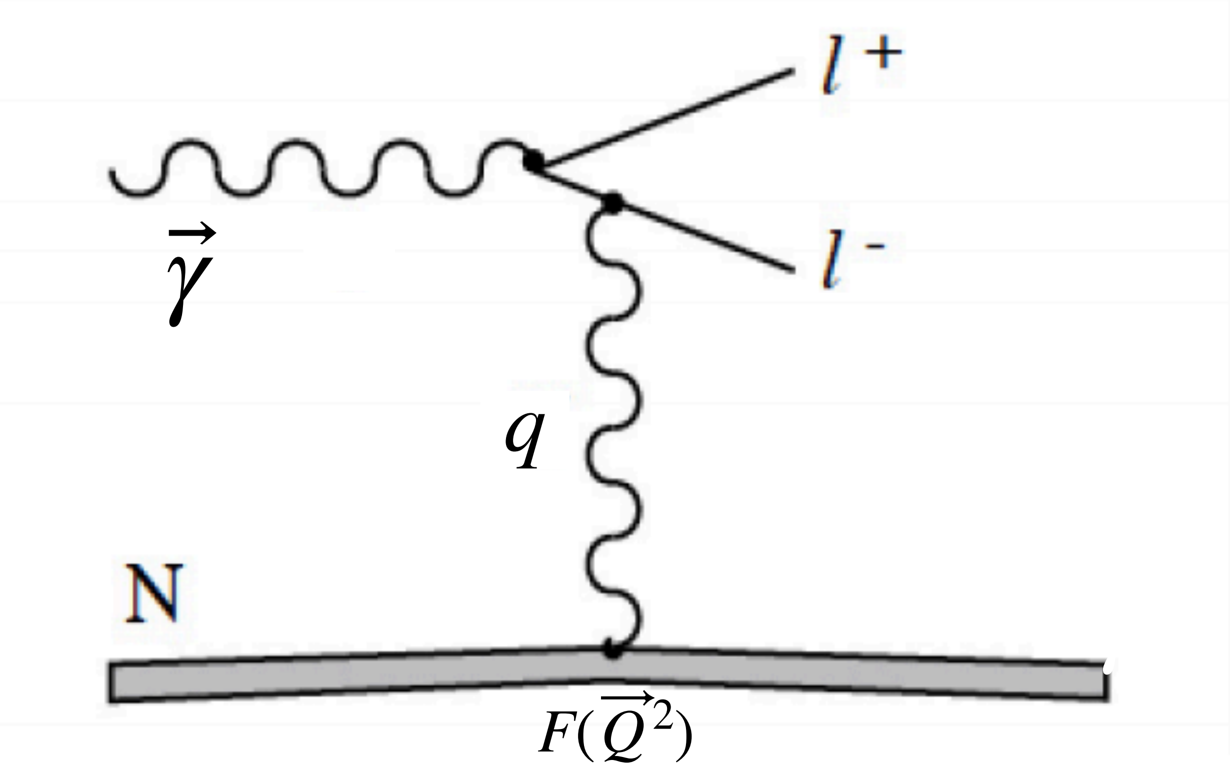
University *of* Massachusetts Amherst

MENP

Using Bethe Heitler Pairs as a Polarimeter in GlueX



Andrew Schick

GlueX Fall Collaboration Meeting, Friday, Sept. 24 2021

Jefferson Lab

Use Bethe-Heitler pairs to measure linear photon polarization.

$$
\begin{aligned}
\mathrm{d}\sigma &= \left(\frac{1+\mathcal{P}}{2}\right)\mathrm{d}\sigma_{||} + \left(\frac{1-\mathcal{P}}{2}\right)\mathrm{d}\sigma_{\perp} \\
&= \underset{\substack{\uparrow \\ \mathrm{d}\sigma_0 \\ \text{Unpolarized}}}{\left(\frac{\mathrm{d}\sigma_{||}+\mathrm{d}\sigma_{\perp}}{2}\right)} + \mathcal{P}\underset{\substack{\uparrow \\ \mathrm{d}\sigma_1 \\ \text{Polarized}}}{\left(\frac{\mathrm{d}\sigma_{||}-\mathrm{d}\sigma_{\perp}}{2}\right)}
\end{aligned}
$$

Use Bethe-Heitler pairs to measure linear photon polarization.

$$
\begin{aligned}
\mathrm{d}\sigma &= \left(\frac{1+\mathcal{P}}{2}\right)\mathrm{d}\sigma_{\parallel} + \left(\frac{1-\mathcal{P}}{2}\right)\mathrm{d}\sigma_{\perp} \\
&= \underbrace{\left(\frac{\mathrm{d}\sigma_{\parallel}+\mathrm{d}\sigma_{\perp}}{2}\right)}_{\substack{\mathrm{d}\sigma_0 \\ \text{Unpolarized}}} + \mathcal{P}\underbrace{\left(\frac{\mathrm{d}\sigma_{\parallel}-\mathrm{d}\sigma_{\perp}}{2}\right)}_{\substack{\mathrm{d}\sigma_1 \\ \text{Polarized}}}
\end{aligned}
$$

Bakmaev et al, Physics Letters B 660 (2008) 494-500

Modern Vectorized Approach

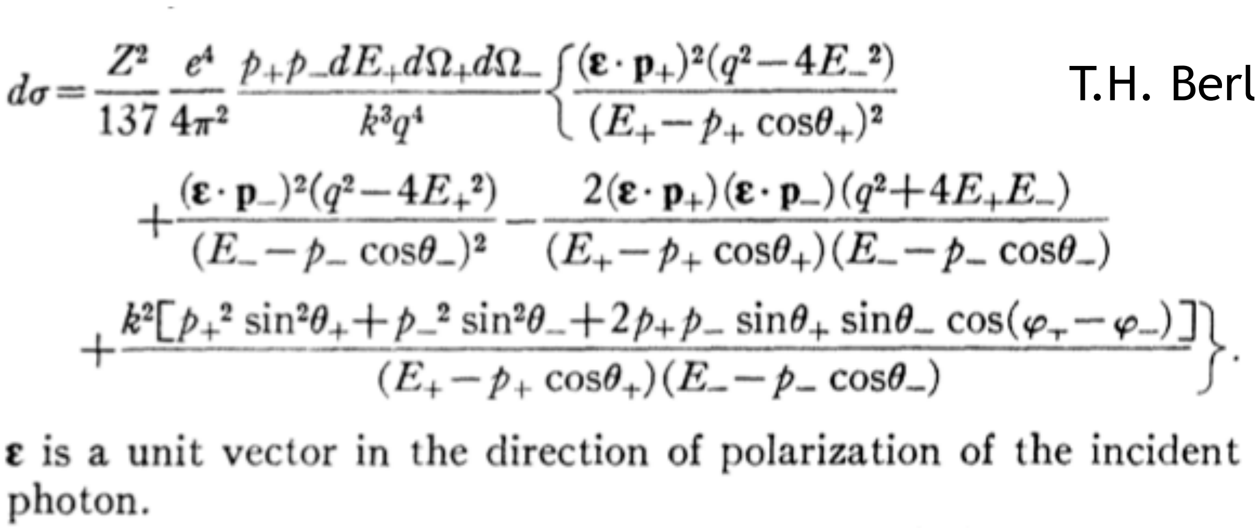
$$\overrightarrow{J}_T = \frac{\overrightarrow{p_1}}{p_1^2 + m^2} + \frac{\overrightarrow{p_2}}{p_2^2 + m^2} = \frac{\overrightarrow{p_1}}{c_1} + \frac{\overrightarrow{p_2}}{c_2}$$

$\overrightarrow{p_1}$, $\overrightarrow{p_2}$ are the lepton's transverse momenta

$$\mathrm{d}\sigma_1 \sim P_\gamma |\overrightarrow{J}_T|^2 \cos(2\phi_{J_T})$$

Bakmaev's formulation is really only valid at very large t

Vectorizing the Classic Bethe-Heitler Formulation



$$d\sigma=\frac{Z^2}{137}\frac{e^4}{4\pi^2}\frac{p_+p_-dE_+d\Omega_+d\Omega_-}{k^3q^4}\left\{\frac{(\boldsymbol{\varepsilon}\cdot\mathbf{p}_+)^2(q^2-4E_-^2)}{(E_+-p_+\cos\theta_+)^2}+\frac{(\boldsymbol{\varepsilon}\cdot\mathbf{p}_-)^2(q^2-4E_+^2)}{(E_--p_-\cos\theta_-)^2}-\frac{2(\boldsymbol{\varepsilon}\cdot\mathbf{p}_+)(\boldsymbol{\varepsilon}\cdot\mathbf{p}_-)(q^2+4E_+E_-)}{(E_+-p_+\cos\theta_+)(E_--p_-\cos\theta_-)}+\frac{k^2[p_+^2\sin^2\theta_++p_-^2\sin^2\theta_-+2p_+p_-\sin\theta_+\sin\theta_-\cos(\varphi_+-\varphi_-)]}{(E_+-p_+\cos\theta_+)(E_--p_-\cos\theta_-)}\right\}.$$

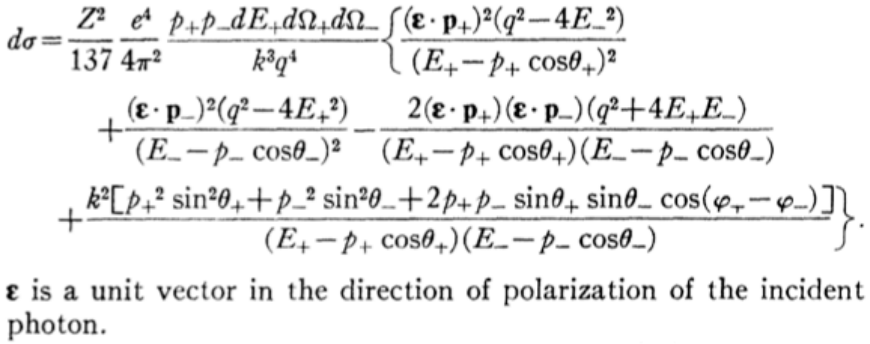
$\boldsymbol{\varepsilon}$ is a unit vector in the direction of polarization of the incident photon.

T.H. Berlin and L. Madansky, Phys. Rev. **78**, 623 (1950)

Vectorizing the Classic Bethe-Heitler Formulation

$$\vec{J}_T = \frac{2E_2}{E_1 - p_1\cos\theta_1}\vec{p}_{1T} + \frac{2E_1}{E_2 - p_2\cos\theta_2}\vec{p}_{2T}$$

$$\vec{K}_T = \frac{\sqrt{q^2}}{E_1 - p_1\cos\theta_1}\vec{p}_{1T} - \frac{\sqrt{q^2}}{E_2 - p_2\cos\theta_2}\vec{p}_{2T}$$



$$d\sigma = \frac{Z^2}{137}\frac{e^4}{4\pi^2}\frac{p_+p_-dE_+d\Omega_+d\Omega_-}{k^3q^4}\left\{\frac{(\boldsymbol{\varepsilon}\cdot\mathbf{p}_+)^2(q^2-4E_-^2)}{(E_+-p_+\cos\theta_+)^2}\right.$$
$$+\frac{(\boldsymbol{\varepsilon}\cdot\mathbf{p}_-)^2(q^2-4E_+^2)}{(E_--p_-\cos\theta_-)^2}-\frac{2(\boldsymbol{\varepsilon}\cdot\mathbf{p}_+)(\boldsymbol{\varepsilon}\cdot\mathbf{p}_-)(q^2+4E_+E_-)}{(E_+-p_+\cos\theta_+)(E_--p_-\cos\theta_-)}$$
$$\left.+\frac{k^2[p_+^2\sin^2\theta_++p_-^2\sin^2\theta_-+2p_+p_-\sin\theta_+\sin\theta_-\cos(\varphi_+-\varphi_-)]}{(E_+-p_+\cos\theta_+)(E_--p_-\cos\theta_-)}\right\}.$$

$\boldsymbol{\varepsilon}$ is a unit vector in the direction of polarization of the incident photon.

