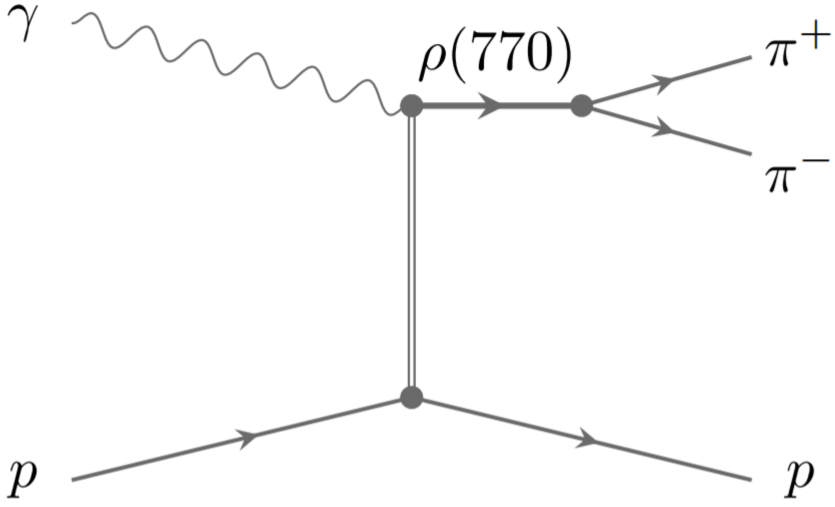
# Measuring ρ(770) SDMEs with the Fourier Coefficient Method

Jamie Fitches

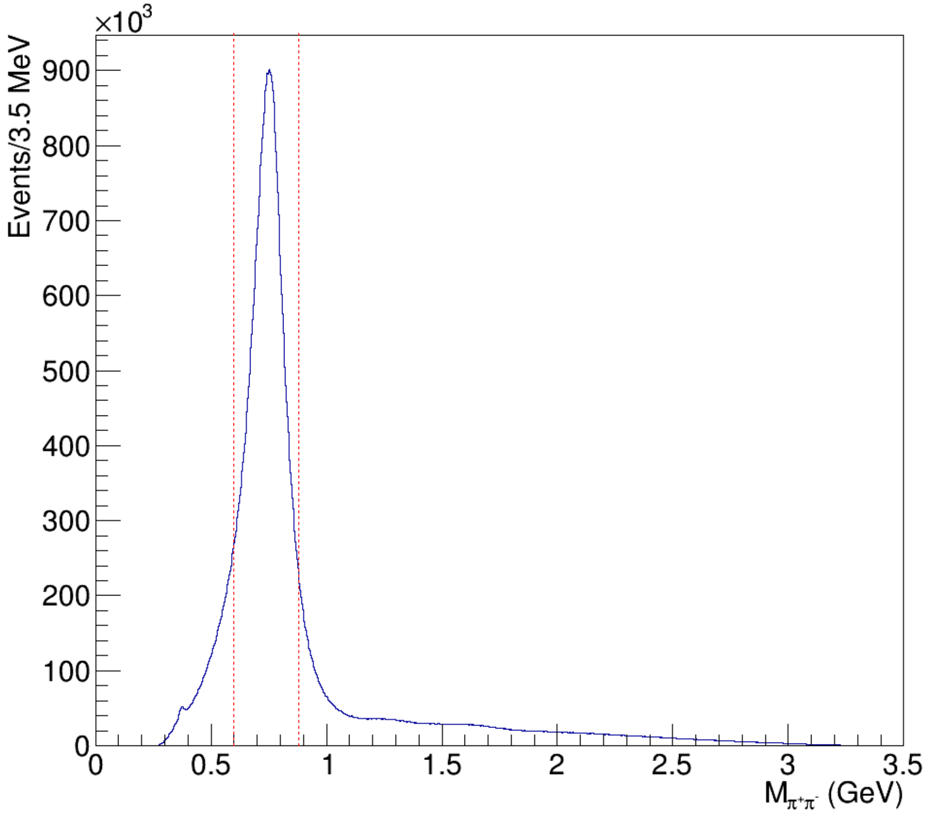
# Motivation

- ρ(770) SDMEs previously measured directly using maximum likelihood fitting with Minuit, MCMC, etc
- SDMEs can also be derived from moments of angular distribution
- Moments parameterise the angular distribution of decay particles from all competing resonance processes
- More free parameters in maximum likelihood fit for moment extraction



