

Associated Strangeness Production in the $\vec{p}p \rightarrow pK^+\Lambda$ Reaction Measured by COSY-TOF

July 20, 2016 | Florian Hauenstein | Seminar at Jefferson Lab, Virginia, USA



Outline

Introduction

COSY-TOF Detector

Data Analysis

Results

Dalitz Plot $p\Lambda$ Scattering Length Λ Polarization Spin Transfer Coefficient D_{NN}

Summary and Outlook



Production and Decay of Λ -Hyperons

Production in $pp \rightarrow pK\Lambda$ or $\gamma p \rightarrow K\Lambda$

Strangeness is conserved in strong interaction \rightarrow Creation of an $s\bar{s}$ pair \Rightarrow Production of Λ and Kaon

 \rightarrow Associate strangeness production

Λ-Decay



Decay: $\Lambda \rightarrow p\pi^-$

- Weak decay in *pπ*⁻ (64%) and *nπ*⁰ (36%)
- Life time 2.63 · 10⁻¹⁰ s
- Separated decay and production vertex (c\u03c0 = 7.89 cm)



Physics of $\vec{p}p \rightarrow pK\Lambda$

- Investigation of production mechanism of associated strangeness close to threshold
 - Which kind of meson-exchange (no perturbative QCD)
 - Role of N^* resonances $(S_{11}(1650), P_{11}(1710), P_{13}(1720))$
 - Dalitz plot and polarization observables e.g. Λ polarization or spin transfer coefficient D_{NN}





Dalitz Plot for $pp \rightarrow pK\Lambda$ see S. Jowzaee et al., Eur. Phys. J. A52, 7 (2016)



- Clear enhancement at low m_p masses from final state interaction
- Full phase space coverage
- pΛ NΣ coupled channel enhancement (cusp effect)



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The NA Interaction

- Limits of SU(3) flavor symmetry in the correlation between NA and NN interactions
- Poor data base on pΛ elastic scattering (no data for pure spin singlet/triplet states as well as nΛ)
- No discrimination between different theoretical calculations
- Understanding of interaction important for
 - Hyperons in neutron stars
 - Hypernuclei (Nuclei with hyperons e.g. $^{3}\mathrm{H}_{\Lambda})$
- Strength of the interaction is given by the scattering length a





Determination of $p\Lambda$ Scattering Length a



- Extraction from $p\Lambda \rightarrow p\Lambda$ scattering
 - Total cross section for $k = p/\hbar \rightarrow 0$ is

$$\lim_{k\to 0}\sigma_{\rm tot}=4\pi a^2$$

- S-wave scattering for $k \to 0$ $(l_{p\Lambda} = 0)$
- Model dependent determination with effective range approximation
- Model independent extraction of scattering length from the shape of the pΛ-FSI for specific spin states

Gasparyan et al., Phys. Rev. C69, 034006 (2004)

- Dispersion relation approach
- Known theoretical precision (0.3 fm) Florian Hauenstein Slide 7



Determination of *p*Λ Scattering Length Method from A. Gasparyan et al., Phys. Rev. C69, 034006 (2004)

- Integral representation of *a* in terms of differential cross section
- Parametrization: $\frac{\mathrm{d}\sigma}{\mathrm{d}m_{p\Lambda}} = PS \cdot \exp\left[C_0 + \frac{C_1}{m_{p\Lambda}^2 C_2}\right]$

•
$$a(C_1, C_2) = -\frac{1}{2}C_1\sqrt{\left(\frac{m_0^2}{m_\rho m_\Lambda}\right) \cdot \frac{(m_{\max}^2 - m_0^2)}{(m_{\max}^2 - C_2) \cdot (m_0^2 - C_2)^3}}\hbar c$$

Spin resolved determination via suitable polarization observable



 COSY-TOF measurement at 2.95 GeV/c (42,000 events)

M. Roeder et al., Eur. Phys. J. A49, 157 (2013)

- Effective scattering length $a_{\rm eff} = (-1.25 \pm 0.08_{\rm stat.} \pm 0.3_{\rm theo.}) \, {\rm fm}$
 - Large systematic error (1 fm) due to kinematical reflection of N^* resonance



Effective $p\Lambda$ Scattering Length for $m_{K\Lambda}$ Regions see M. Roeder et al., Eur. Phys. J. A49, 157 (2013)