Then:

$$\mathrm{d}\sigma = \mathrm{d}\sigma_0 + P_\gamma \mathrm{d}\sigma_1$$

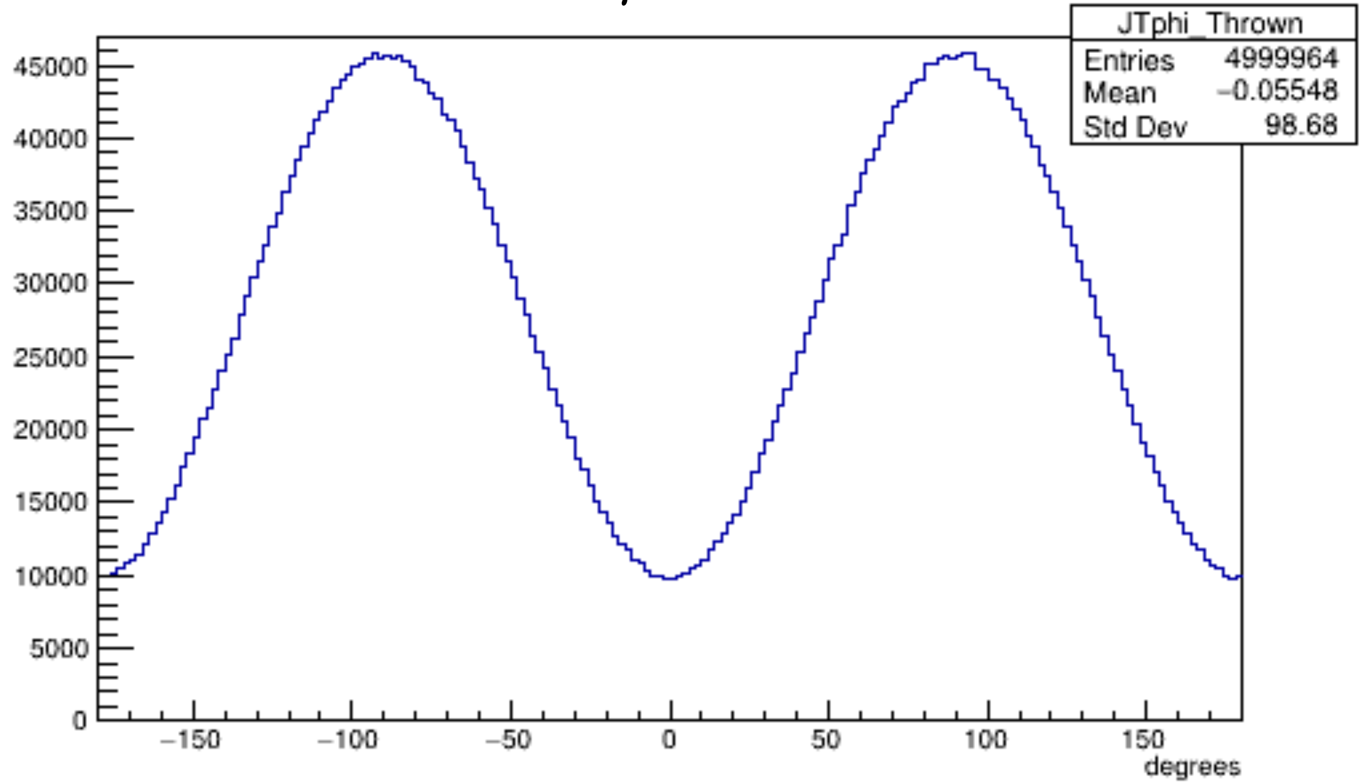
$$\mathrm{d}\sigma_0 = \frac{\mathrm{d}\sigma_\parallel + \mathrm{d}\sigma_\perp}{2} = k\left[-\left|\vec{J}_T\right|^2 + \left|\vec{K}_T\right|^2 + 2E_0^2\frac{\left|\vec{p}_{1T} + \vec{p}_{2T}\right|^2}{\left(E_1 - p_1\cos\theta_1\right)\left(E_2 - p_2\cos\theta_2\right)}\right]$$

$$\mathrm{d}\sigma_1 = \frac{\mathrm{d}\sigma_\parallel - \mathrm{d}\sigma_\perp}{2} = k\left[\boxed{-\left|\vec{J}_T\right|^2\cos 2\phi_{J_T}} + \left|\vec{K}_T\right|^2\cos 2\phi_{J_T}\right]$$

$$\left|\vec{J}_T\right|^2 \gg \left|\vec{K}_T\right|^2$$

$$\mathrm{d}\sigma = \mathrm{d}\sigma_0 + P_\gamma \mathrm{d}\sigma_1 \qquad \mathrm{d}\sigma_1 = \sim \left|\vec{J}_T\right|^2 \cos 2\phi_{J_T}$$

$$P_\gamma = 1$$



MC with BH Cross-Section

$$d\sigma = \frac{Z^2}{137}\frac{e^4}{4\pi^2}\frac{p_+p_-dE_+d\Omega_+d\Omega_-}{k^3q^4}\left\{\frac{(\boldsymbol{\varepsilon}\cdot\mathbf{p}_+)^2(q^2-4E_-^2)}{(E_+-p_+\cos\theta_+)^2} + \frac{(\boldsymbol{\varepsilon}\cdot\mathbf{p}_-)^2(q^2-4E_+^2)}{(E_--p_-\cos\theta_-)^2} - \frac{2(\boldsymbol{\varepsilon}\cdot\mathbf{p}_+)(\boldsymbol{\varepsilon}\cdot\mathbf{p}_-)(q^2+4E_+E_-)}{(E_+-p_+\cos\theta_+)(E_--p_-\cos\theta_-)} + \frac{k^2[p_+^2\sin^2\theta_+ + p_-^2\sin^2\theta_- + 2p_+p_-\sin\theta_+\sin\theta_-\cos(\varphi_+-\varphi_-)]}{(E_+-p_+\cos\theta_+)(E_--p_-\cos\theta_-)}\right\}.$$

1. Generate e+e- 4 vectors using this cross section

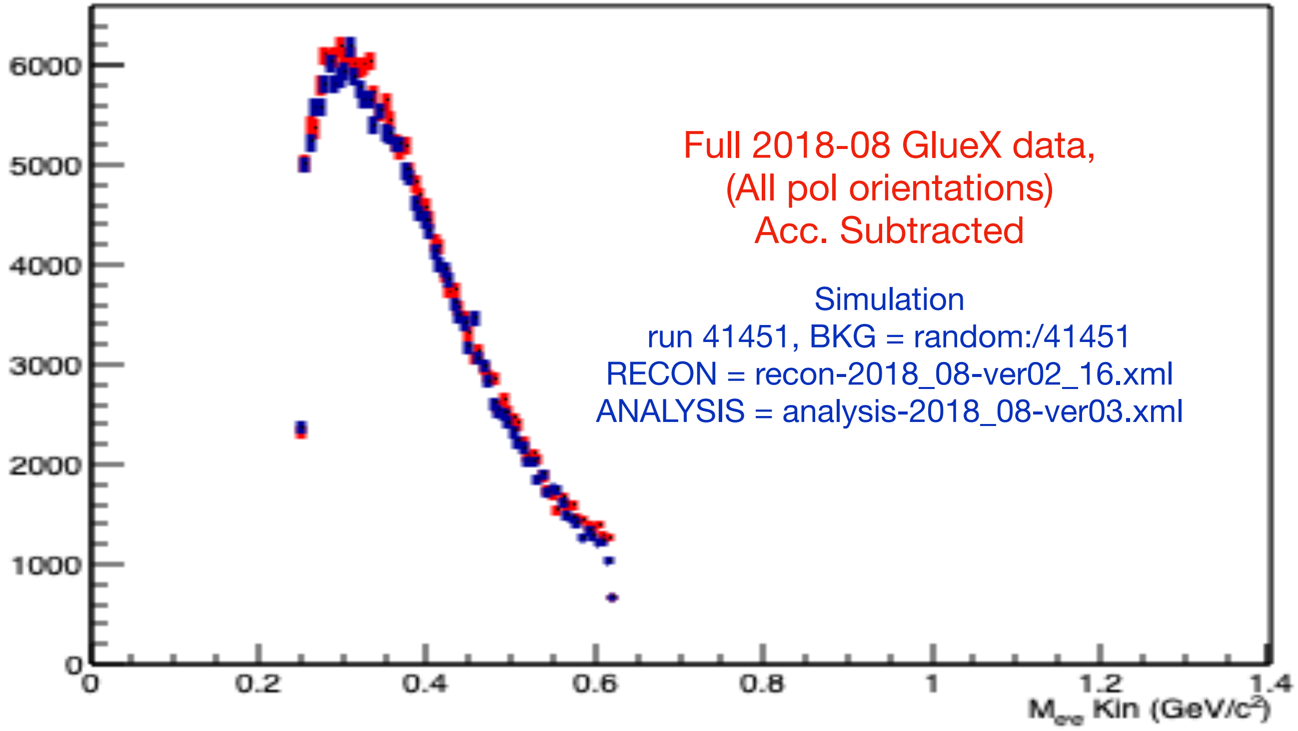
2. Plot ϕ_{J_T} from the 4 vectors

3. Measuring ϕ_{J_T} allows you to infer the beam polarization

2018-01, 2018-08 GlueX data
$\gamma p \rightarrow e^+e^-(p)$ Reaction Filter

Neural Net Cuts:
Neural Net Classification Cuts (NN1, NN2 > 0.8)
Fiducial Cuts:
$8.2\,\mathrm{GeV} < E_\gamma < 8.8\,\mathrm{GeV}$
$0.25\,\mathrm{GeV} < W_{ee} < 0.621\,\mathrm{GeV}$
Both tracks have hits in the TOF
$\theta_1, \theta_2 > 1.5\,\mathrm{deg}$
Vertex cut (Window free): 52 < z < 78 cm

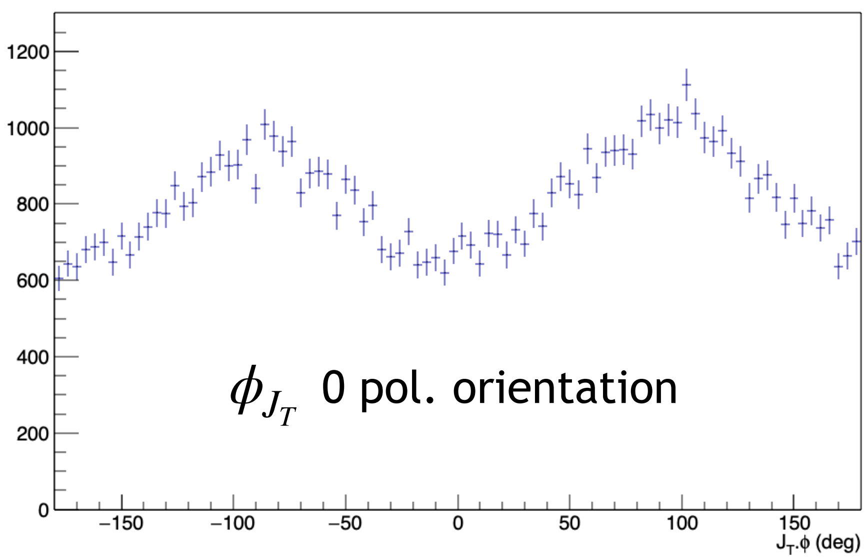
e^+e^- Invariant Mass

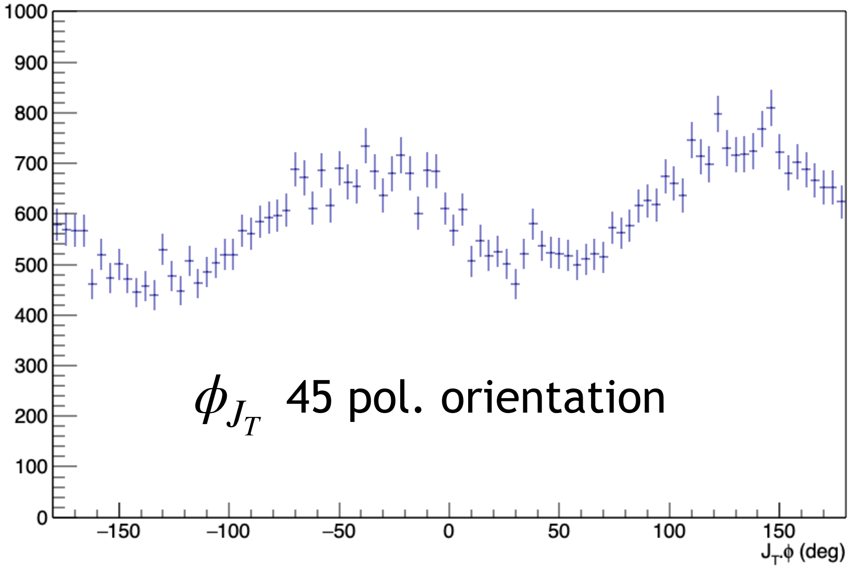


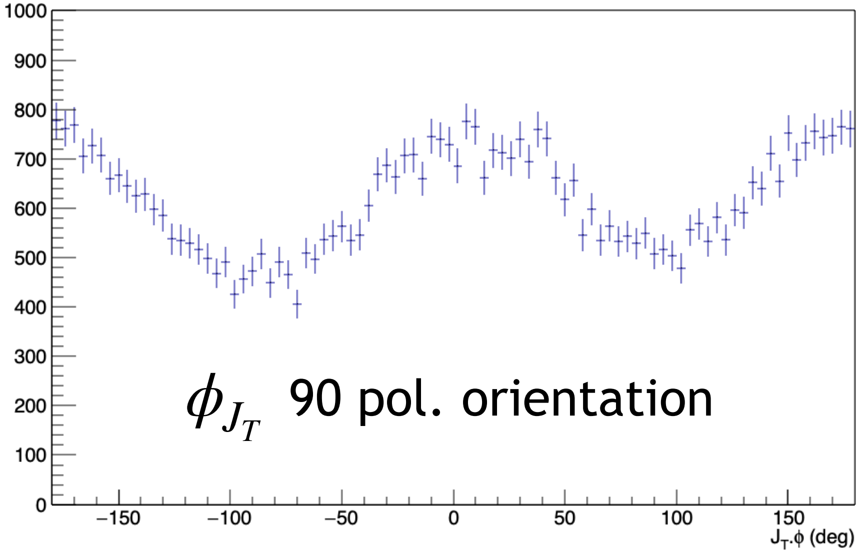
Full 2018-08 GlueX data,
(All pol orientations)
Acc. Subtracted