# Event Selection

- Data set:
  - tree_pippim__B4, ver36 (2017-01)
  - Kinematic fit: Vertex and 4-momentum
  - 4 beam bunches on either side of prompt peak
- Simulation:
  - Generator: gen_amp
  - Flat angular distribution
- Dselector cuts:
  - Default proton dE/dx
  - |Missing mass squared| < 0.02 $GeV^2$
  - 0.60 GeV < $M_{\pi^+\pi^-}$ < 0.88 GeV
  - 55cm < $Z_{vertex}$ < 75cm
  - 8.2 GeV < $E_\gamma$ < 8.8 GeV



# SDME Extraction

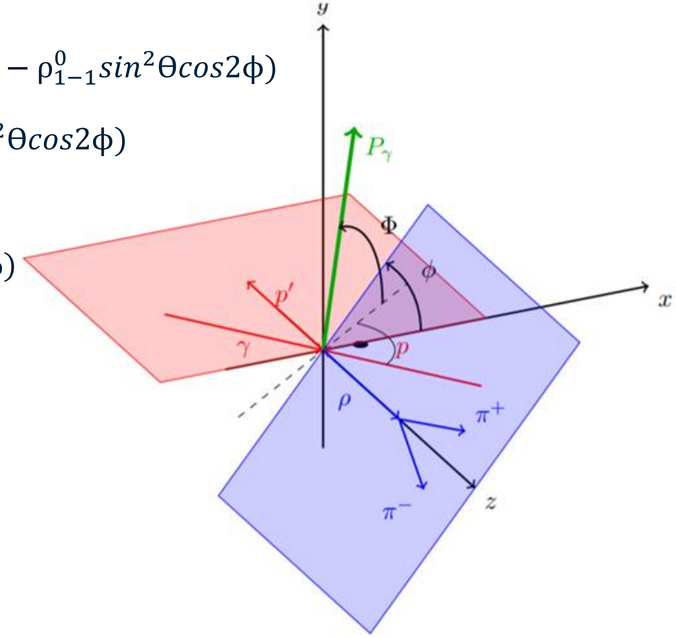
Intensity function:

$$W^0(\cos\Theta,\phi) = \frac{3}{4\pi}\left(\frac{1}{2}(1-\rho^0_{00}) + \frac{1}{2}(3\rho^0_{00}-1)\cos^2\Theta - \sqrt{2}Re\rho^0_{10}\sin2\Theta\cos\phi - \rho^0_{1-1}\sin^2\Theta\cos2\phi\right)$$

$$W^1(\cos\Theta,\phi) = \frac{3}{4\pi}\left(\rho^1_{11}\sin^2\Theta + \rho^1_{00}\cos^2\Theta - \sqrt{2}Re\rho^1_{10}\sin2\Theta\cos\phi - \rho^1_{1-1}\sin^2\Theta\cos2\phi\right)$$

$$W^2(\cos\Theta,\phi) = \frac{3}{4\pi}\left(\sqrt{2}\mathrm{Im}\,\rho^2_{10}\sin2\Theta\,\sin\phi + Im\rho^2_{1-1}\sin^2\Theta\sin2\phi\right)$$

$$W(\cos\Theta,\phi) = W^0(\cos\Theta,\phi) - P_\gamma\cos2\Phi\,W^1(\cos\Theta,\phi) - P_\gamma\sin2\Phi W^2(\cos\Theta,\phi)$$




Extended maximum likelihood fitting:

$$Measured\ Intensity\ \mathrm{I}(\Omega) \propto W(\cos\Theta,\phi)$$

$$lnL = observed\ events\ (weighted) - normalisation\ integral$$

$$= \sum_{i=1}^{N} w_i lnI(\Omega_i) - \int d\Omega I(\Omega)\eta(\Omega),$$

where $w_i = 1$ (prompt) or -1/8 (background)