- $a_{\rm eff} = (-1.25 \pm 0.08_{\rm stat.} \pm 0.3_{\rm theo.}) \, {\rm fm}$ (full data)
- $a_{\rm eff} = (-2.06 \pm 0.16_{\rm stat.} \pm 0.3_{\rm theo.}) \, \text{fm}$ (upper region)
- $a_{\text{eff}} = (-0.86 \pm 0.06_{\text{stat.}} \pm 0.3_{\text{theo.}}) \text{ fm}$ (lower region)

- Strong influence of *N*^{*} resonances
- Error in the order of 1 fm



Dalitz Plot Dependence on Beam Momentum





- Contributions of N* change with beam momenta
- Expected smaller systematic effect for measurement at 2.7 GeV/c?
- \Rightarrow Comparison of results from the COSY-TOF measurements at 2.7 GeV/c and 2.95 GeV/c beam momentum

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COSY Facility

COoler SYnchroton





- Circumference: 184 m
- Beam momentum: 0.3 GeV/c -3.7 GeV/c
- Stochastic and electron cooling
- (Un-)Polarized proton and deuteron beams

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COSY-TOF Detector

Time Of Flight



Features:

- Full phase space coverage
- Clear signature for $pK\Lambda \rightarrow pK \{p\pi\}$ (2 primary and 2 secondary tracks)
- Primary and delayed hyperon decay vertex (cτ(Λ) = 7.89 cm)

Measurements of $\vec{p}p \rightarrow pK\Lambda$:

- = 2.95 GeV/c with (61.0 \pm 1.7) % polarization \rightarrow 42,000 events
- = 2.95 GeV/c with (87.5 \pm 2.0) % polarization \rightarrow 132,000 events
- = 2.70 GeV/c with (77.9 \pm 1.2) % polarization \rightarrow 220,000 events



Straw-Tube-Tracker (STT)



- 2704 straw tubes (*l* = 1 m, *d* = 1 cm) arranged in 13 double layers
- Ar : CO₂ gas mixture with ratio 8 : 2 at 1.2 bar overpressure
- Drift time information used for track to wire distance
- Obtained averaged spatial resolution $\sigma = (137 \pm 9) \, \mu \mathrm{m}$





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Event Reconstruction Steps for $pp \rightarrow pK\Lambda$ reconstruction

- 1 Track finding (Hough transformation) and fitting
- 2 Vertex finding and fitting
- **3** Geometric fit of $pp \rightarrow pK\Lambda$ event topology
- **4** Kinematic fit of $pp \rightarrow pK\Lambda$ (two overconstraints)
 - \rightarrow Kinematically complete events
 - ightarrow pA mass resolution $\sigma = 1.1 \, {\rm MeV}/c^2$





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Event Selection at $2.7 \, \text{GeV}/c$

Selection criteria

- $\chi^2_{\rm kin.fit}/{
 m NDF} < 5$
- Λ decay length > 3 cm
- $\measuredangle(\Lambda, \text{decayproton}) > 2^{\circ}$
- Similar for 2.95 GeV/c

Monte Carlo simulations

- Low background from other reactions $(pp \rightarrow pK\Sigma^0 < 1\%)$
- Reconstruction efficiency $\sim 15\,\%~(20\,\%$ for $2.95\,{\rm GeV}/c)$





$\vec{p}p \rightarrow pK\Lambda$ Dalitz Plot





- Full phase space acceptance
- Reconstruction efficiency relatively flat
- Strong *p*A final state interaction for both data sets
- More substructures for 2.95 GeV/c

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Effective $p\Lambda$ Scattering Length at 2.7 GeV/cHauenstein et al., nucl-ex:1607.04783, submitted to PRL





Systematic Error from N^* Resonances (1) Dalitz plot slices



- Dalitz plot sliced by cuts on helicity angle $(\cos \theta_{\rm pK}^{\rm Rp\Lambda})$ \rightarrow same $m_{p\Lambda}$ phase space acceptance but different N^* fraction
- Determination of effective scattering length for each slice
 - \rightarrow Access to systematic error from $\mathit{N}^* s$ in the $\mathit{K} \Lambda$ channel



Systematic Error from N^* Resonances (2)