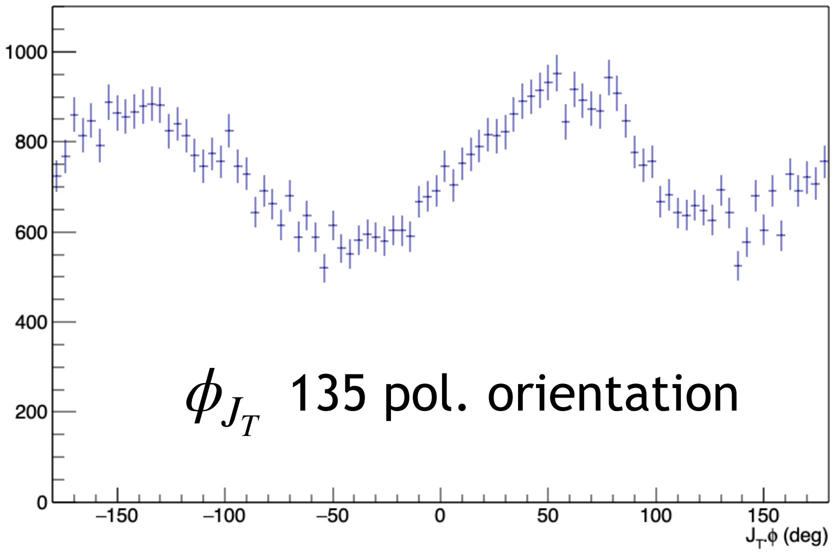
Simulation
run 41451, BKG = random:/41451
RECON = recon-2018_08-ver02_16.xml
ANALYSIS = analysis-2018_08-ver03.xml

$\gamma p \rightarrow e^+e^-(p)$ 2018-01 GlueX data, w/ fiducial+N.N. cuts + pion subtraction









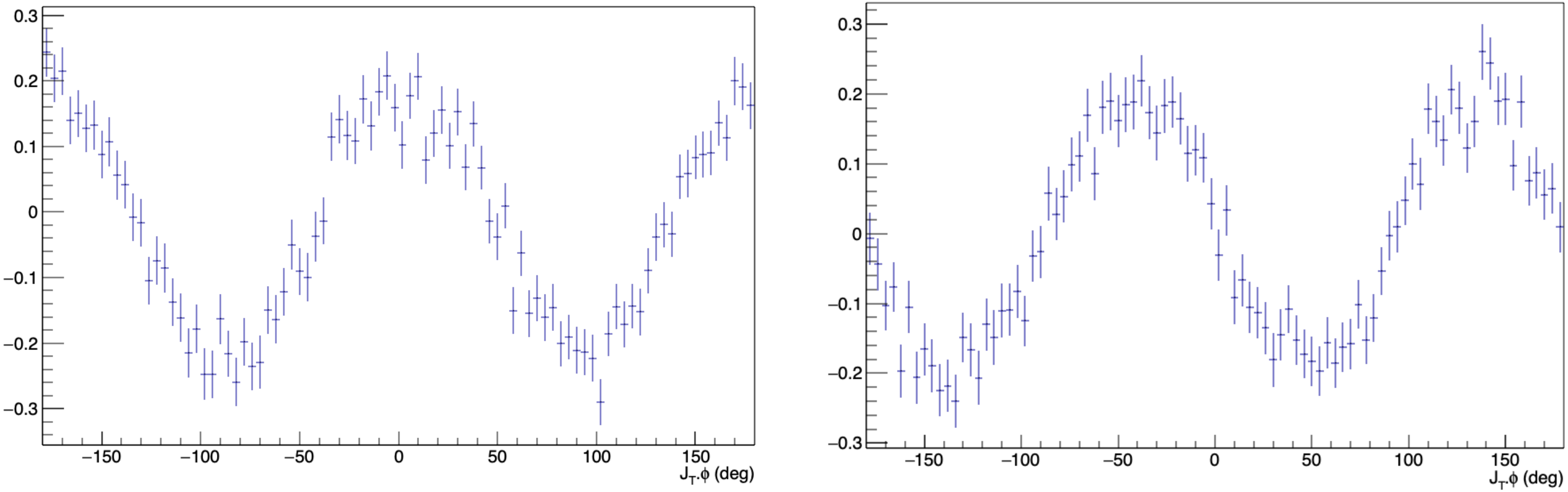
$$\frac{Y_\perp(\phi) - \frac{N_\perp}{N_\parallel} Y_\parallel(\phi)}{Y_\perp + \frac{N_\perp}{N_\parallel} Y_\parallel(\phi)} = \frac{\Sigma \cos 2\phi (P_\perp + P_\parallel)}{2 + \Sigma \cos 2\phi (P_\perp - P_\parallel)}$$

2018-01 GlueX data (pion subtracted), $\gamma p \to e^+ e^- (p)$

Yield Asymmetry

0/90 runs

45/135 runs

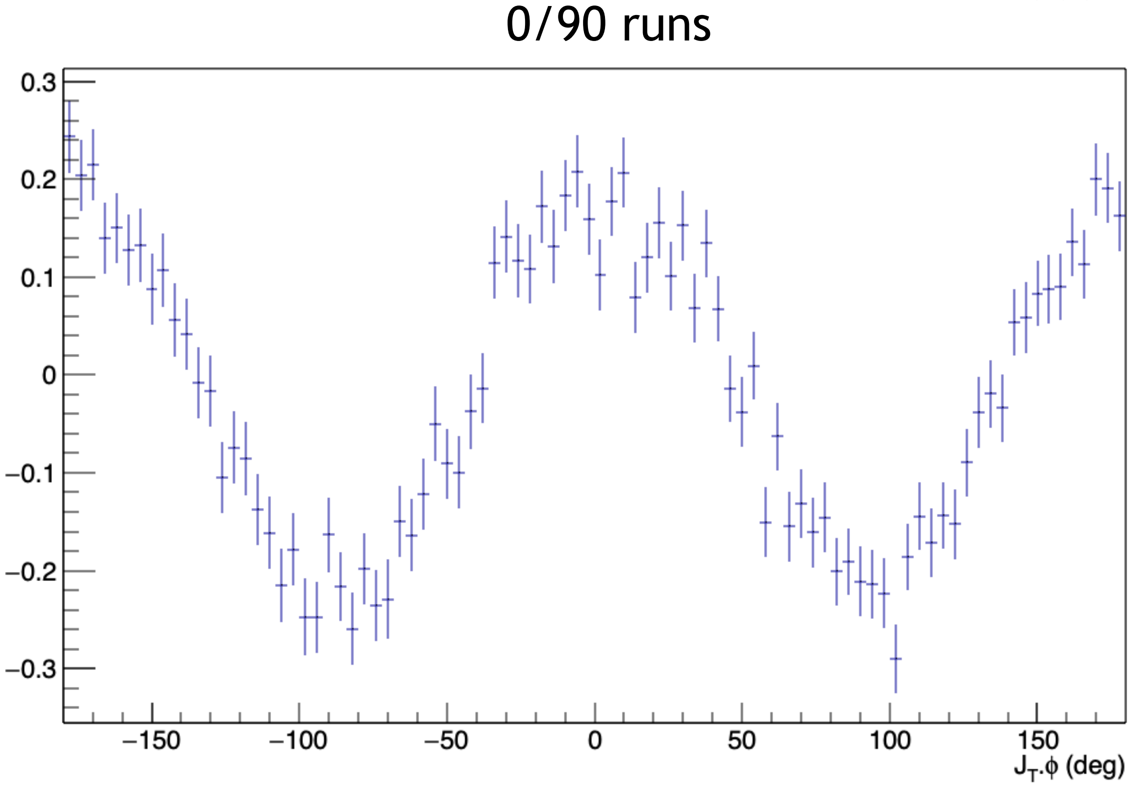


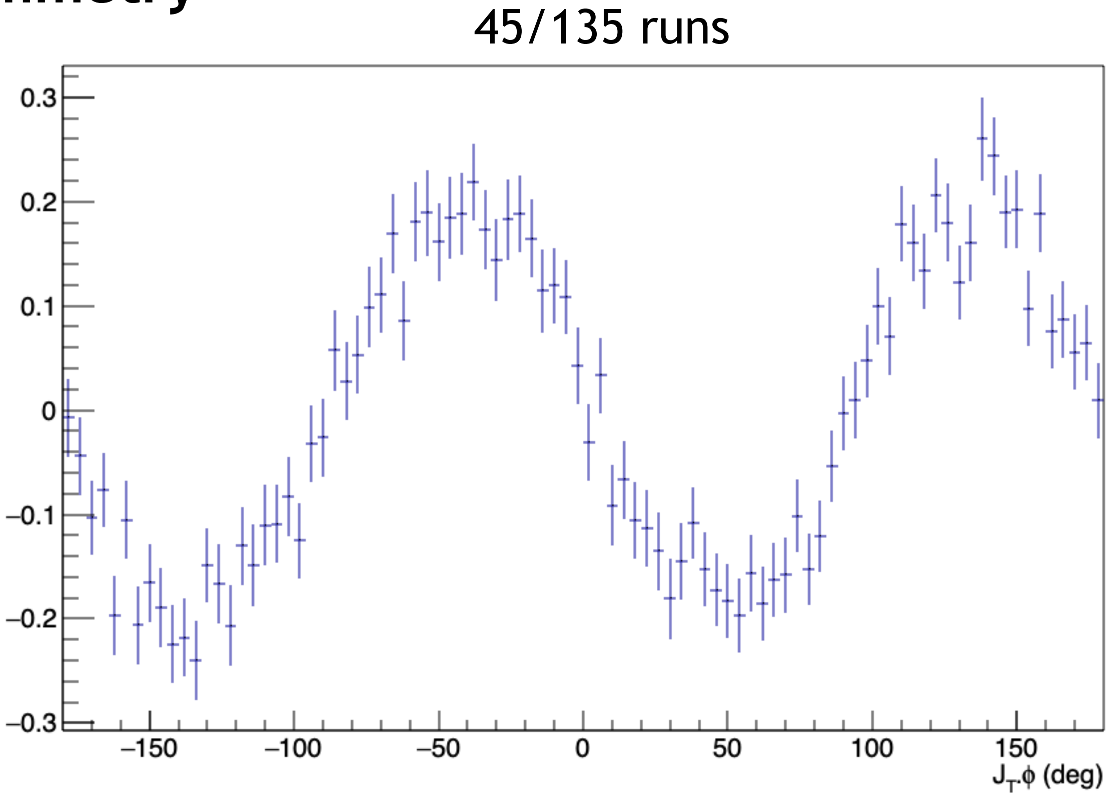
$$\frac{Y_{\perp}(\phi)-\frac{N_{\perp}}{N_{\parallel}}Y_{\parallel}(\phi)}{Y_{\perp}+\frac{N_{\perp}}{N_{\parallel}}Y_{\parallel}(\phi)} = \frac{\Sigma\cos 2\phi(P_{\perp}+P_{\parallel})}{2+\Sigma\cos 2\phi(P_{\perp}-P_{\parallel})}$$

Assume $P_{\perp} \approx P_{\parallel}$
And allow for phase

2018-01 GlueX data (pion subtracted), $\gamma p \rightarrow e^{+}e^{-}(p)$

Yield Asymmetry





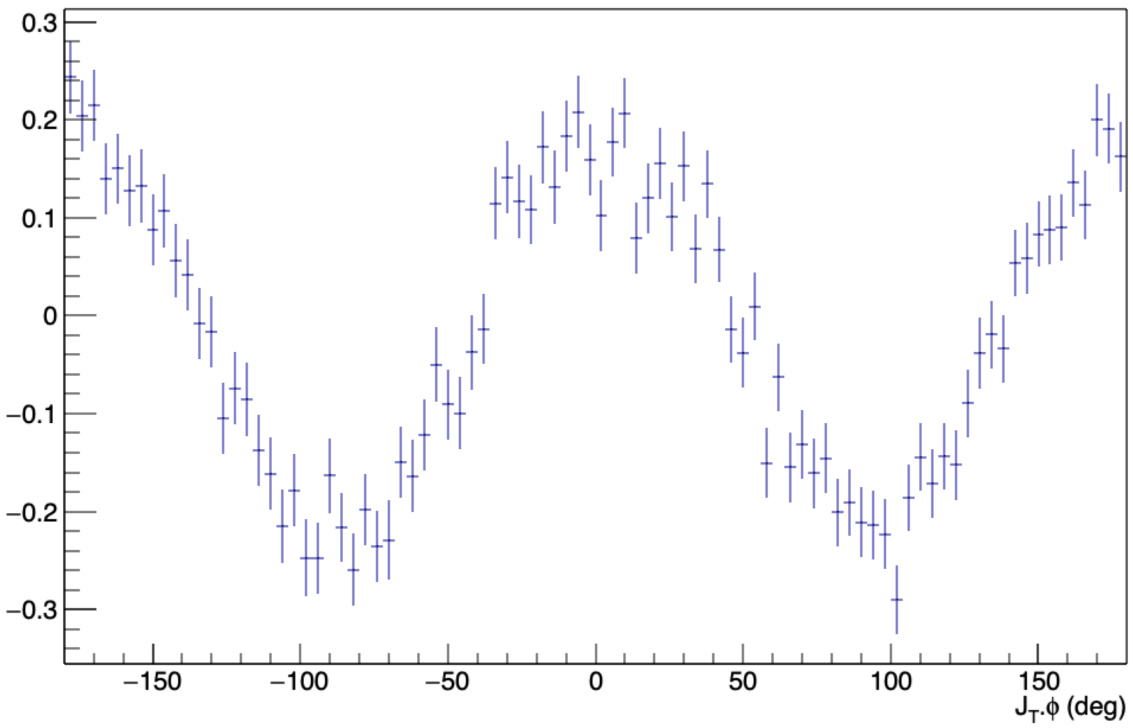
$$\frac{Y_\perp(\phi) - \frac{N_\perp}{N_\parallel} Y_\parallel(\phi)}{Y_\perp + \frac{N_\perp}{N_\parallel} Y_\parallel(\phi)} = \frac{\Sigma(P_\perp + P_\parallel)\cos 2(\phi + \alpha)}{2}$$