# Moment Extraction

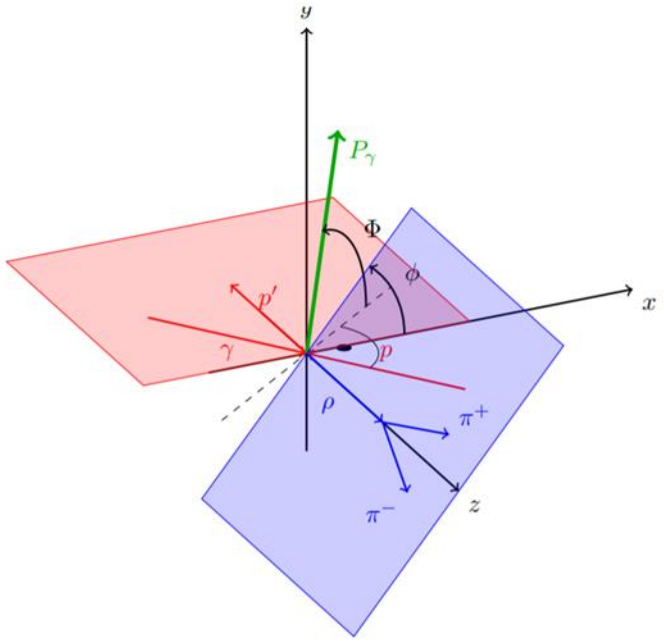
Intensity function:

$$W^0(\cos\Theta,\phi) = \sum_{L,M\geq 0} \frac{2L+1}{4\pi}\left(2-\delta_{M,0}\right)H^0(LM)d^L_{M0}(\theta)\cos M\phi$$

$$W^1(\cos\Theta,\phi) = -\sum_{L,M\geq 0} \frac{2L+1}{4\pi}\left(2-\delta_{M,0}\right)H^1(LM)d^L_{M0}(\theta)\cos M\phi$$

$$W^2(\cos\Theta,\phi) = 2\sum_{L,M>0} \frac{2L+1}{4\pi} ImH^2(LM)d^L_{M0}(\theta)\sin M\phi$$

$$W(\cos\Theta,\phi) = W^0(\cos\Theta,\phi) - P_\gamma \cos 2\Phi\, W^1(\cos\Theta,\phi) - P_\gamma \sin 2\Phi W^2(\cos\Theta,\phi)$$

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# The Fourier Coefficient Method

- Match Fourier coefficients in moments expansion and vector meson SDME equation
- From SDME equation we have:

$$I^0(cos\Theta,\phi) = \frac{3}{4\pi}(\frac{1}{2}(1-\rho^0_{00}) + \frac{1}{2}(3\rho^0_{00}-1)cos^2\Theta - \sqrt{2}Re\rho^0_{10}sin2\Theta cos\phi - \rho^0_{1-1}sin^2\Theta cos2\phi)$$

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- Intensity component for each L,M is:

$$I^0_{L,M}(cos\Theta,\phi) = H^0(LM)\frac{2L+1}{4\pi}\sqrt{\frac{(L-M)!}{(L+M)!}}P_L^M(\cos(\theta))e^{iM\phi}$$

- E.g. $I^0_{2,1}(cos\Theta,\phi) = -H^0(21)\frac{5}{4\pi}\sqrt{\frac{1}{6}\frac{3}{2}}\sin(2\theta)\cos(\phi)$

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$$Re\rho^0_{10} = \frac{5}{\sqrt{12}}H^0(21)$$

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# Fourier Coefficient Method

- Direct extraction from vector meson SDME equation assumes that all measured final states in data sample are ρ(770) events
- Higher order L, M terms can introduce a bias without model for full angular distribution
- With the Fourier coefficient method, a fit is performed to the full angular distribution