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Systematic Error from N^* Resonances (3)

$\cos \theta_{pK}^{Rp\Lambda}$ range	$a_{ m eff}$ [fm]
1) $\cos \theta_{pK}^{Rp\Lambda} > 0.5$	$-1.51\substack{+0.09\\-0.10}$
2) $0 < \cos \theta_{ ho K}^{R ho \Lambda} < 0.5$	$-1.33\substack{+0.08\\-0.08}$
$3) -0.5 < \cos \theta_{pK}^{Rp\Lambda} < 0$	$-1.43\substack{+0.08\\-0.10}$
4) $\cos \theta_{pK}^{Rp\Lambda} < -0.5$	$-1.33\substack{+0.06\\-0.07}$
full range	$-1.38\substack{+0.04\\-0.05}$

- Systematic error from N*s is about 0.1 fm
- Systematic error about factor ten weaker than for 2.95 GeV/c (1 fm)
- Assume similar error for spin triplet scattering length



Spin Triplet $p\Lambda$ Scattering Length see Appendix B in Gasparyan et al., Phys. Rev. C69, 034006 (2004)

- {p∧} in S-wave ⇒ {p∧} in spin triplet configuration only for odd kaon partial waves
- Kaon angular distribution flat \rightarrow Use analyzing power A_y^K from kaon asymmetry
- A_v^K sensitive to interferences of kaon partial waves
- Expand in associated Legendre Polynomials $P_I^1(\cos \theta)$

 $A_y^{\mathcal{K}}(\cos\theta, m_{p\Lambda}) \approx \alpha(m_{p\Lambda})P_1^1(\cos\theta) + \beta(m_{p\Lambda})P_2^1(\cos\theta)$

- For $A_y^K(\cos\theta = 0) = -\alpha$ only spin triplet scattering contributes
- Determination of $a_{\rm t}$ from

$$|lpha(m_{p\Lambda})| \cdot \left| \tilde{A}(FSI)_{ ext{eff}}(m_{p\Lambda}) \right|^2 = \exp\left[C_0 + \frac{C_1}{m_{p\Lambda}^2 - C_2} \right] = |b_1(m_{p\Lambda})|$$





Analyzing Power - Determination Principle see also F. Hauenstein et al., Nucl. Inst. Meth. A817, 42 (2016)

Angular distribution with beam polarization P_Y : $I(\vartheta^*, \phi) = I_0(\vartheta^*) \cdot (1 + A_N(\vartheta^*)P_Y \cos \phi)$



 ϑ^* : cm scattering angle ϕ : azimuthal angle

Formula:

 $A_{N}(\vartheta^{*}) = \frac{2}{P_{Y}} \cdot \epsilon_{A}(\vartheta^{*}) = \frac{2}{P_{Y}} \cdot \frac{(N_{L}^{\uparrow}(\vartheta^{*}) + N_{R}^{\downarrow}(\vartheta^{*})) - (N_{R}^{\uparrow}(\vartheta^{*}) + N_{L}^{\downarrow}(\vartheta^{*}))}{N_{L}^{\uparrow}(\vartheta^{*}) + N_{R}^{\downarrow}(\vartheta^{*}) + (N_{R}^{\uparrow}(\vartheta^{*}) + N_{L}^{\downarrow}(\vartheta^{*}))}$

- Beam polarization P_Y
- $N_{L,R}^{\uparrow\downarrow}$ countrates left or right with polarization directions



Analyzing Power at $2.7 \, \text{GeV}/c$

Fit with associated Legendre polynomials and dependence on $m_{p\Lambda}$



- Reasonable fit of analyzing power by $A_v^K = \alpha P_1^1 + \beta P_2^1$
- β decreases for higher m_{pΛ} masses (expected due to lower kaon momentum)
- α non zero for low m_{pΛ} mass → extraction of spin triplet scattering length possible



Spin Triplet Scattering Length a_t

Hauenstein et al., nucl-ex:1607.04783, submitted to PRL



- · Fit limit and parametrization as for effective scattering length
- Value and statistical errors determined with bootstrapping
- $a_{\rm t} = (-2.55^{+0.72}_{-1.39 {\rm stat.}} \pm 0.6_{\rm syst.} \pm 0.3_{\rm theo.}) \, {\rm fm}$
- First direct model-independent determination of a_t