Assume $P_\perp \approx P_\parallel$
And allow for phase

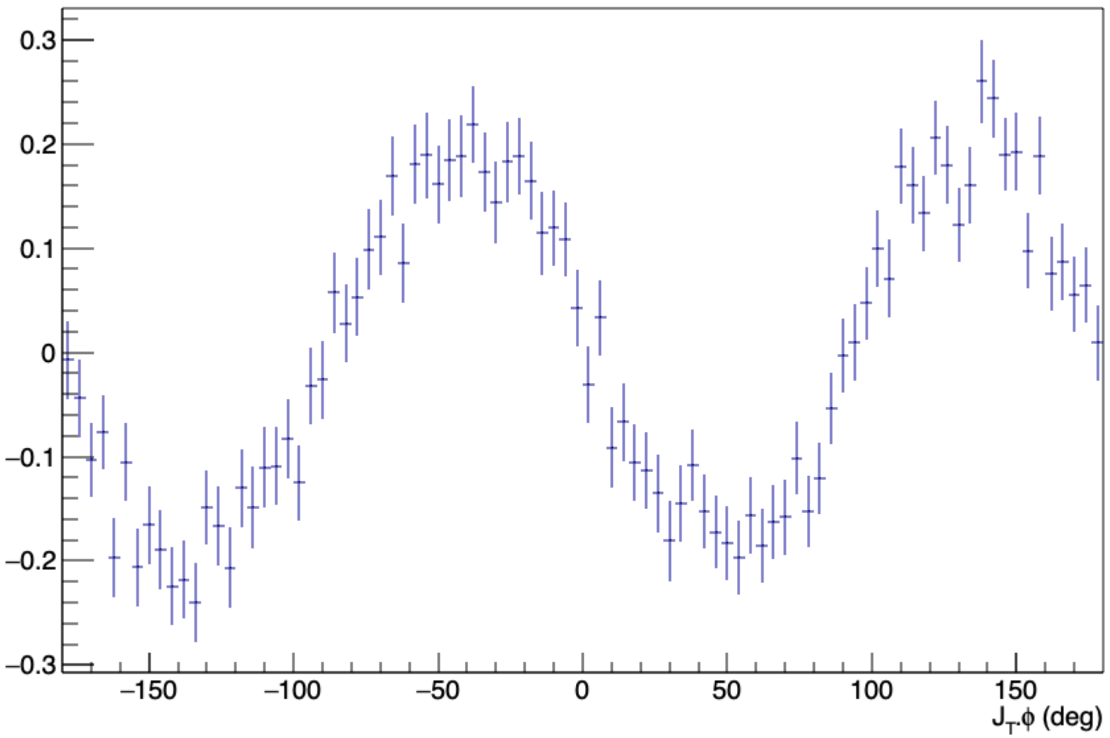
2018-01 GlueX data (pion subtracted), $\gamma p \rightarrow e^+ e^-(p)$

Yield Asymmetry

0/90 runs

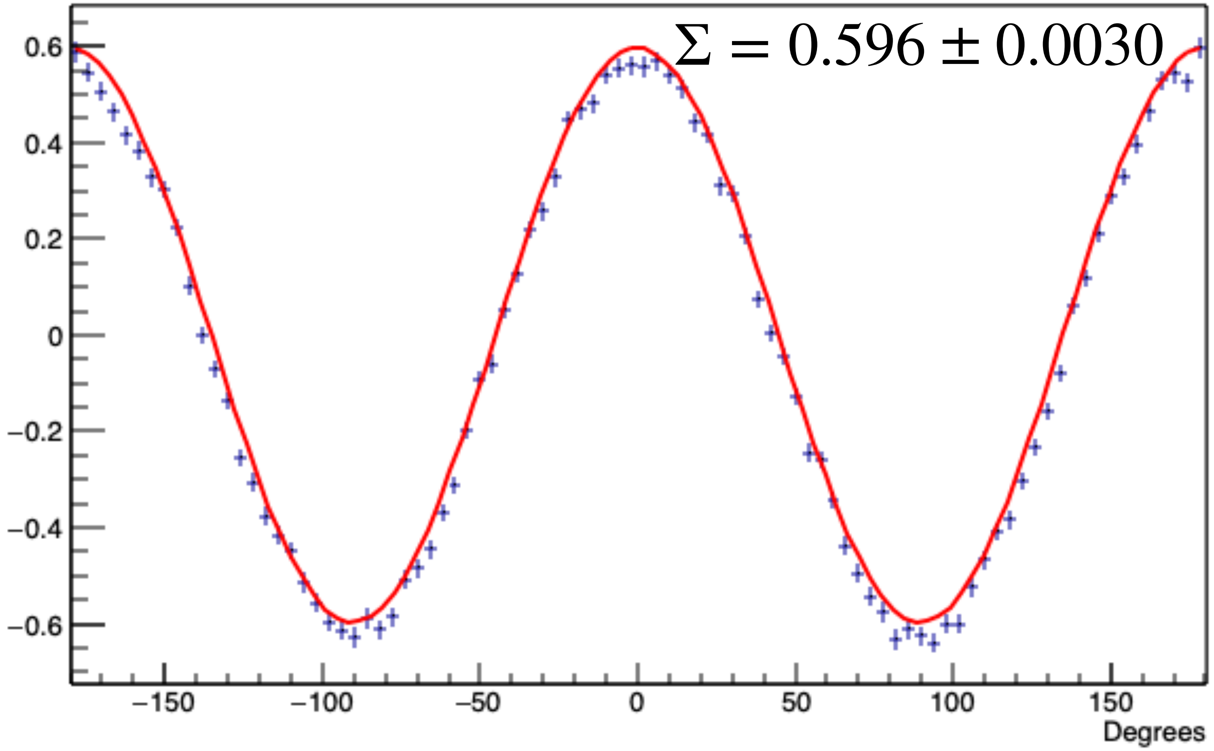


45/135 runs



$$\frac{Y_{\perp}(\phi) - Y_{\|}(\phi)}{Y_{\perp} + Y_{\|}(\phi)} = \Sigma \cos 2\phi$$

Simulated Yield Asymmetry

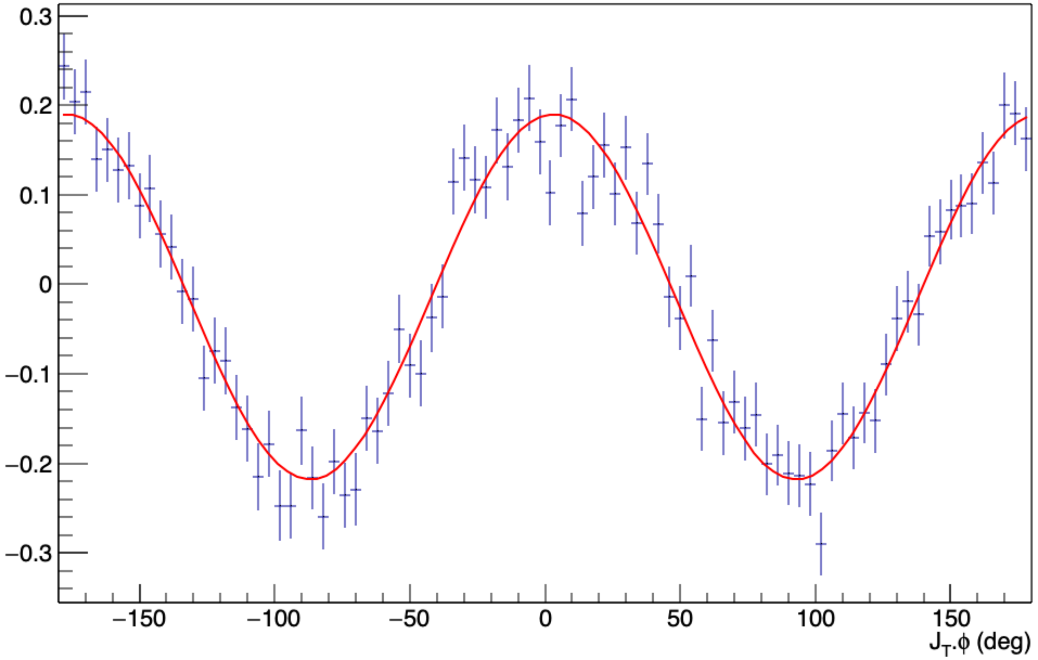


2018-01 Spring GlueX data, Average Polarization

0 and 90

TPOL expected average polarization: $\frac{\mathscr{P}_{\perp} + \mathscr{P}_{\parallel}}{2} = 0.341 \pm 0.004$

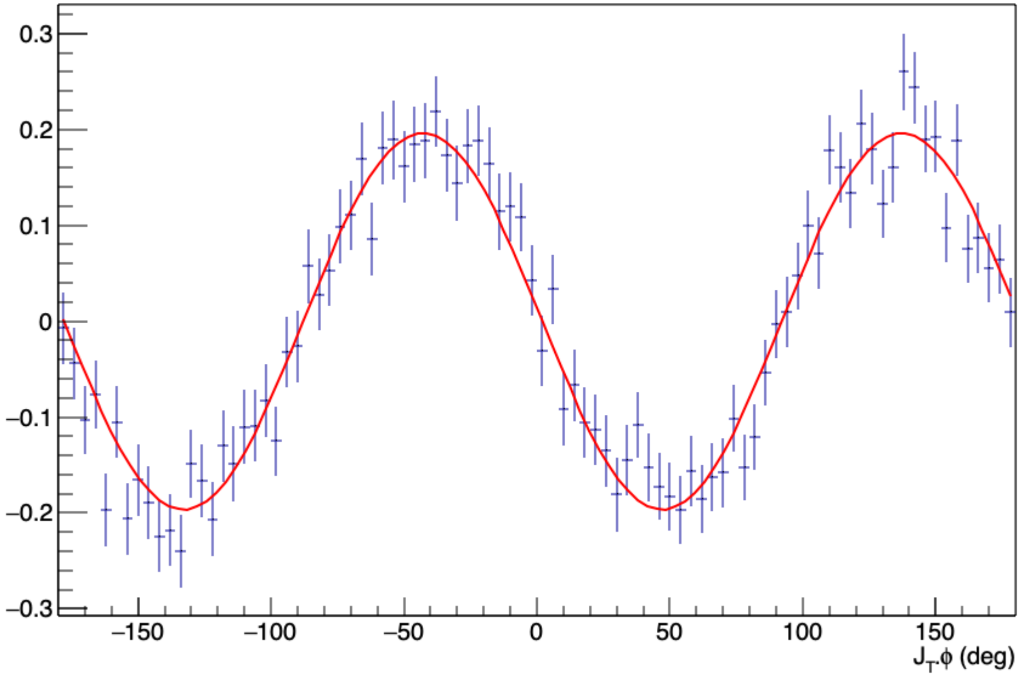
BH average polarization: $\frac{\mathscr{P}_{\perp} + \mathscr{P}_{\parallel}}{2} = 0.342 \pm 0.009$



45 and 135

TPOL expected average polarization: $\frac{\mathscr{P}_{\perp} + \mathscr{P}_{\parallel}}{2} = 0.344 \pm 0.004$

BH average polarization: $\frac{\mathscr{P}_{\perp} + \mathscr{P}_{\parallel}}{2} = 0.336 \pm 0.009$

