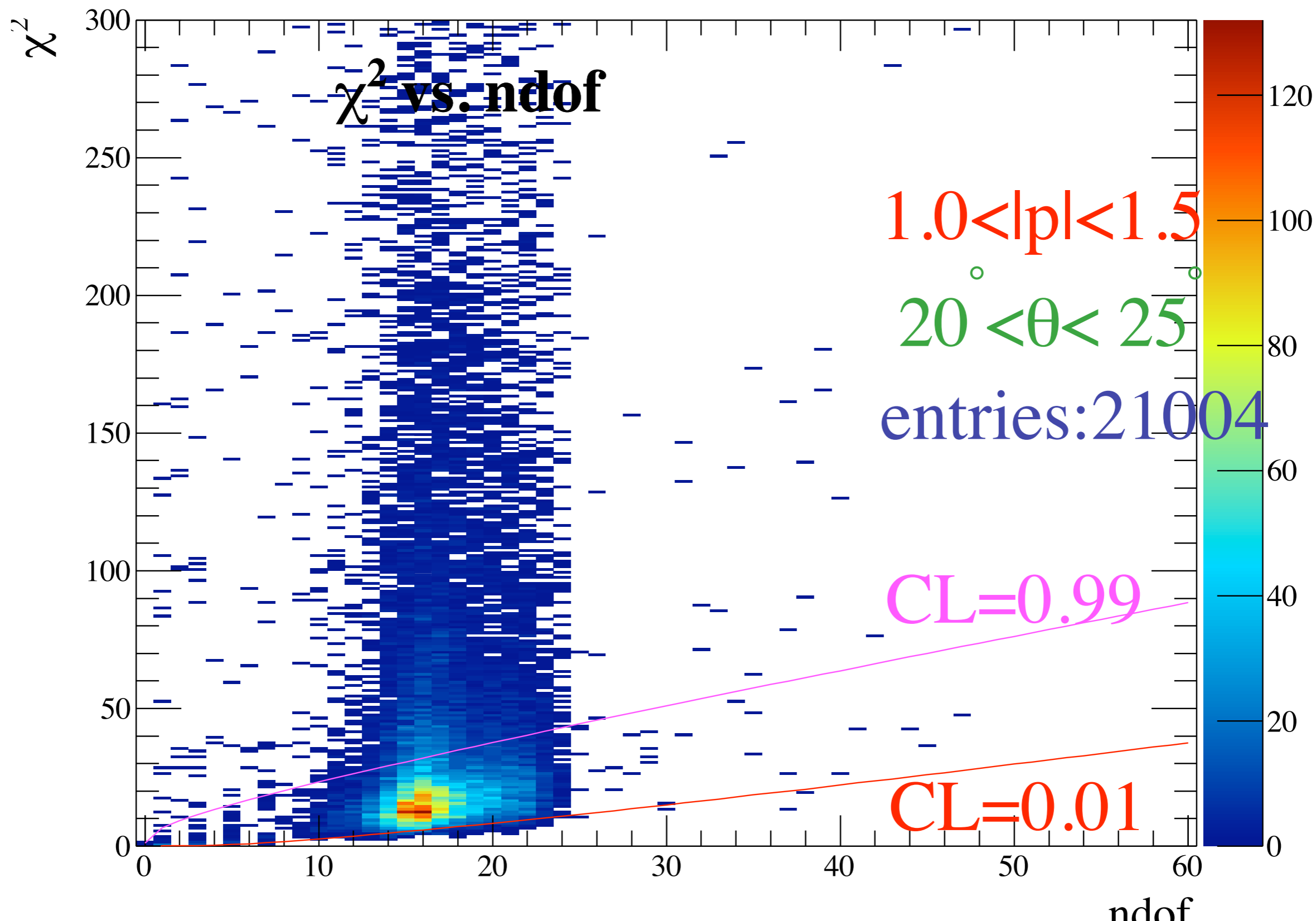
# Distributions

- $\chi^2$  vs. ndof for each track



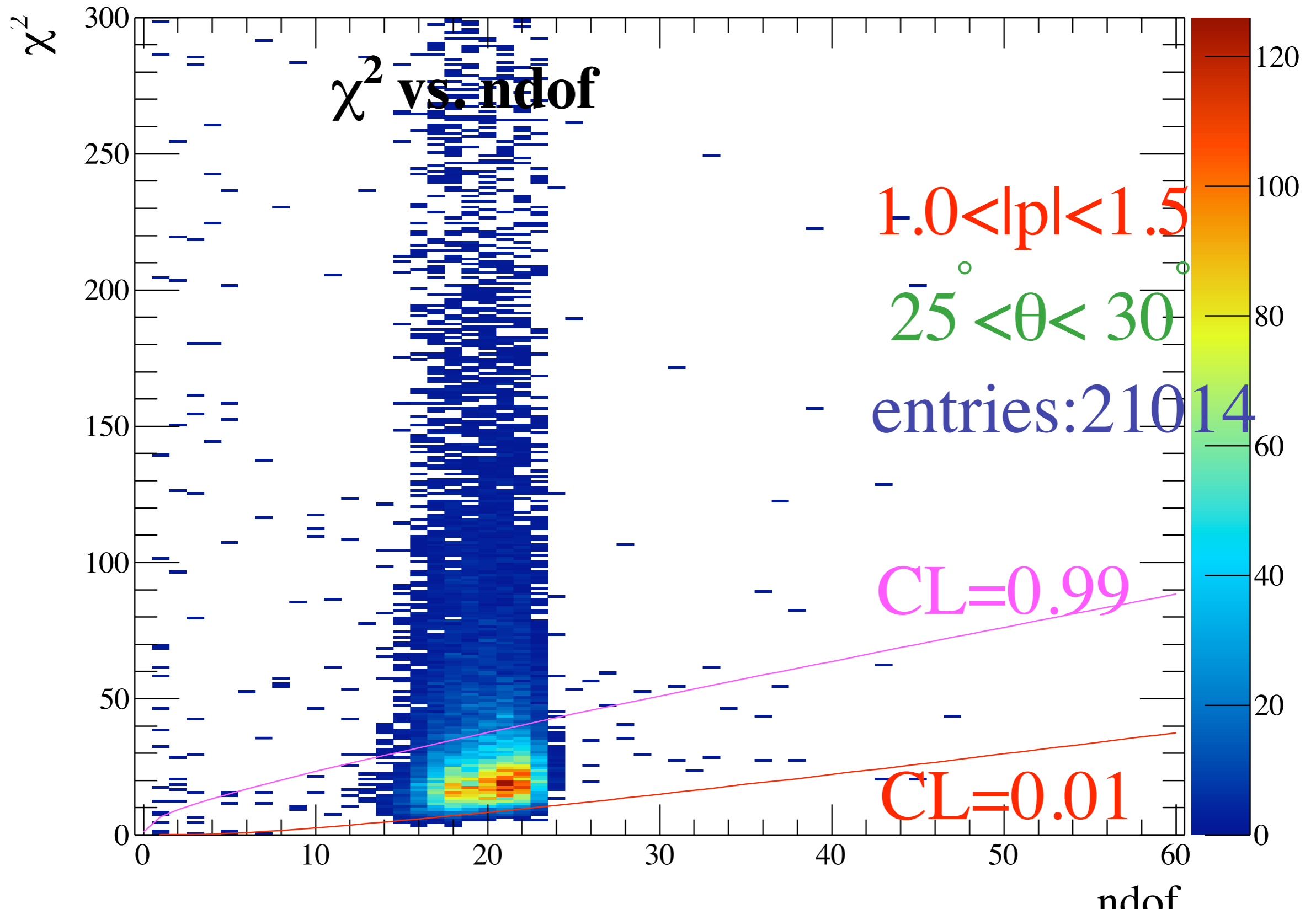
# Distributions

- $\chi^2$  vs. ndof for each track



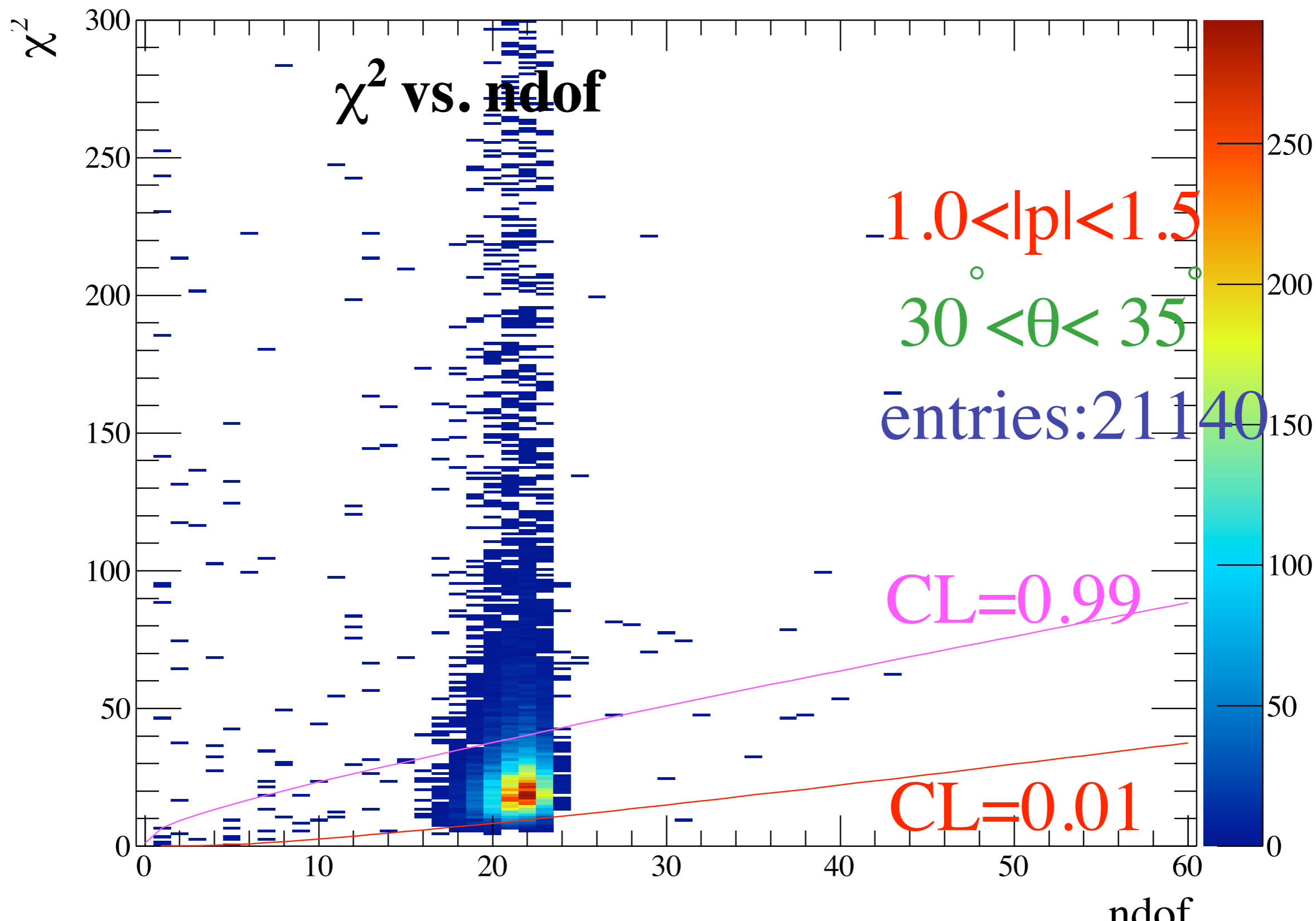
# Distributions

- $\chi^2$  vs. ndof for each track