Comparison with Theory and Other Measurements

	$a_t(\mathrm{fm})$	stat.(fm)	sys.(fm)	theo.(fm)
COSY-TOF	-2.55	$^{+0.72}_{-1.39}$	±0.6	±0.3
$pp ightarrow K^+ + (\Lambda p) \ ^1$	-1.56	$^{+0.19}_{-0.22}$		±0.4
$p\Lambda$ scattering ²	-1.6	$\substack{+1.1\\-0.8}$		
$K^- d ightarrow \pi^- p \Lambda^{-3}$	-2.0	± 0.5		
χ EFT NLO (500)	-1.61			
$\chi {\sf EFT}$ NLO (700)	-1.48			
Jülich 04 model	-1.66			
Nijmegen NSC97f	-1.75			

¹Combined fit of inclusive data and elastic data with constraint from $\mathcal{K}^- d \rightarrow \pi^- p\Lambda$; Budzanowski et al., Phys. Lett. B687, 31 (2010) ²Alexander et al., Phys. Rev. 173, 1452 (1968) ³Tan, Phys. Rev. Lett. 23, 395 (1969)

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∧ Polarization - Determination Principle see also F. Hauenstein et al., Nucl. Inst. Meth. A817, 42 (2016)



$$P_{\Lambda} = \frac{2}{\alpha} \frac{N^A - N^B}{N^A + N^B}$$

- A polarization along $\vec{n}_{\rm sp}$ axis
- Measurement via self analyzing Λ decay
- Distribution of decay protons: $I = I_0 (1 + \alpha P_{\Lambda} \cos \theta^{**})$
- $\alpha = 0.642 \pm 0.013$ (weak asymmetry parameter)
- A = same hemisphere \vec{n}_{sp}
- $B = \text{opposite hemisphere } \vec{n}_{sp}$



Results for the Λ Polarization





- Expected point symmetry at cos ϑ* = 0 and x_F = 0
- A polarization changes sign
- No explanation available

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Λ Polarization Comparison

Similar Energy Regime



- DISTO $(pp \rightarrow pK\Lambda, Nucl. Phys. A639, 1 (1998))$ and HADES $(p + Nb \rightarrow \Lambda X, Eur. Phys. J. A50, 81 (2014))$ cover large part of the phase space
- Compatible results with the COSY-TOF data at 2.95 GeV/c
- Λ polarization probably independent of target material



A Spin Transfer Coefficient D_{NN} see also F. Hauenstein et al., Nucl. Inst. Meth. A817, 42 (2016)

 $I = I_0 \cdot (1 + A_N P_B \cos \phi + \alpha P_\Lambda \cos \theta^{**} + D_{NN} \alpha P_B \cos \phi \cos \theta^{**})$



- 8 countrates depending on beam spin (↑↓), φ angle (LR) and hemisphere in Λ decay (AB)
- $N_{\text{same}} = N_L^{A\uparrow} + N_R^{B\uparrow} + N_R^{A\downarrow} + N_L^{B\downarrow}$ (P_{Λ} & P_B same direction)
- $N_{\rm op} = N_L^{A\downarrow} + N_R^{B\downarrow} + N_R^{A\uparrow} + N_L^{B\uparrow}$ (P_Λ & P_B opposite direction)



Results for D_{NN}





D_{NN} Theoretical Explanation

see also M. Maggiora, Nucl. Phys. A691, 329 (2001)



- $x_F \rightarrow -1$: Λ from unpolarized target proton $\rightarrow D_{NN} = 0$
- $x_F \rightarrow 1$: Λ stems from polarized beam proton
- Pion exchange ightarrow no spin-flip at Λ -vertex ightarrow $D_{NN}=+1$
- Kaon exchange ightarrow spin-flip at Λ -vertex ightarrow $D_{NN}=-1$
- Data exhibits combination of both exchanges
- Missing contributions from vector mesons like K*?