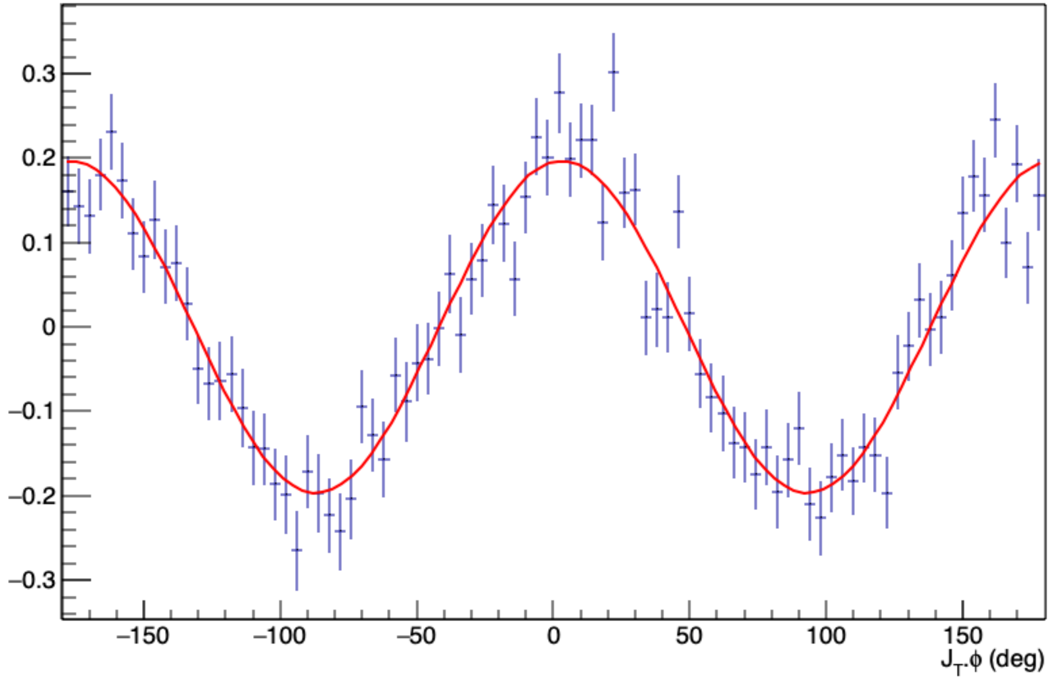


2018-08 Fall GlueX data, Average Polarization

0 and 90

TPOL expected average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.345 \pm 0.005$

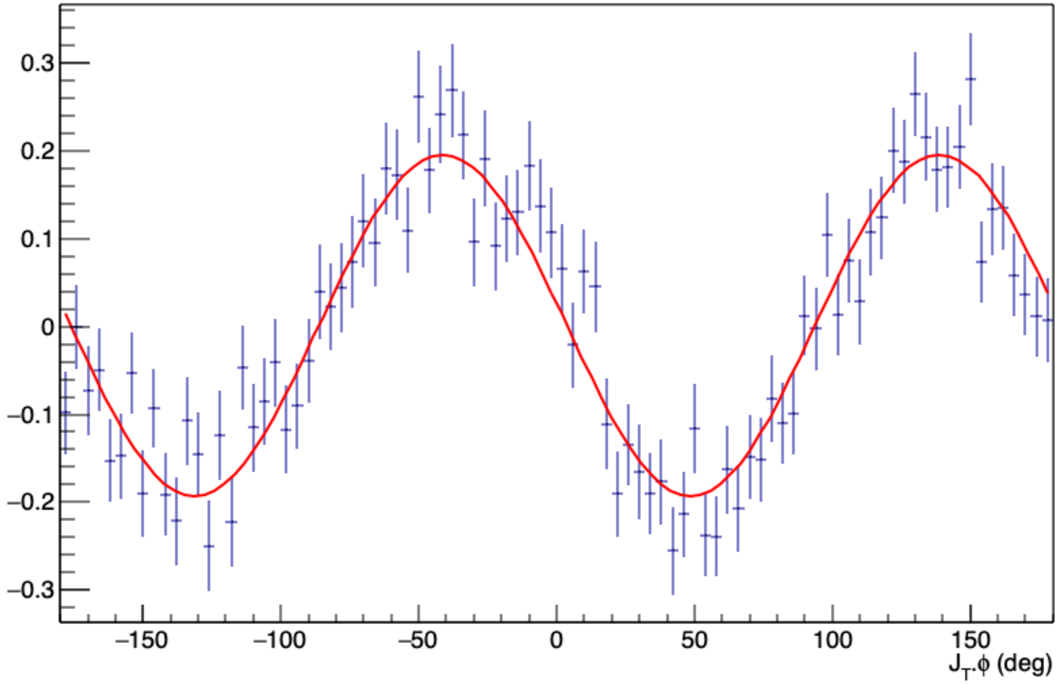
BH average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.337 \pm 0.011$



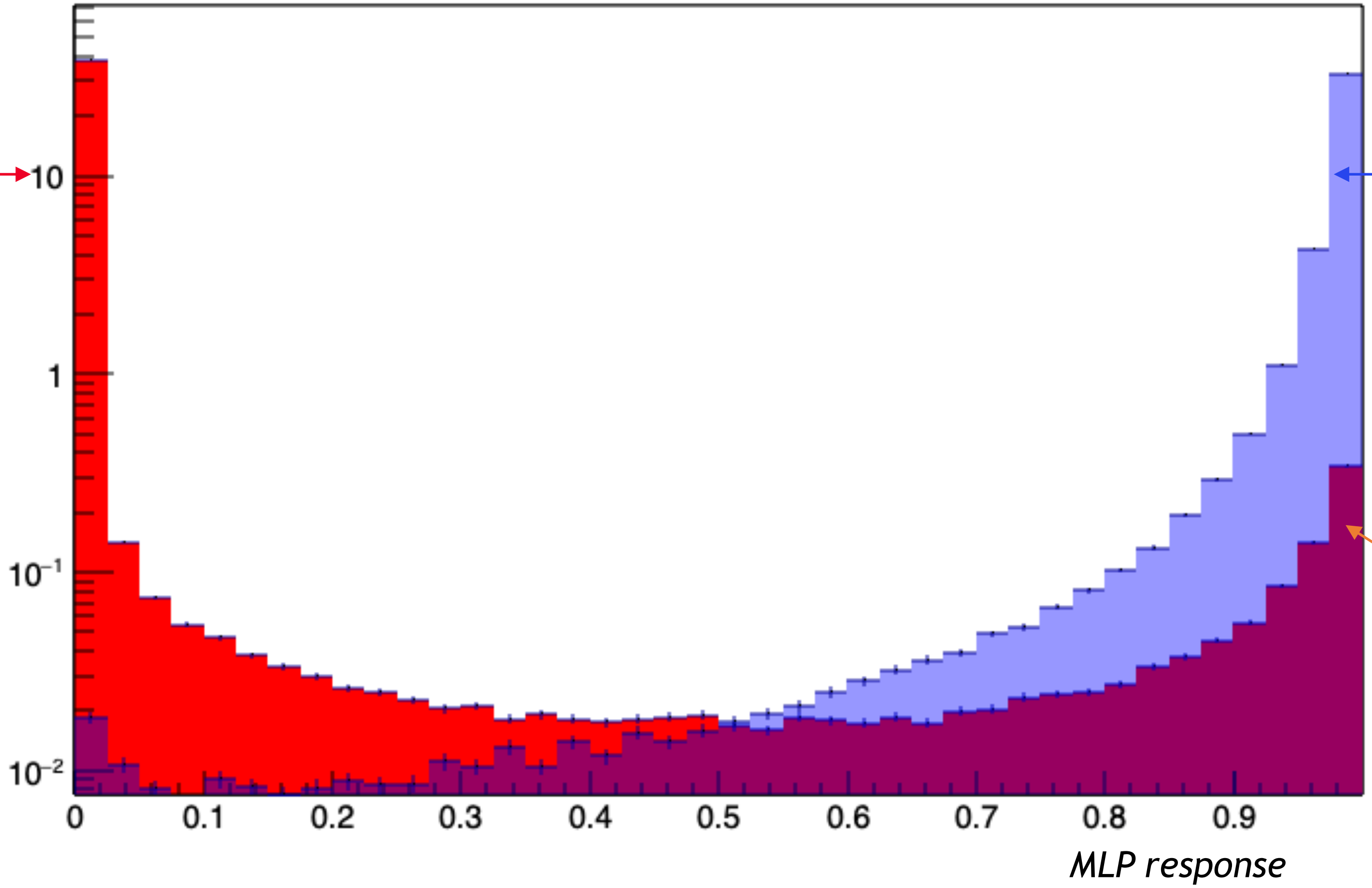
45 and 135

TPOL expected average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.342 \pm 0.005$

BH average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.332 \pm 0.013$



e-/π- MLP response from training samples



π- have low MLP response

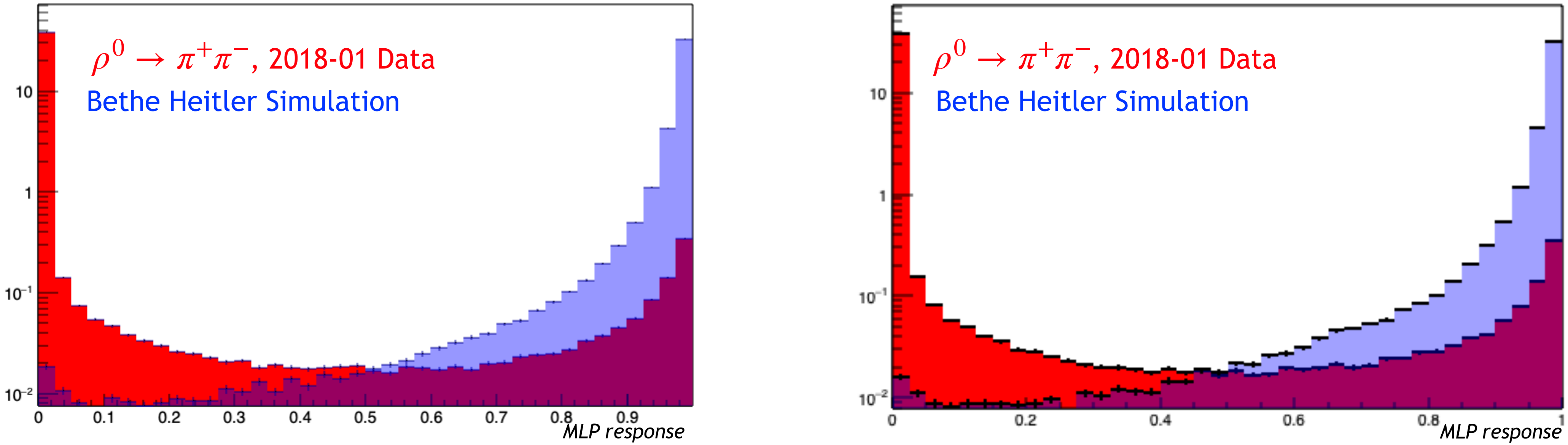
e- have high MLP response

A fraction of pions from the training sample will look like electrons in calorimeters due to charge exchange reaction, and $\rho^0 \rightarrow e^+e^-$

Two neural nets, one for classifying the positive track, one for the negative track

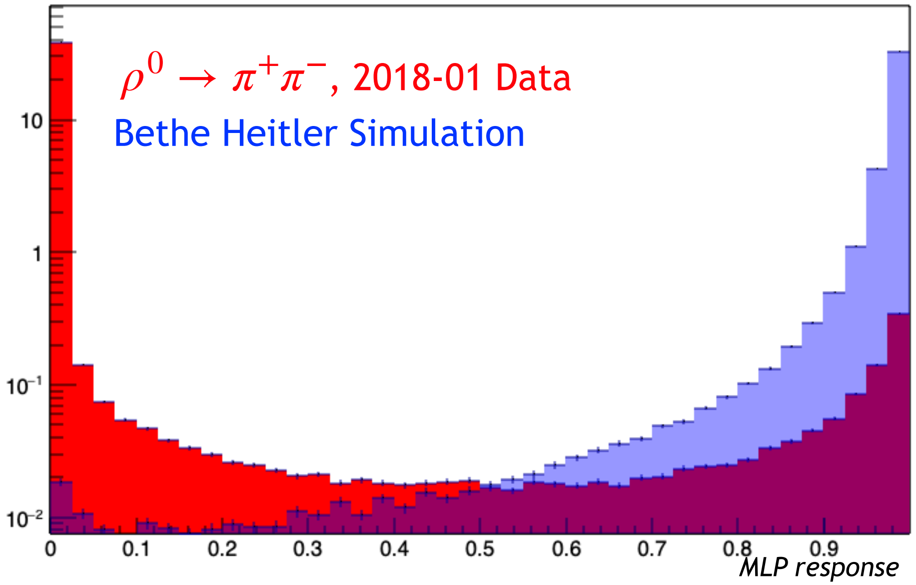
e-/π- MLP response from training

e+/π+ MLP response from training

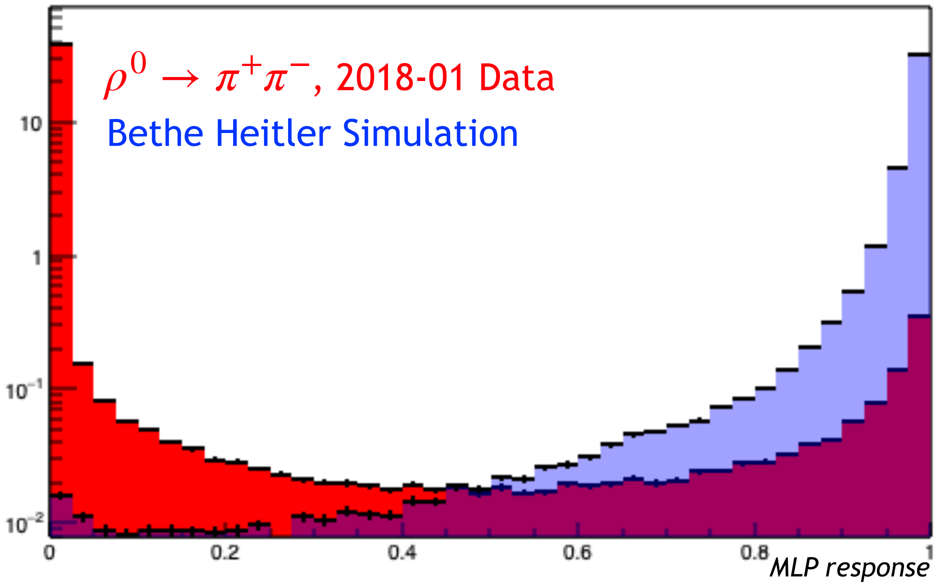


Two neural nets, one for classifying the positive track, one for the negative track