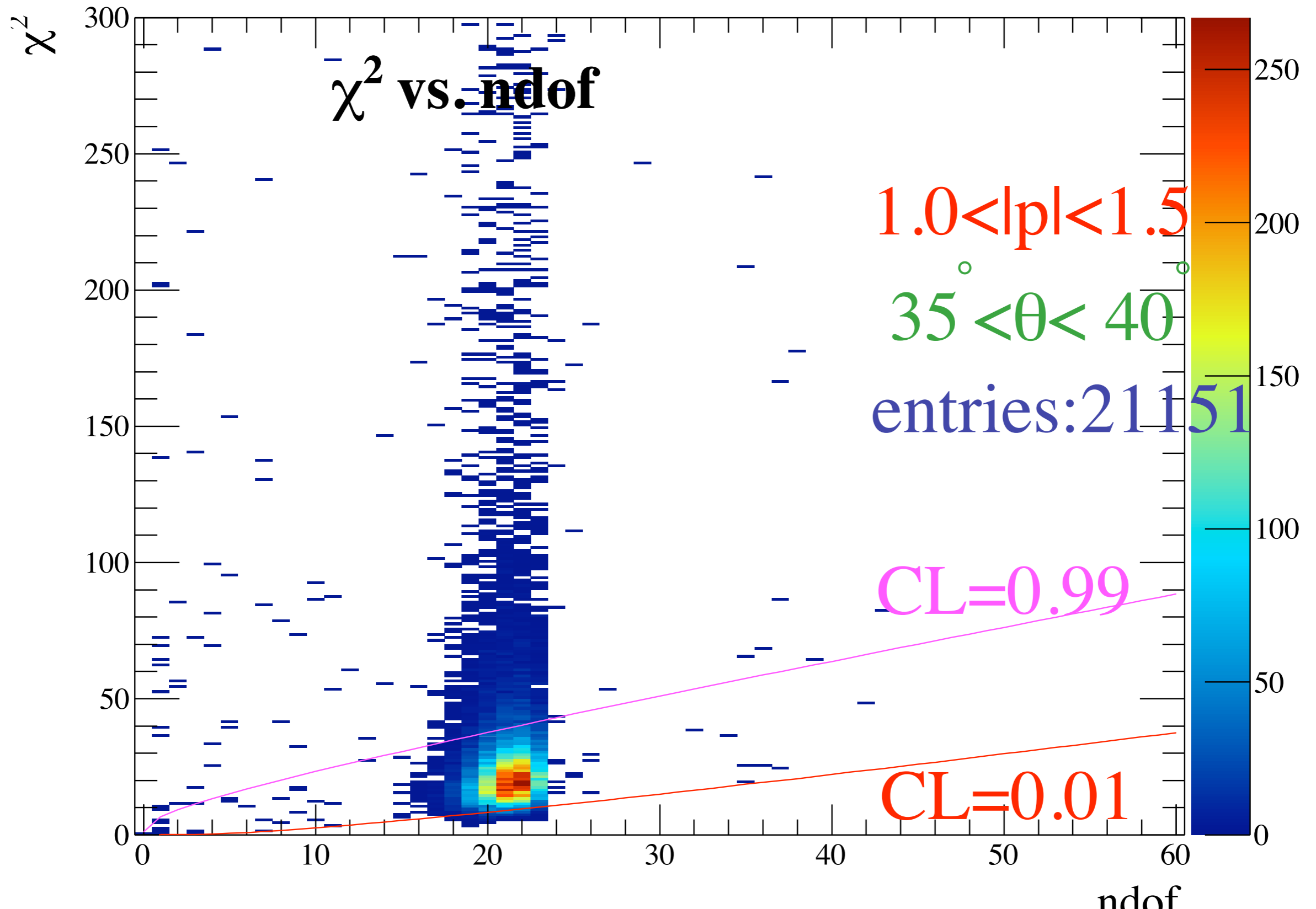
# Distributions

- $\chi^2$  vs. ndof for each track



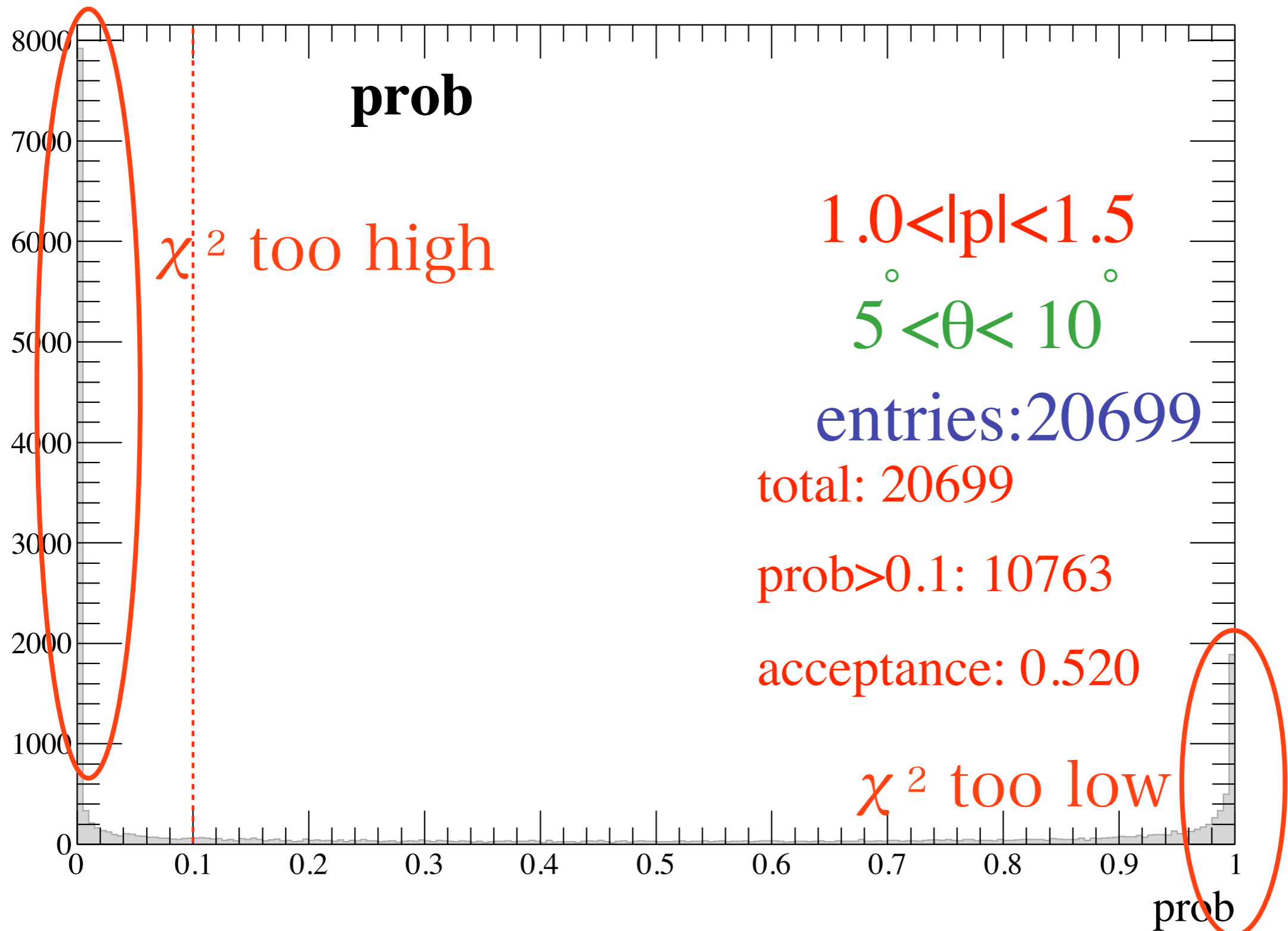
# Distributions

- $\chi^2$  vs. ndof for each track



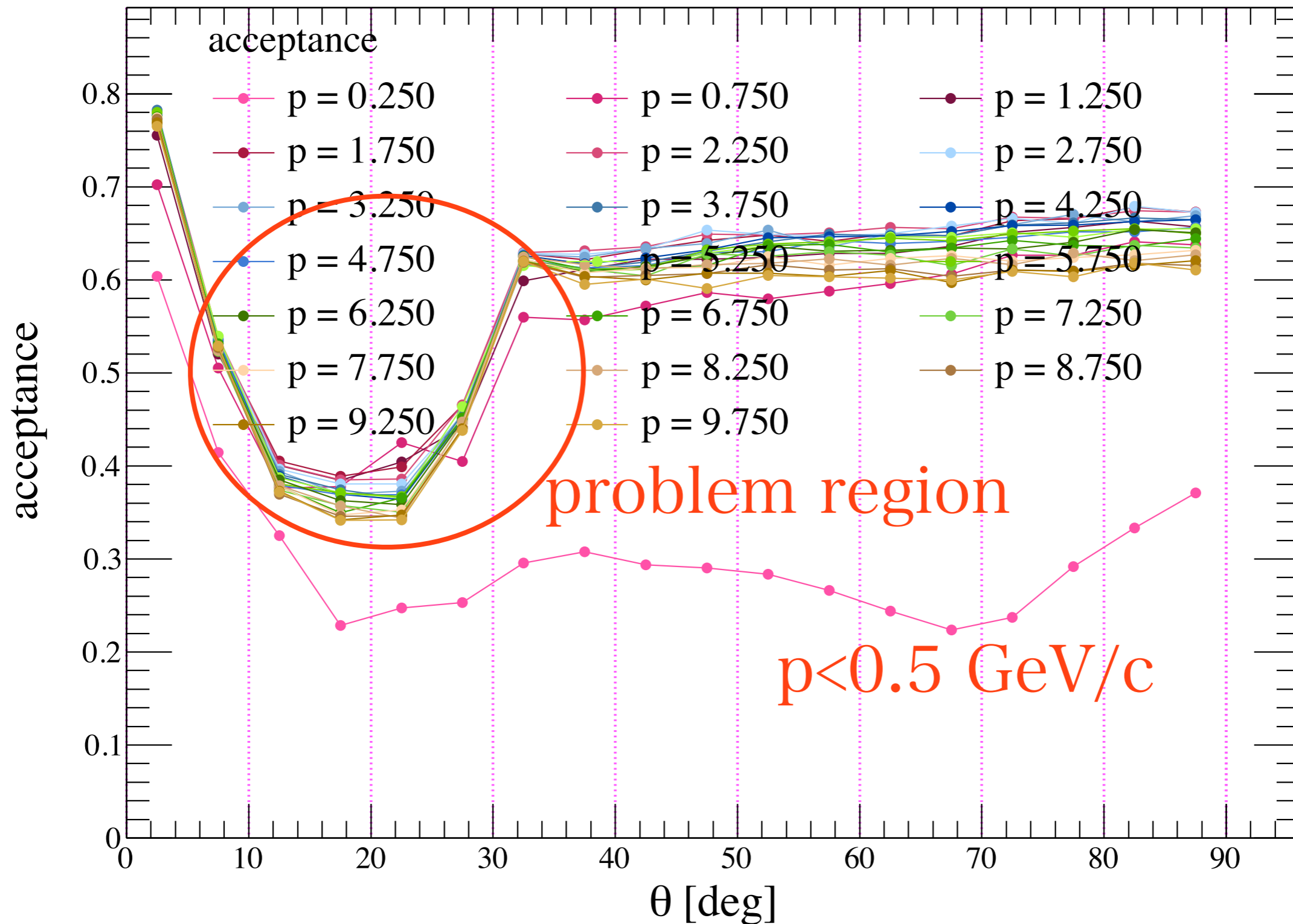
# Turn Into prob

- `prob = TMath::Prob( $\chi^2$ , ndof)`