Summary

- High resolution measurement with full phase space acceptance of the $\vec{p}p \rightarrow pK\Lambda$ reaction at 2.7 GeV/c and 2.95 GeV/c
- Determination of *p*Λ scattering length from *p*Λ-FSI at 2.7 GeV/*c* (Hauenstein et al., nucl-ex:1607.04783)
 - Compatible result for effective $p\Lambda$ scattering length with previous TOF result at 2.95 GeV/c
 - Systematic error from N^* resonances factor ten weaker
 - First direct measurement of spin triplet $p\Lambda$ scattering length $\rightarrow a_{t} = (-2.55^{+0.72}_{-1.39\text{stat.}} \pm 0.6_{\text{syst.}} \pm 0.3_{\text{theo.}}) \text{ fm}$
- Λ polarization
 - Changes sign from 2.7 GeV/c to 2.95 GeV/c
 - Results at 2.95 GeV/c compatible to DISTO and HADES results
- Spin transfer coefficient D_{NN}
 - Results for both momenta similar and compatible with DISTO
 - Combination of kaon and pion exchange in the production?



Outlook

- Partial wave analysis of the data under way
- Need: Theoretical description for the behavior of the Λ polarization
- Publishing of the results for the polarization observables soon



Backup Slides

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Overview of Considered Systematic Errors

Error	$a_{ m eff}$	a _t
Fit limit	negligible	negligible
Wrong beam polarization	negligible	negligible
Improper acceptance correction	0.2 fm	0.2 fm
Influence of N^*s	0.1 fm	0.1 fm
Binning of $m_{p\Lambda}$	0.02 fm	0.56 fm
Total	0.22 fm	0.6 fm



COSY-TOF Target





N* Resonances



coupling of N^* to hyperons little known

Baryon	Status	Mass	Width	ΛK	ΣK
S_{11}	****	1645-1670	145-185	3-11	?
D_{15}	****	1670-1680	130-165	<1	?
F_{15}	****	1680-1690	120-140	?	?
D_{13}	***	1650-1750	50-150	<3	?
P_{11}	***	1680-1740	50 - 250	5-25	?
P_{13}	****	1700-1750	150-300	1-15	?
P ₃₃	***	1550-1700	250-450	-	?
D_{33}	****	1670-1750	200-400	-	?

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CMS Distributions for $2.7 \,\mathrm{GeV}/c$





- Distributions almost symmetric
- Small deviations at boarders due to acceptance correction



Measurement of lpha at 2.95 GeV/c

see M. Roeder et al., Eur. Phys. J. A49, 157 (2013)



- Unexpected: α is < 11% (3σ) for low invariant mass
 → no sufficient precision for extraction of spin triplet pΛ scattering length
- β behavior reasonable (high $p\Lambda$ mass \rightarrow low momentum kaons)
- \rightarrow Additional measurement at 2.95 ${\rm GeV}/c$ to reduce statistical error

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Dalitz Plot for Measurements at 2.95 GeV/c



- Left: Published in M. Roeder et al., Eur. Phys. J. A49, 157 (2013)
- Right (preliminary): S. Jowzaee, PhD Thesis, Jagiellonian University Crakow (2014)
- Enhancement at pΛ-FSI and NΣ threshold (cusp effect)



Dalitz Plot Projections on $m_{p\Lambda}$



- Green: Scaled phase space distribution
- Small enhancement at NΣ threshold



- Brown: Scaled phase space distribution
- Large enhancement at NΣ threshold compared to 2.7 GeV/c



$p\Lambda - N\Sigma$ Cusp



- Study of cusp at 2.95 GeV/c (Jowzaee et al., EPJA 52, 7 (2016))
- Reasonable description of spectrum by FSI + cusp(Flatté) + N* reflections
- Further theoretical description necessary

- Cusp described by Flatté distribution
- Angular distributions in cusp region point to S-wave in K – pΣ and subsequent Λ – p



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Self Analyzing Λ Decay

- Quantum numbers of particles in the decay $\Lambda \rightarrow p\pi^ J^P(\Lambda) = \frac{1}{2}^+$, $J^P(p) = \frac{1}{2}^+$ and $J^{\pi^-}(\Lambda) = 0^- \rightarrow \Delta I = 0, 1 \rightarrow$ S-wave or P-wave in the decay system
- Decay proton distribution I = I₀(1 + αP_Λ cos(θ^{**}_p))
- Asymmetry parameter $\alpha = \frac{2\Re(a_s^*a_p)}{|a_s|^2 + |a_p|^2}$
- *a_s* and *a_p* S-wave and P-wave amplitudes
- If parity conservation holds in decay no S-wave possible $\rightarrow a_s = 0 \rightarrow \alpha = 0 \rightarrow$ no measurement of P_{Λ}
- Parity violation in weak decay $\rightarrow \alpha \neq \mathbf{0}$