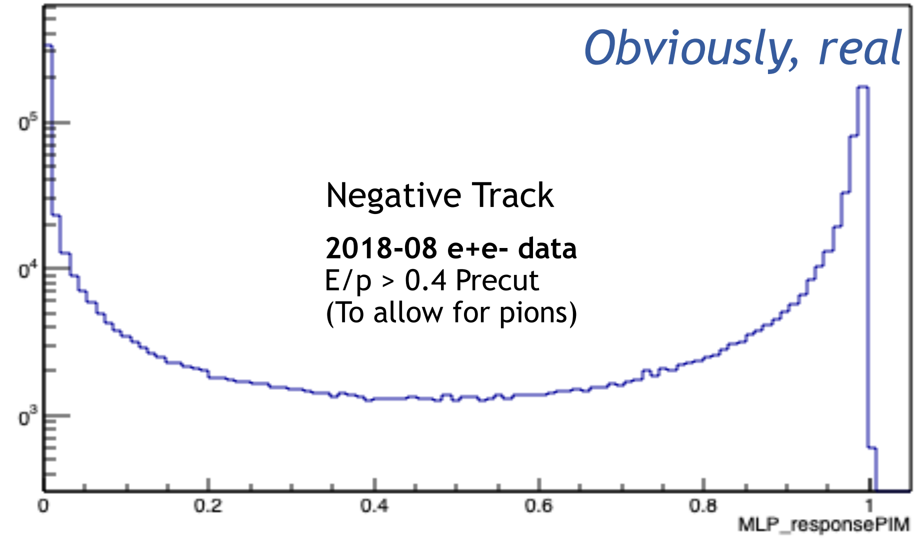
e-/π- MLP response from training



e+/π+ MLP response from training

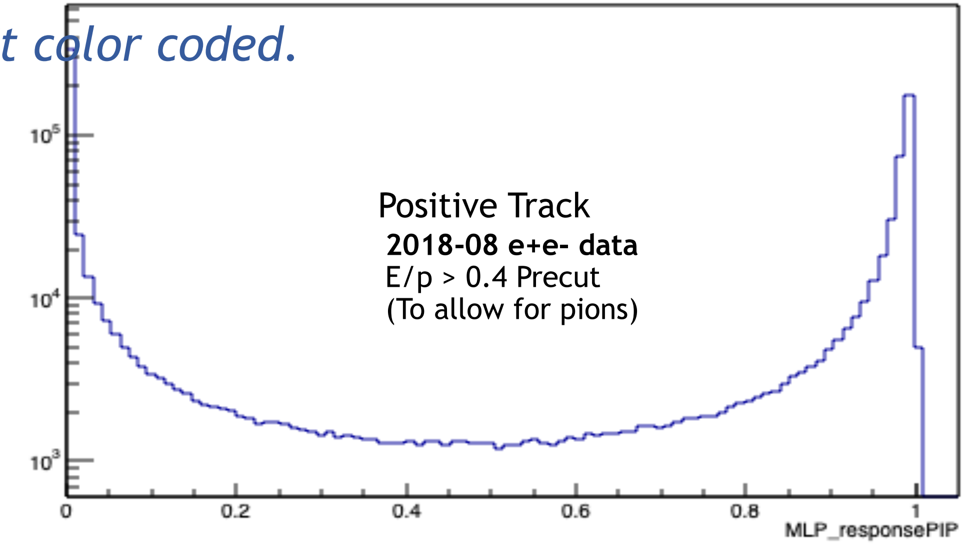


Obviously, real data isn't color coded.



Negative Track

2018-08 e+e- data
E/p > 0.4 Precut
(To allow for pions)



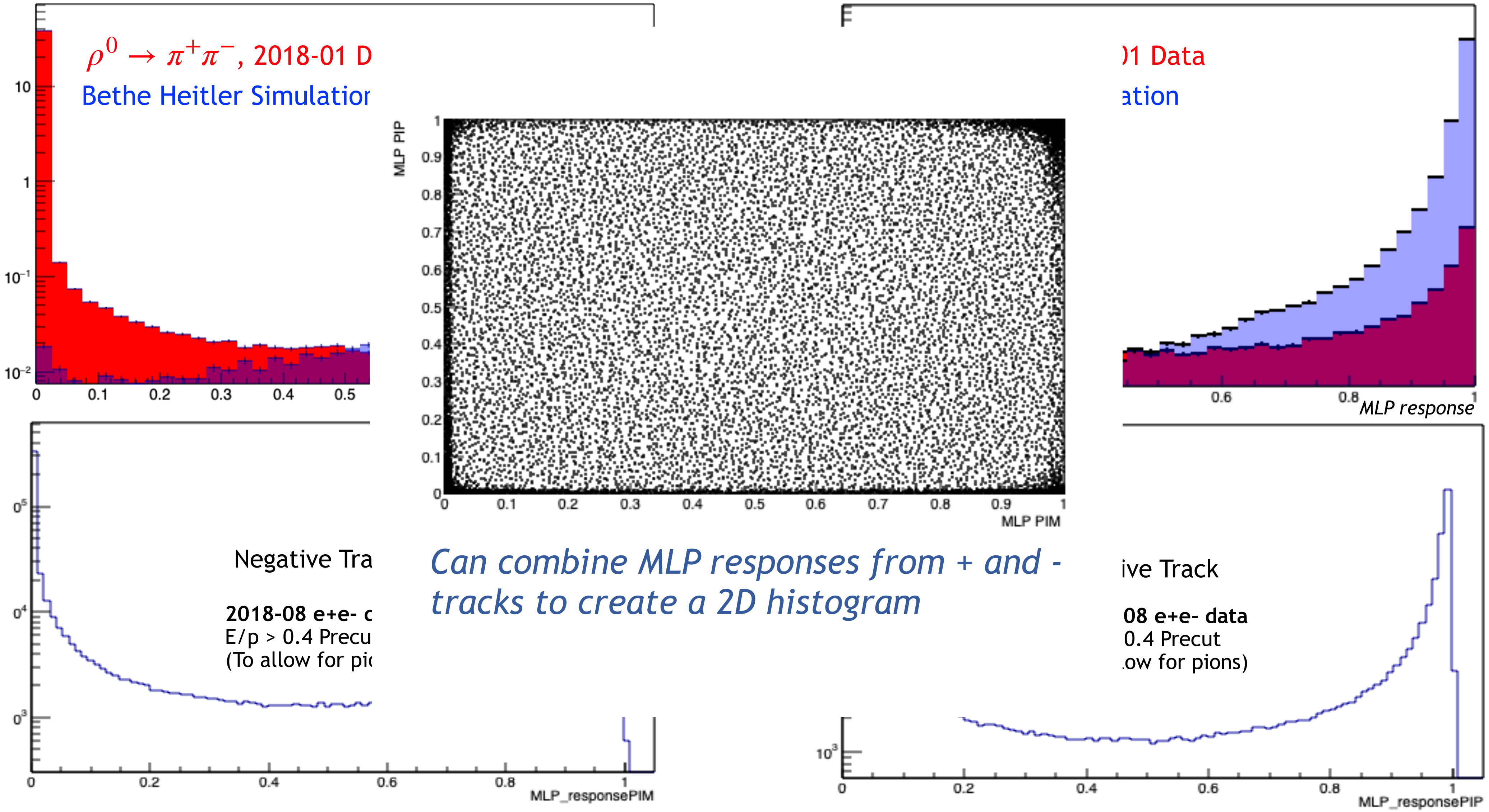
Positive Track

2018-08 e+e- data
E/p > 0.4 Precut
(To allow for pions)

Two neural nets, one for classifying the positive track, one for the negative track

e-/π- MLP response from training

e+/π+ MLP response from training

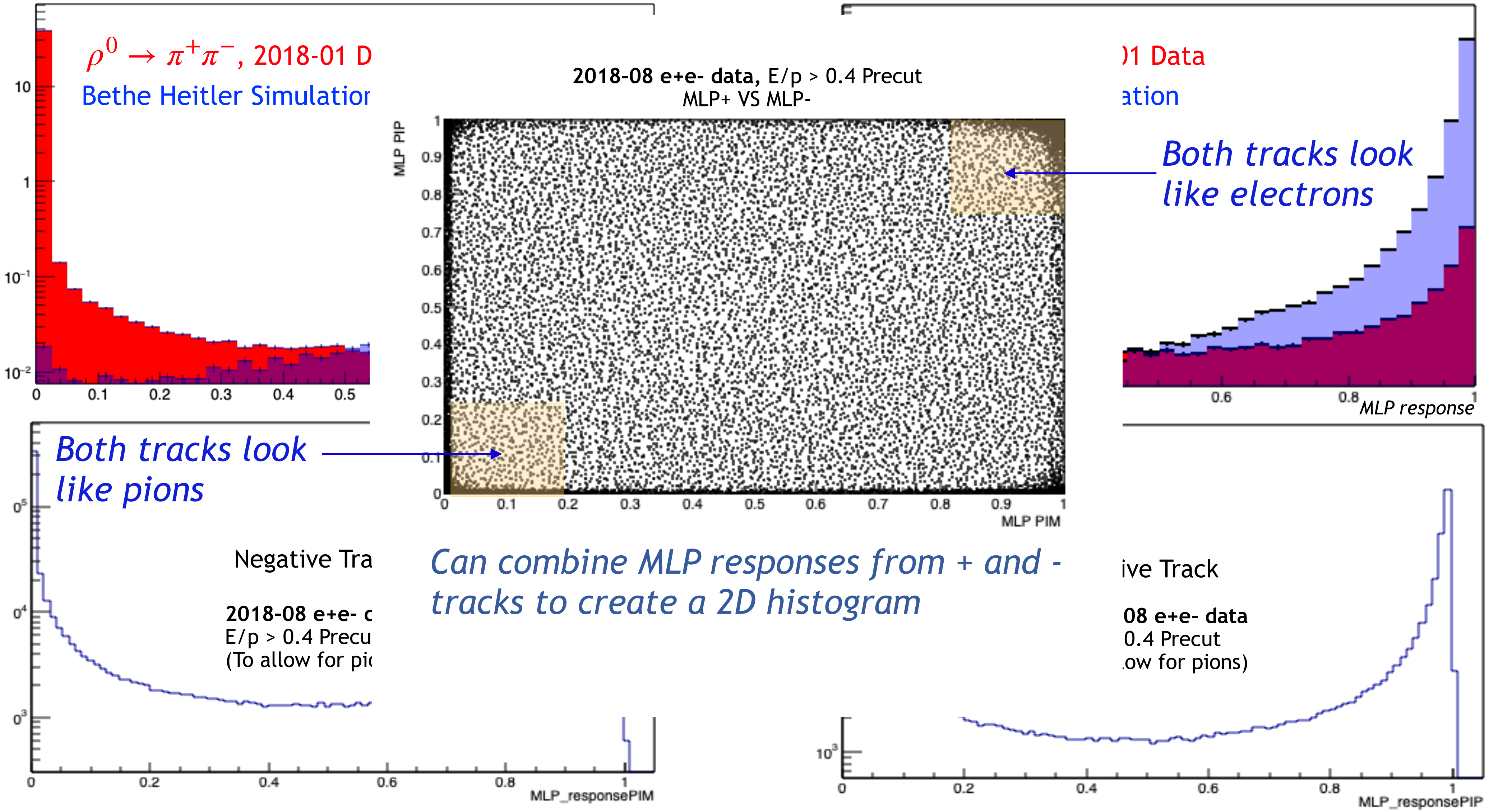


Can combine MLP responses from + and - tracks to create a 2D histogram

Two neural nets, one for classifying the positive track, one for the negative track

e-/π- MLP response from training

e+/π+ MLP response from training



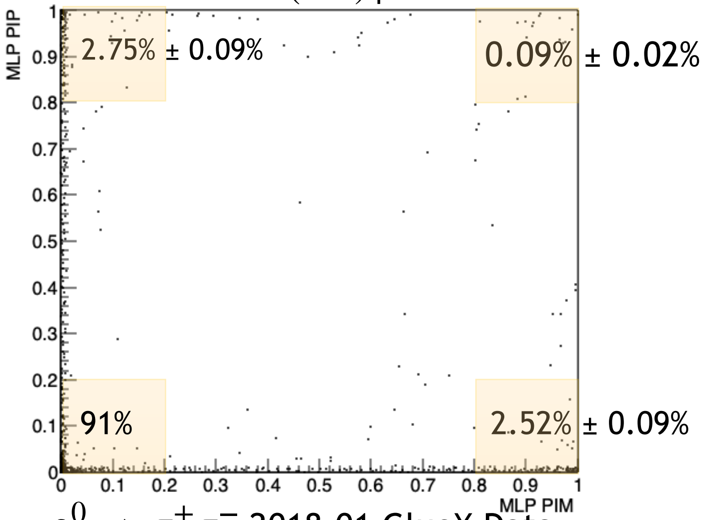
Both tracks look like electrons

Both tracks look like pions

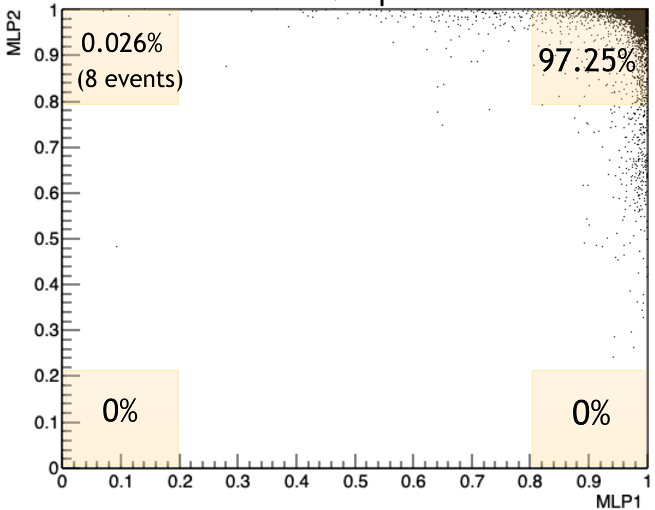
Can combine MLP responses from + and - tracks to create a 2D histogram