$$\rho^0_{00} = \frac{1}{3}(5H^0(20) + 1)$$

$$Re\rho^0_{10} = \frac{5}{\sqrt{12}}H^0(21)$$

$$\rho^0_{1-1} = -\frac{5}{\sqrt{6}}H^0(22)$$

$$\rho^1_{00} = -2.5H^1(20) - \frac{1}{3}H^1(00)$$

$$\rho^1_{11} = -\frac{1}{3}H^1(00)$$

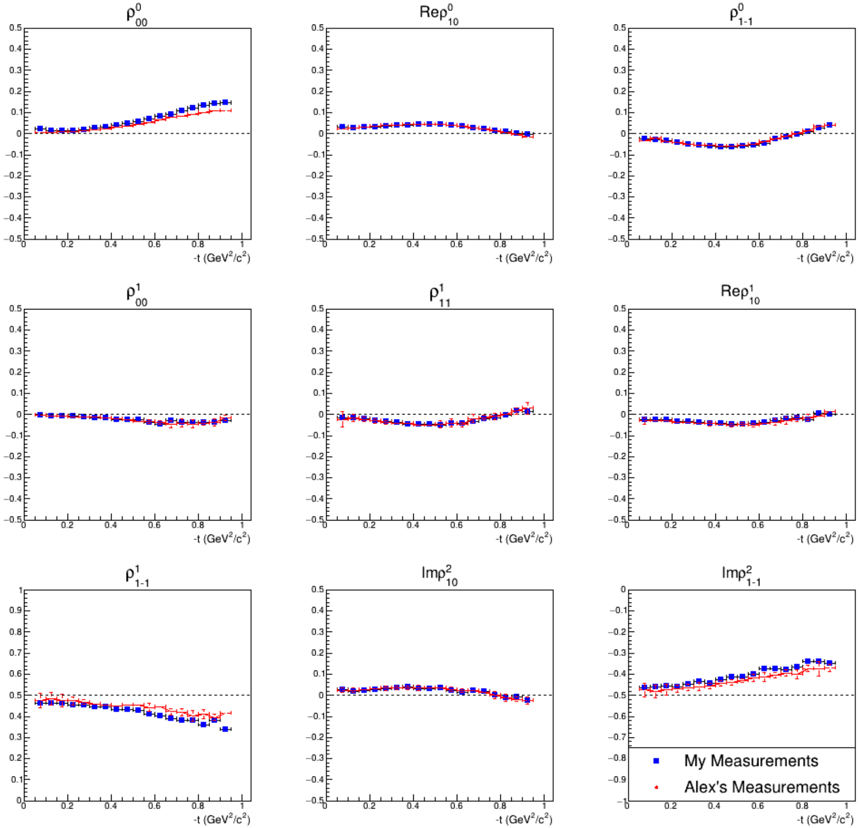
$$Re\rho^1_{10} = -\frac{5}{\sqrt{12}}H^1(21)$$

$$\rho^1_{1-1} = \frac{5}{\sqrt{6}}H^1(22)$$

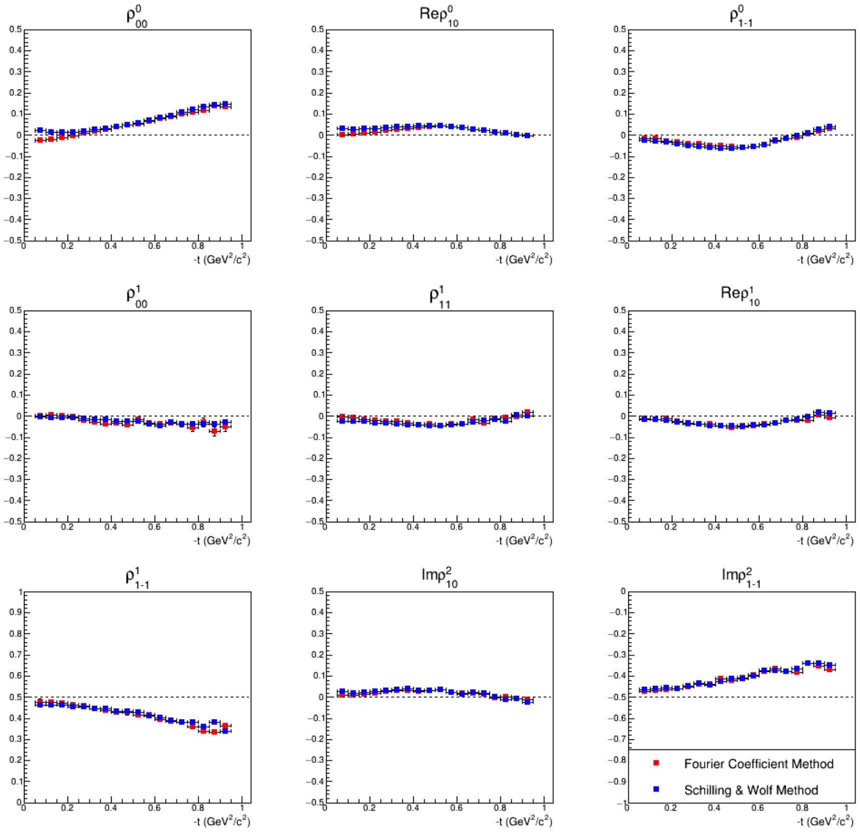
$$Im\rho^2_{10} = -\frac{5}{\sqrt{12}}H^2(21)$$