Results for the Λ Polarization



[Roe11] M. Roeder, PhD Thesis, University Bochum, 2012

[Piz07] C. Pizzolotto, PhD Thesis, University Erlangen, 2007

2.7 GeV/c Hauenstein, PhD Thesis, University Erlangen, 2014

- Λ polarization changes sign
- Expected point symmetry at $\cos \vartheta^* = 0$
- Further study by fitting of associated Legendre polynomials



A Polarization

Associated Legendre Polynomials Fits







A Polarization

Associated Legendre Polynomials Contributions



- As expected C₁ compatible with zero for all beam momenta
- C₂ strong variation with beam momentum. Linear increase?
- No theoretical calculations available

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Λ Polarization High Energy Data Sets



- Measurements with very limited kinematic regions
- Different target materials
- Reactions from inclusive to exclusive Λ production



Λ Polarization Comparison High Energy Data Limited x_f Regions



- Most data exhibit different behavior
- Common Feature: Linear increase of the polarization with p_t
- Increase of polarization with higher x_F



Analyzing Power of Final State Particles





- Proton and kaon analyzing power: Similar behavior for different momenta
- Λ analyzing power: for cos(θ_Λ^{CMS} > 0) different behavior



pp Elastics Event Selection and Beam Polarization Determination



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Track Reconstruction with Hough Transformation















Analyzing Power - Determination Principle see also F. Hauenstein et al., Nucl. Inst. Meth. A817, 42 (2016)

Angular distribution with beam polarization P_B : $I(\vartheta^*, \phi) = I_0(\vartheta^*) \cdot (1 + A_N(\vartheta^*)P_B \cos \phi)$



 ϑ^* : cm scattering angle ϕ : Angle between polarization direction (Y) and normal on production plane (N)

Formula:

 $A_{N}(\vartheta^{*}) = \frac{2}{P_{B}} \cdot \epsilon_{A}(\vartheta^{*}) = \frac{2}{P_{B}} \cdot \frac{(N_{L}^{\uparrow}(\vartheta^{*}) + N_{R}^{\downarrow}(\vartheta^{*})) - (N_{R}^{\uparrow}(\vartheta^{*}) + N_{L}^{\downarrow}(\vartheta^{*}))}{N_{L}^{\uparrow}(\vartheta^{*}) + N_{R}^{\downarrow}(\vartheta^{*}) + (N_{R}^{\uparrow}(\vartheta^{*}) + N_{L}^{\downarrow}(\vartheta^{*}))}$

- Beam polarization P_B
- $N_{L,R}^{\uparrow\downarrow}$ countrates left or right with polarization directions



Analyzing Power

Determination Principle

Angular distribution for particles with polarization
$$P_Y$$
:
 $(\frac{d\sigma}{d\Omega})_{\text{pol.}} = (\frac{d\sigma}{d\Omega})_0 \cdot (1 + A_N P_N) = (\frac{d\sigma}{d\Omega})_0 \cdot (1 + A_N P_Y \cos \phi)$

$$A_N(\cos \theta^{\text{CMS}}) = \frac{\epsilon_{LR}(\cos \theta^{\text{CMS}}, \phi)}{\cos(\phi) \cdot p_Y}$$

- Azimuthal left-right asymmetry $\epsilon_{LR}(\cos\theta^{\text{CMS}},\phi) = \frac{L(\theta_{\rho}^{\text{CMS}},\phi) - R(\theta_{\rho}^{\text{CMS}},\phi)}{L(\theta_{\rho}^{\text{CMS}},\phi) + R(\theta_{\rho}^{\text{CMS}},\phi)}$
- Count rates $L(\theta_p^{\text{CMS}}, \phi)) = \sqrt{N^+(\phi) \cdot N^-(\phi + \pi)} \text{ and }$ $R(\theta_p^{\text{CMS}}, \phi) = \sqrt{N^+(\phi + \pi) \cdot N^-(\phi)}$
- Beam polarization p_Y