# Calculate Acceptance

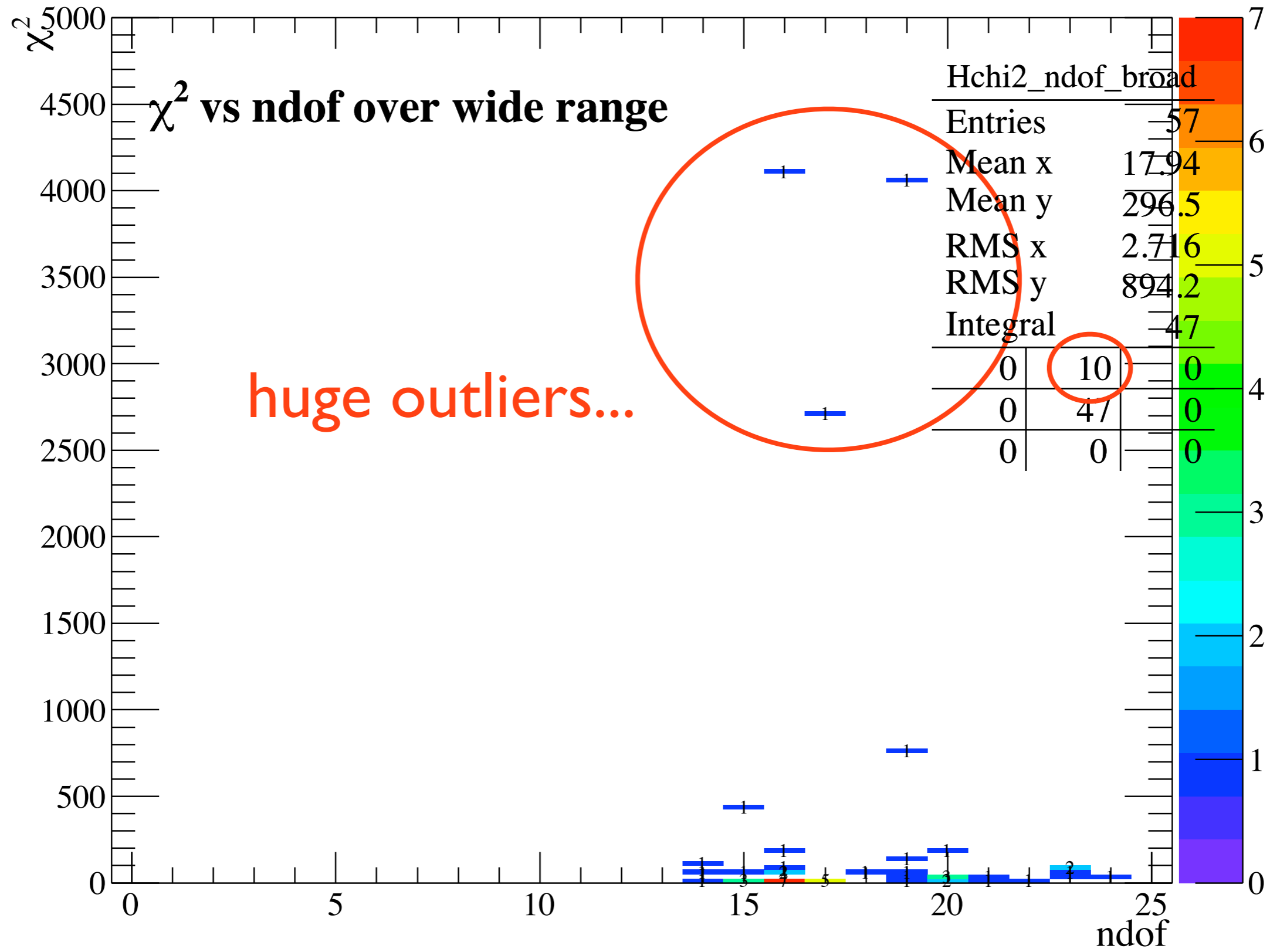
- define acceptance as prob  $> 0.10$





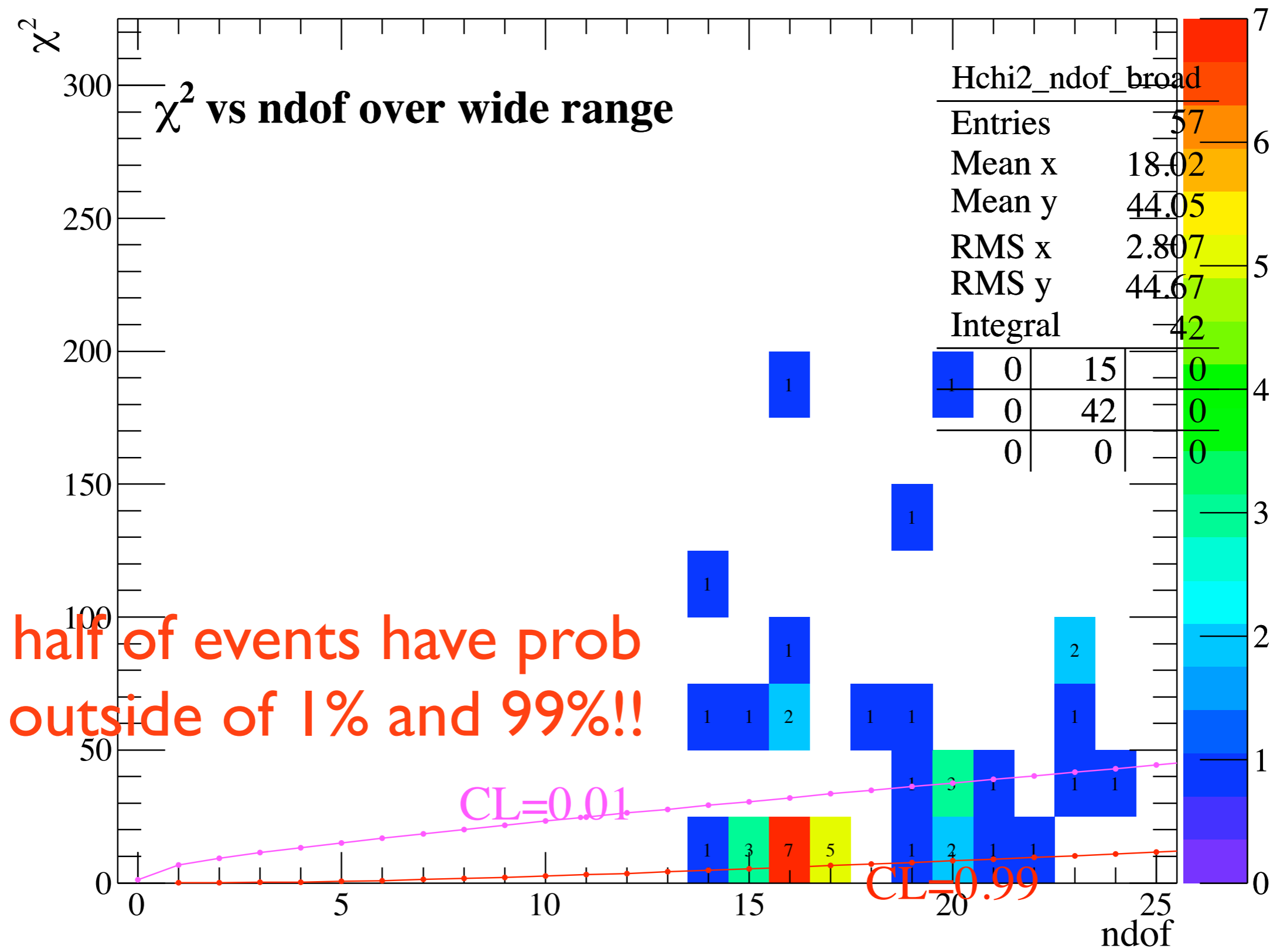
# Shockingly High $\chi^2$

- for  $20^\circ < \theta < 25^\circ$ ,



# Shockingly High $\chi^2$

- for  $20^\circ < \theta < 25^\circ$ , closer look