$$Im\rho^2_{1-1} = \frac{5}{\sqrt{6}}H^2(22)$$

Preliminary

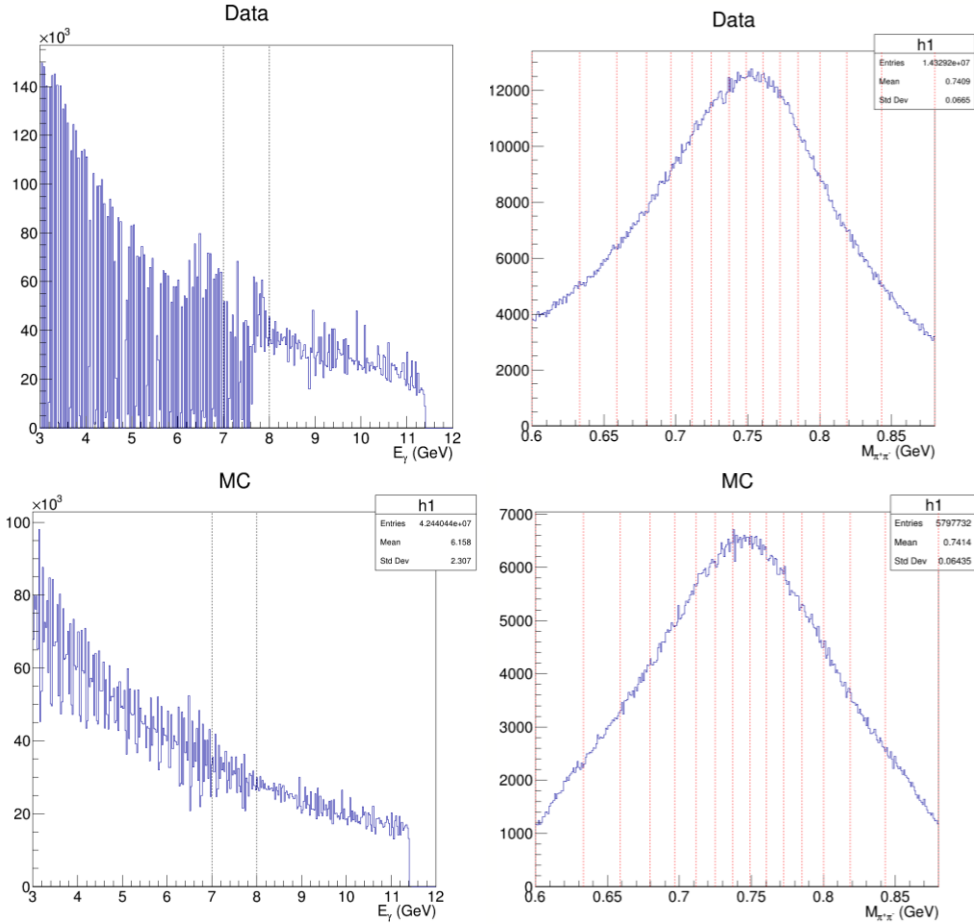


Preliminary



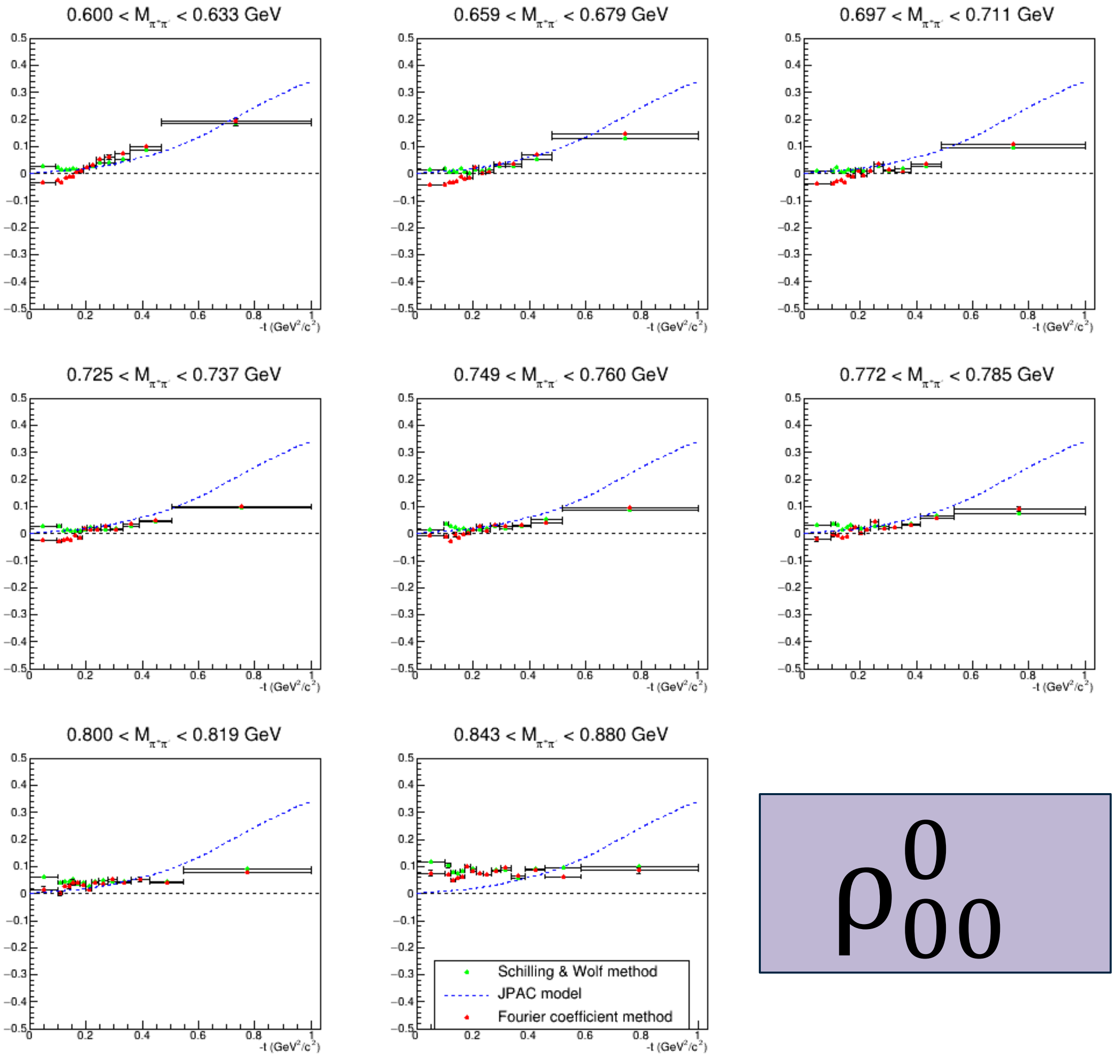
# Polarisation Independent SDMEs

- Unpolarised ρ(770) SDMEs ($\rho^0_{00}$, $Re\rho^0_{10}$, $\rho^0_{1-1}$) can be measured over the full $E_\gamma$ range
- This will be attempted in the coming months using spring 2017 amorphous data
- 15 bins in $M_{\pi^+\pi^-}$ and 15 bins in $|t|$, each with fixed stats
- 9 bins in $E_\gamma$ with fixed bin width of 1 GeV (3-12 GeV)