What Box Plots look like for pure samples

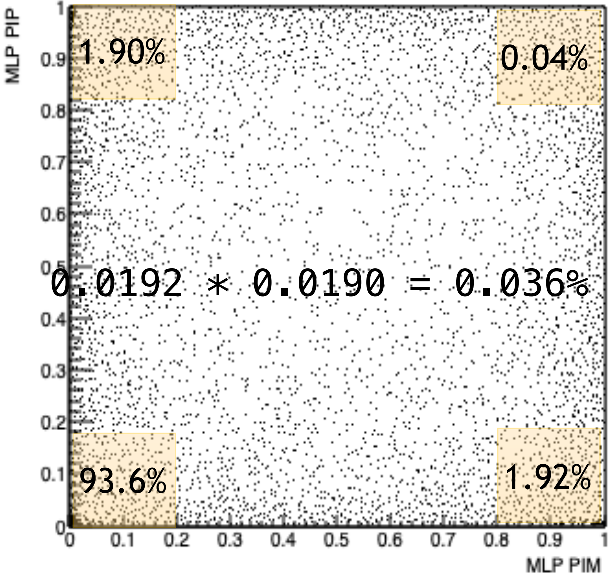
$\Omega(782) \rightarrow \pi^+\pi^-\pi^0$, 2018-01 GlueX Data,
Cut on $\Omega(782)$ peak



$\pi^0 \rightarrow \gamma e^+e^-$, 2018-01 GlueX Data,
cut on π^0 peak

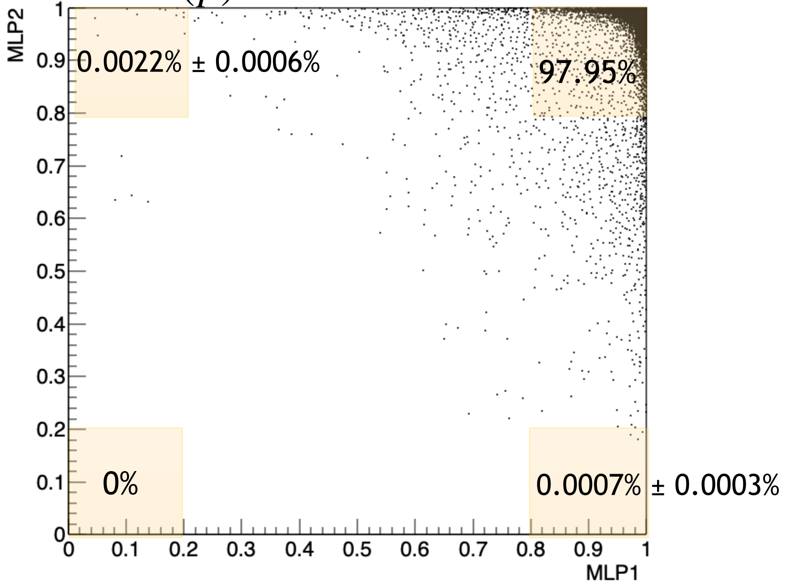


$\rho^0 \rightarrow \pi^+\pi^-$ 2018-01 GlueX Data



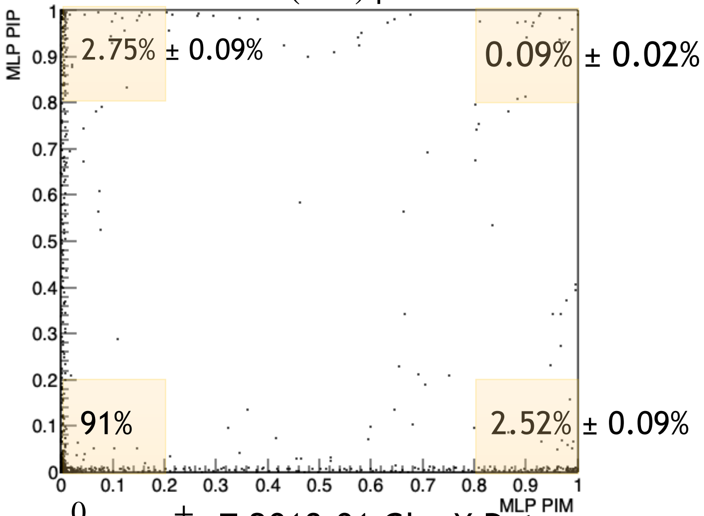
0.0192 * 0.0190 = 0.036%

$e^+e^-(p)$ Bethe Heitler Simulation

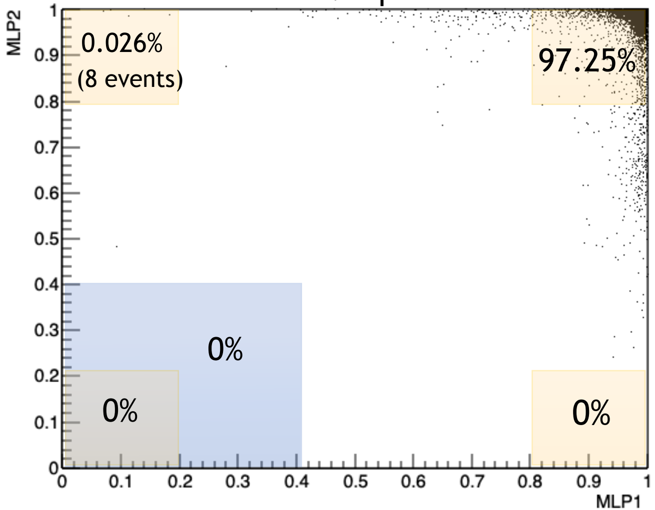


What Box Plots look like for pure samples

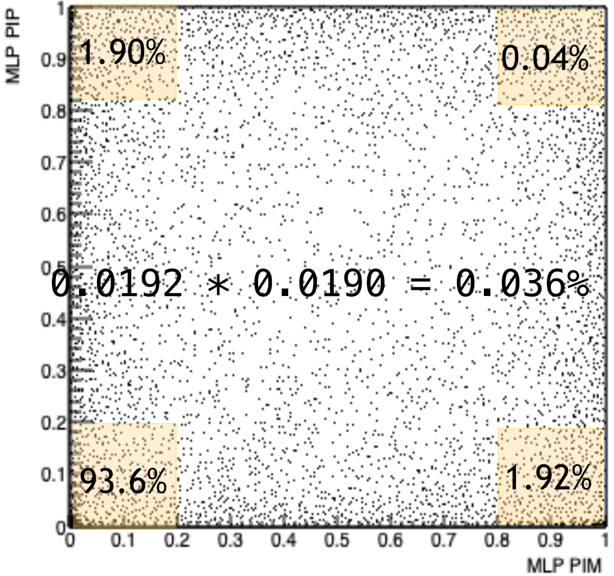
$\Omega(782) \rightarrow \pi^+\pi^-\pi^0$, 2018-01 GlueX Data,
Cut on $\Omega(782)$ peak



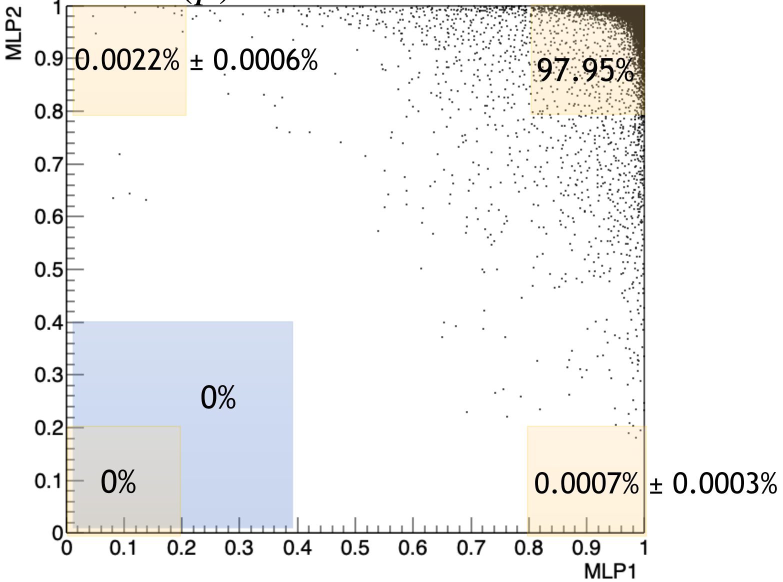
$\pi^0 \rightarrow \gamma e^+e^-$, 2018-01 GlueX Data,
cut on π^0 peak



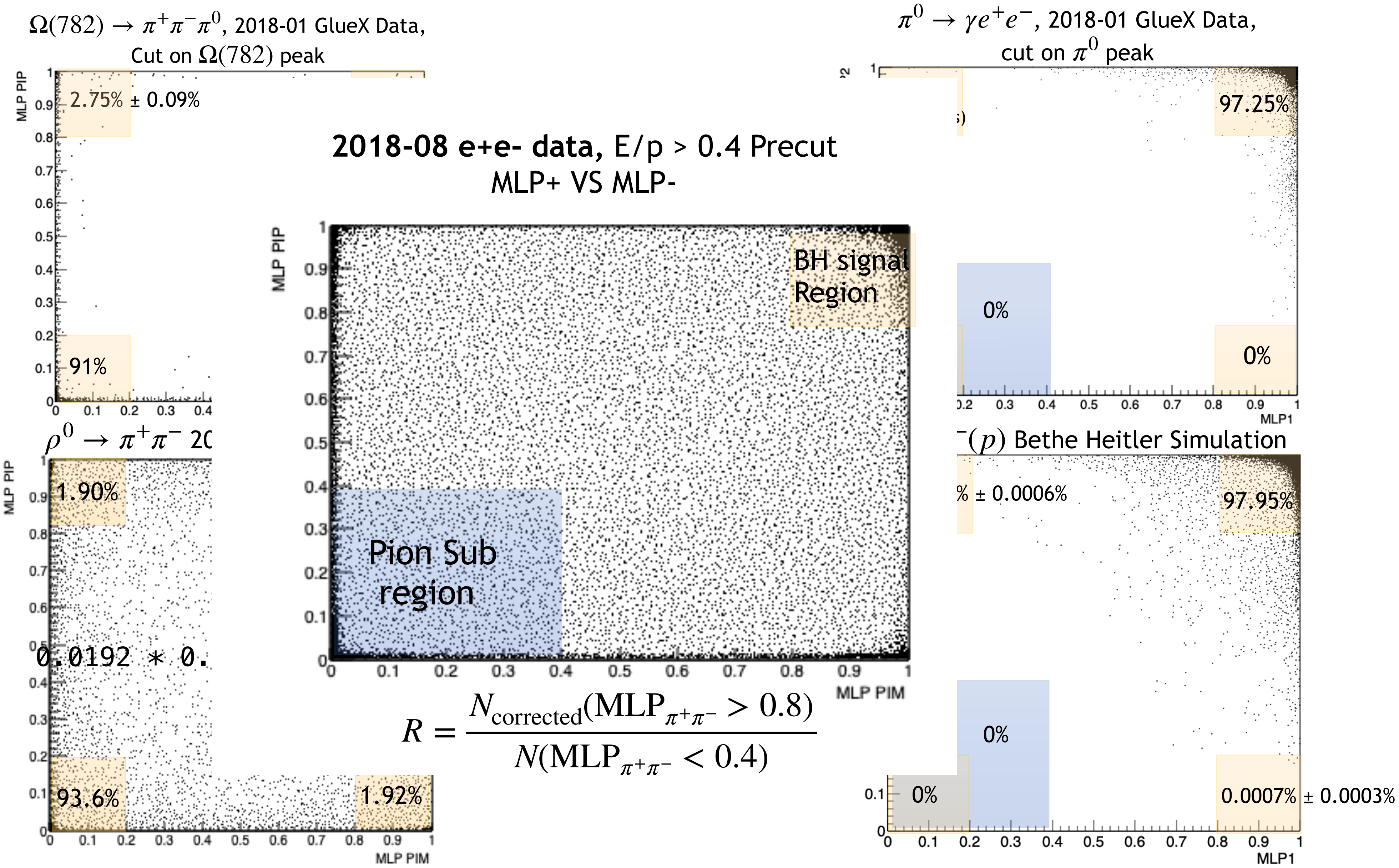
$\rho^0 \rightarrow \pi^+\pi^-$ 2018-01 GlueX Data