# Improving Results

- want to improve acceptance
- want better  $\chi^2$  values
- where does  $\chi^2$  come from?
- have spent past few days understanding the Kalman filter, and the actual code that implements it

# Origin of $\chi^2$

- within analysis code:

```
vector< const DChargedTrack* > tracks;  
loop->Get(tracks);  
vector< const DTrackTimeBased* > hypo  
    = tracks[i]->hypotheses;
```

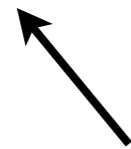
```
Double_t chisq = hypo[j]->chisq;  
Double_t ndof = hypo[j]->Ndof;
```

# Origin of $\chi^2$

- within `DTrackTimeBased_factory::evnt`

```
DTrackFitter::fit_status_t status
```

```
= fitter->FindHitsAndFitTrack(*track, rt, loop,  
                             track->mass(),mStartTime);
```



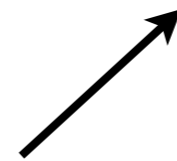
`DTrackFitter*`



```
fit_status = FitTrack(pos, mom,q, mass,t0);
```



```
DTrackFitter::fit_status_t status = FitTrack();
```



virtual function

# Origin of $\chi^2$

- within `DTrackFitter_factory.h`

```
vector<const DTrackFitter*> fitters;  
loop->Get(fitters, "KalmanSIMD");
```

use `DTrackFitterKalmanSIMD`



- within `DTrackFitterKalmanSIMD::FitTrack()`

```
error = KalmanLoop();
```



almost there....