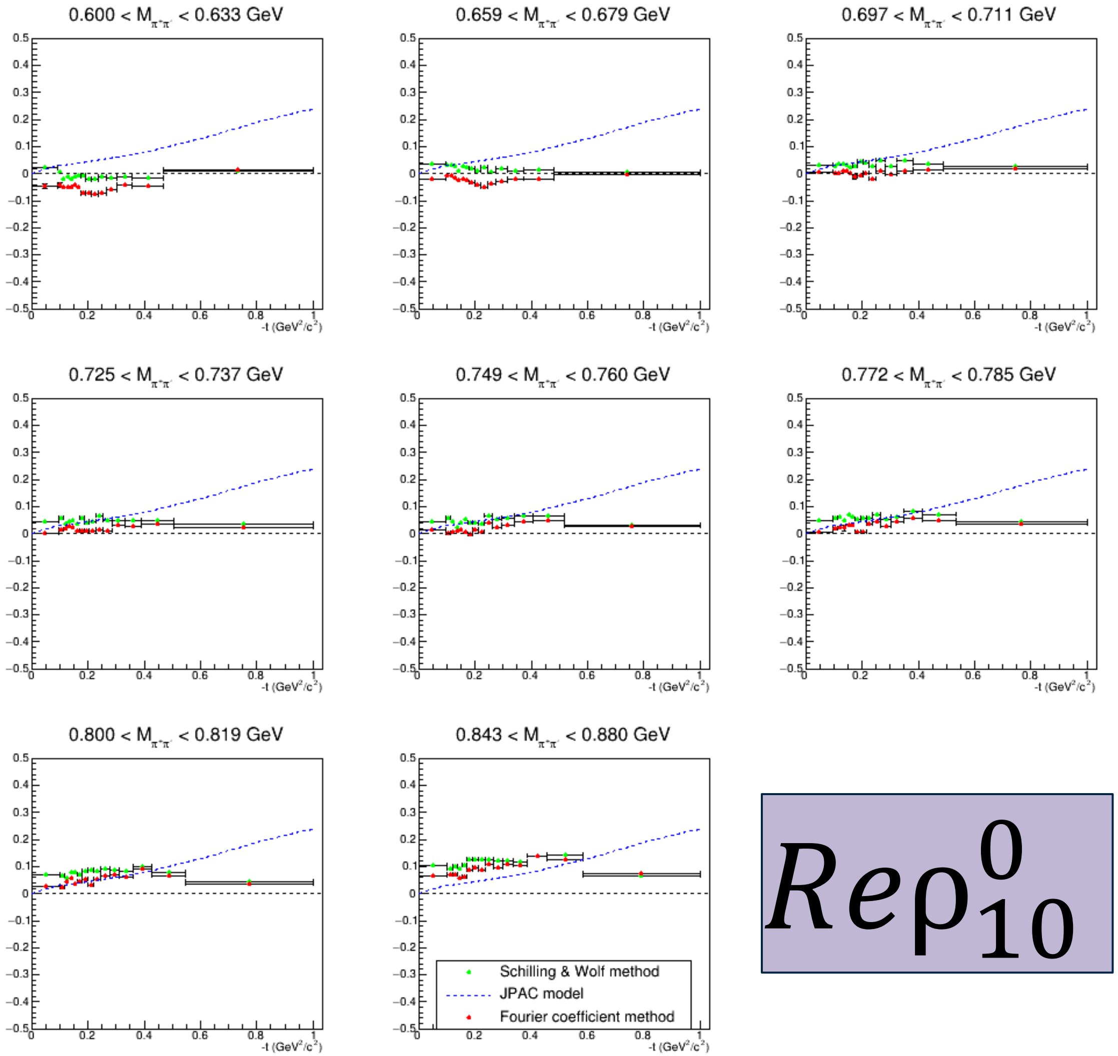
University of Glasgow

GlueX Preliminary

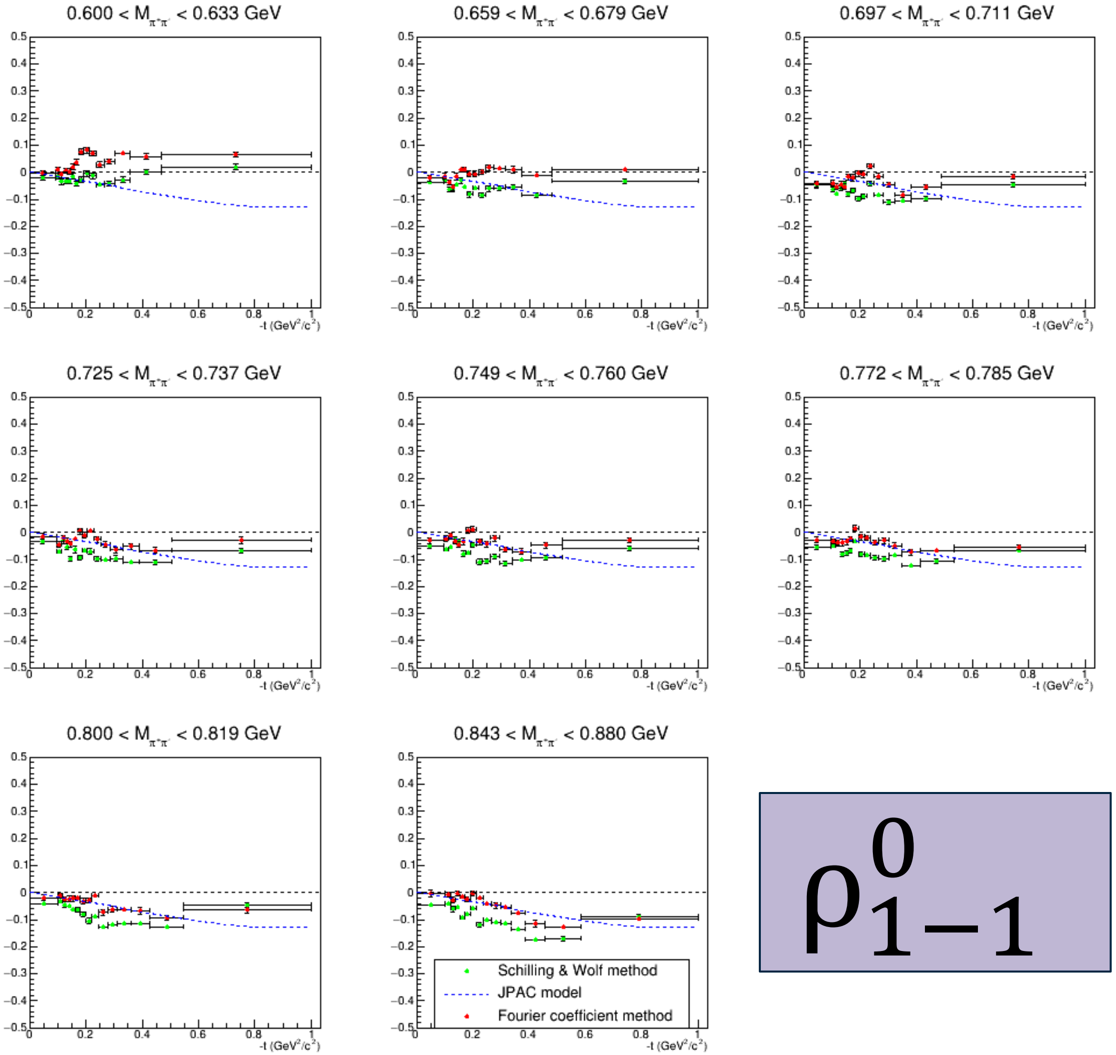



$\rho^0_{00}$

Preliminary




$Re\rho^0_{10}$




GlueX Preliminary

$$\rho^0_{1-1}$$

# Summary

- ρ(770) polarised SDMEs have been measured using two independent methods applied to the GlueX I data-set
- Strong agreement between both sets of results, however some discrepancies visible at low t
- Polarisation independent SDME measurements ongoing
- Future work:
  - Measure polarisation independent SDMEs across 3-12 GeV beam energy range using Fourier coefficient method
  - Measure same SDMEs using standard 'direct extraction' method for comparison
  - Toy studies to evaluate which method is more effective