# Origin of $\chi^2$

- within `DTrackFitterKalmanSIMD::FitTrack()`  
error = `KalmanLoop()`;

1. if there are FDC hits

```
error = KalmanForward(anneal_factor, S, C,  
                      chisq, my_ndf);
```

vector of params

cov. matrix

2. if there are CDC hits and  $\theta < 60^\circ$

```
error = KalmanForwardCDC(anneal_factor, S, C,  
                          chisq, my_ndf);
```

3. if there are CDC hits

```
error = KalmanCentral(anneal_factor, Sc, Cc, pos,  
                      chisq_central, my_ndf);
```

# Origin of $\chi^2$

- within `DTrackFitterKalmanSIMD::KalmanForwardCDC()`
- loop over CDC hits{
  - get J (Jacobian), Q (noise cov. matrix), S0 from kth CDC hit

set within `PropagateForwardCDC`

`GetProcessNoise(...,Q)`

Lynch-Dahl algorithm

`StepJacobian(...,J)`

`CalcJacobian(...,J1)`

$$J = \Delta z * J_1$$

called within `SetCDCForwardReferenceTrajectory`

$$- S = S_0 + J (S - S_0)$$

$$- C = Q + J C J^T$$

$$- dx = S.x\text{-wire}.x, dy = S.y\text{-wire}.y$$

$$- doca = \sqrt{dx^2 + dy^2}$$



# Origin of $\chi^2$

- within `DTrackFitterKalmanSIMD::KalmanForwardCDC()`
- loop over CDC hits{
  - if(doca doesn't keep decreasing){
    - ▶ calculate new dz
    - ▶ `Step(z,newz=z+dz,dedx,S)`
    - ▶ `StepJacobian(z,newz,S0,dedx,J)`
      - ▶  $J=J+\Delta z * J_1$ ,  $J_1$  set in `CalcJacobian`
    - ▶  $C = J C J^T$
    - ▶ define H (track projection matrix) 1x5
    - ▶  $H_x = dx \cos^2 \theta_{\text{stereo}} / \sqrt{dx^2 + dy^2}$ , same for  $H_y$

# Origin of $\chi^2$

- within `DTrackFitterKalmanSIMD::KalmanForwardCDC()`
- loop over CDC hits{
  - if(docca doesn't keep decreasing){
    - ▶  $dm = v_{\text{CDC}} [(cdchit \rightarrow t_{\text{drift}}) - (\text{vertex time}) - (t \text{ of } k\text{th hit})]$
    - ▶  $V = \sigma^2_{\text{CDC}}(dm) + v_{\text{CDC}}^2 \sigma_{t_0}^2$
    - ▶  $\text{Inv}V = 1 / (V + H^T C H)$
    - ▶  $K$  (Kalman gain) =  $\text{Inv}V * (C H)$
    - ▶  $S = S + (dm - d) K$
    - ▶  $C = C - K H C$

# Origin of $\chi^2$

- within `DTrackFitterKalmanSIMD::KalmanForwardCDC()`
- loop over CDC hits{
  - if(docca doesn't keep decreasing){
    - ▶  $\text{res\_scale} = 1 - H K = V / (V + HCH^T)$
    - ▶  $\text{res} = (\text{dm} - d) * \text{res\_scale}$
    - ▶  $\Delta \text{chisq} = \text{anneal} * \text{res}^2 / V / \text{res\_scale}$   
 $= \text{anneal} * (\text{dm} - d)^2 / (V + HCH^T)$
    - ▶ `ndof++`
    - ▶ `anneal` is a factor currently set to 1

# Origin of $\chi^2$

- within `DTrackFitterKalmanSIMD::KalmanForwardCDC()`
- loop over CDC hits{
  - if(doca doesn't keep decreasing){
    - ▶ `StepJacobian(newz,z,S0,dedx,J)`
    - ▶  $C = J C J^T$
    - ▶ `Step(newz,z,dedx,S)`
    - ▶ calculate new doca
  - }
  - set kth CDC state  $S$  and cov. matrix  $C$
- }

# Conclusion

- looking into Kalman filter code
- looking for where  $\chi^2$  values are too small/too large
- may need some help in understanding Kalman filtering in general