$e^+e^-(p)$ Bethe Heitler Simulation

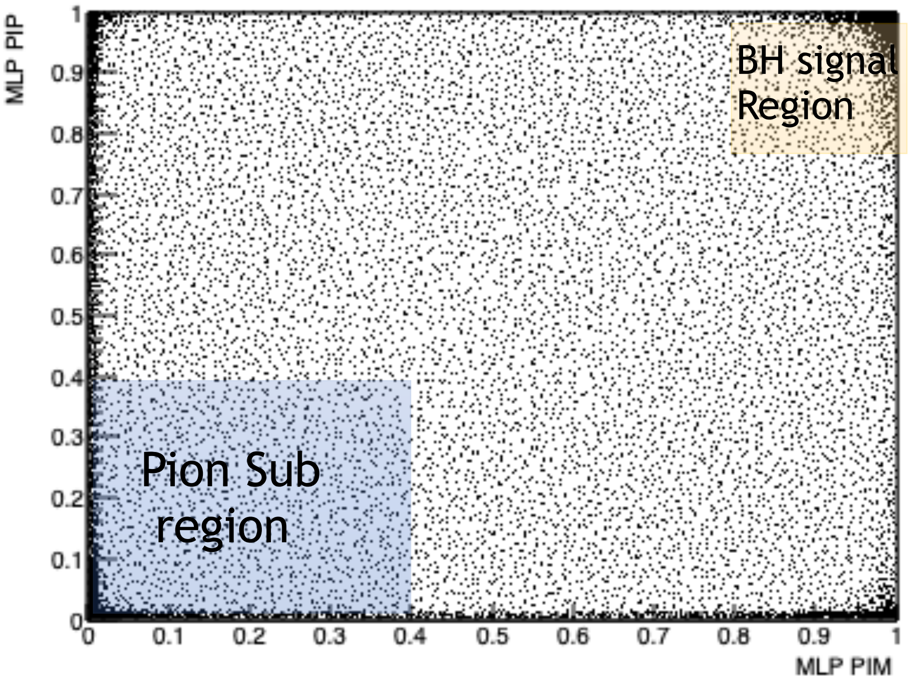


What Box Plots look like for pure samples

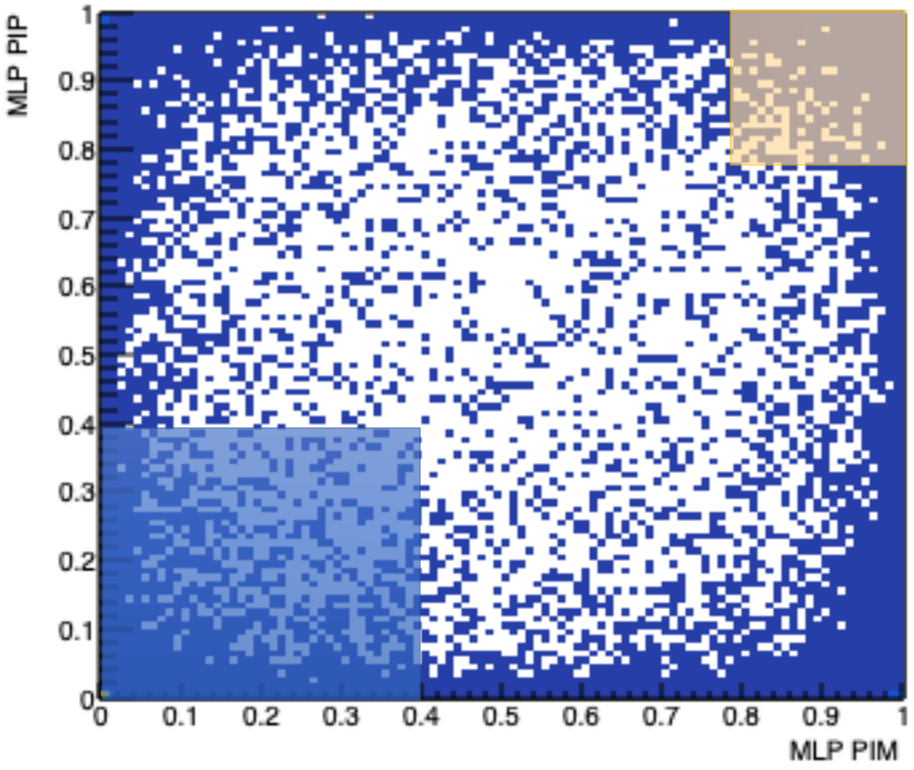


$$R = \frac{N_{\text{corrected}}(\text{MLP}_{\pi^+\pi^-} > 0.8)}{N(\text{MLP}_{\pi^+\pi^-} < 0.4)}$$

2018-08 e+e-(p) data, E/p > 0.4 Precut
MLP+ VS MLP-



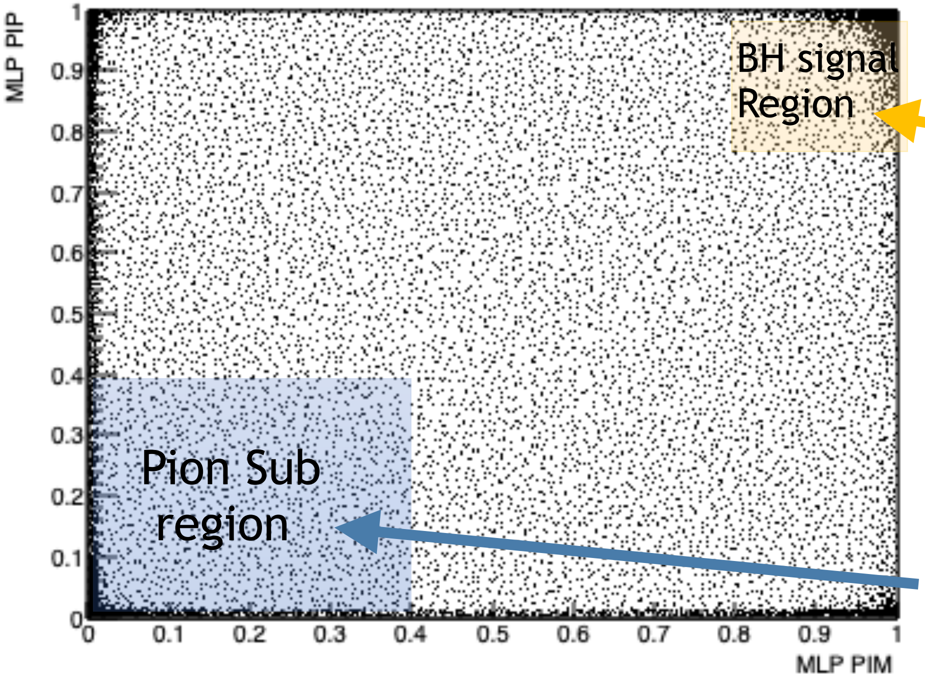
2018-08 π+π- data, E/p > 0.4 Precut
MLP+ VS MLP-



$$R = \frac{N_{\text{corrected}}(\text{MLP}_{\pi^+\pi^-} > 0.8)}{N(\text{MLP}_{\pi^+\pi^-} < 0.4)} = 0.07317$$

$$N_{\text{corrected}}(\text{MLP}_{\pi^+\pi^-} > 0.8) = N(\text{MLP}_{\pi^+\pi^-} > 0.8) - N_{\rho^0}\, f_{\text{BHsim}}\, \Gamma_{\rho^0 \to e^+e^-}$$

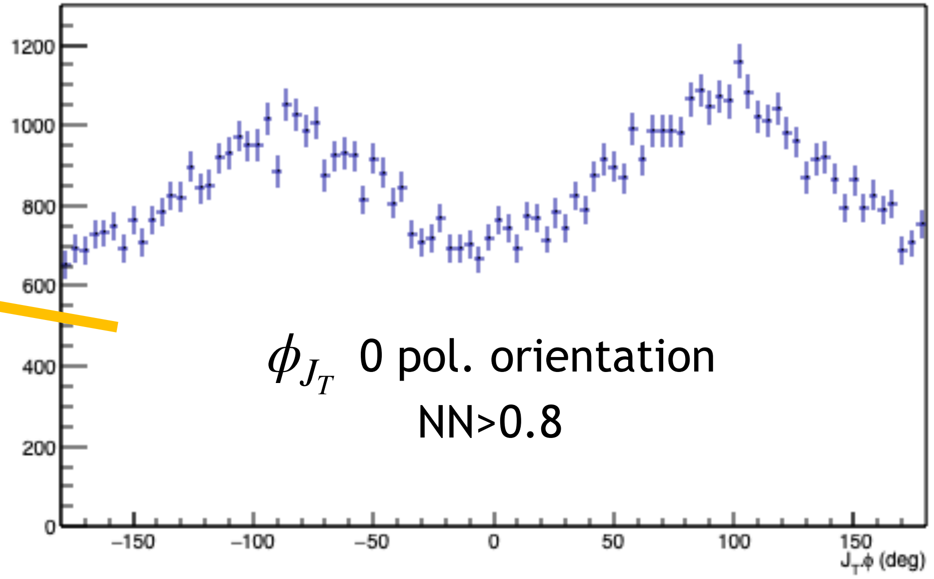
2018-08 e+e-(p) data, E/p > 0.4 Precut
MLP+ VS MLP-



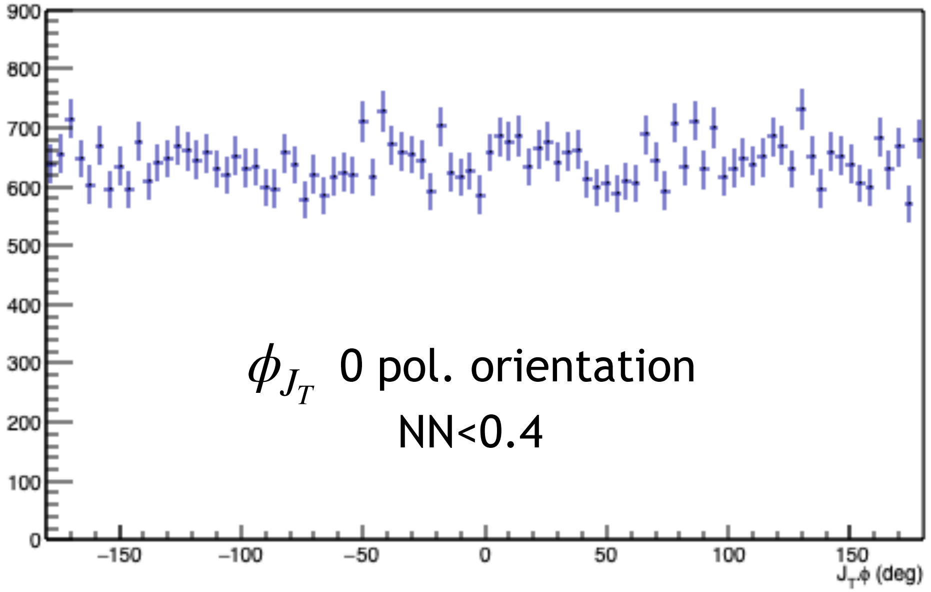
BH signal Region

Pion Sub region

$$R = \frac{N_{\text{corrected}}(\text{MLP}_{\pi^+\pi^-} > 0.8)}{N(\text{MLP}_{\pi^+\pi^-} < 0.4)} = 0.07317$$



ϕ_{J_T} 0 pol. orientation
NN>0.8



ϕ_{J_T} 0 pol. orientation
NN<0.4

ata, E/p > 0.4 Precut
+ VS MLP-

$.8) - N_{\rho^0} f_{\text{BHsim}} \Gamma_{\rho^0 \to e^+e^-}$

SUMMARY

2018-01 Spring GlueX data, Average Polarization

0 and 90

TPOL expected average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.341 \pm 0.004$

BH average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.342 \pm 0.009$

45 and 135

TPOL expected average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.344 \pm 0.004$

BH average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.336 \pm 0.009$

2018-08 Fall GlueX data, Average Polarization

0 and 90

TPOL expected average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.345 \pm 0.005$

BH average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.337 \pm 0.011$

45 and 135

TPOL expected average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.342 \pm 0.005$

BH average polarization: $\frac{\mathscr{P}_\perp + \mathscr{P}_\parallel}{2} = 0.332 \pm 0.013$

In-depth systematics study in progress: expect at next BWG meeting

Backups

Rho0 File.
700 MeV < W < 770 MeV
Theta > 1.5 Deg
FCAL E > 0
TOF dEdx > 0

1511050.9

$N_\rho = 1597963 =$ # of $\pi^+\pi^-$ pairs

W/ Electron Cuts (E/p > 0.4)

$$N(\text{MLP}_{\pi^+\pi^-} > 0.8) = 709$$

$$N(\text{MLP}_{\pi^+\pi^-} < 0.4) = 8680$$

$$f_{\text{BHsim}} = .9795$$

$$\Gamma_{\rho^0 \to e^+e^-} = 0.00005$$

$4.72 \pm 0.05 \times 10^{-5} = 0.0000472$

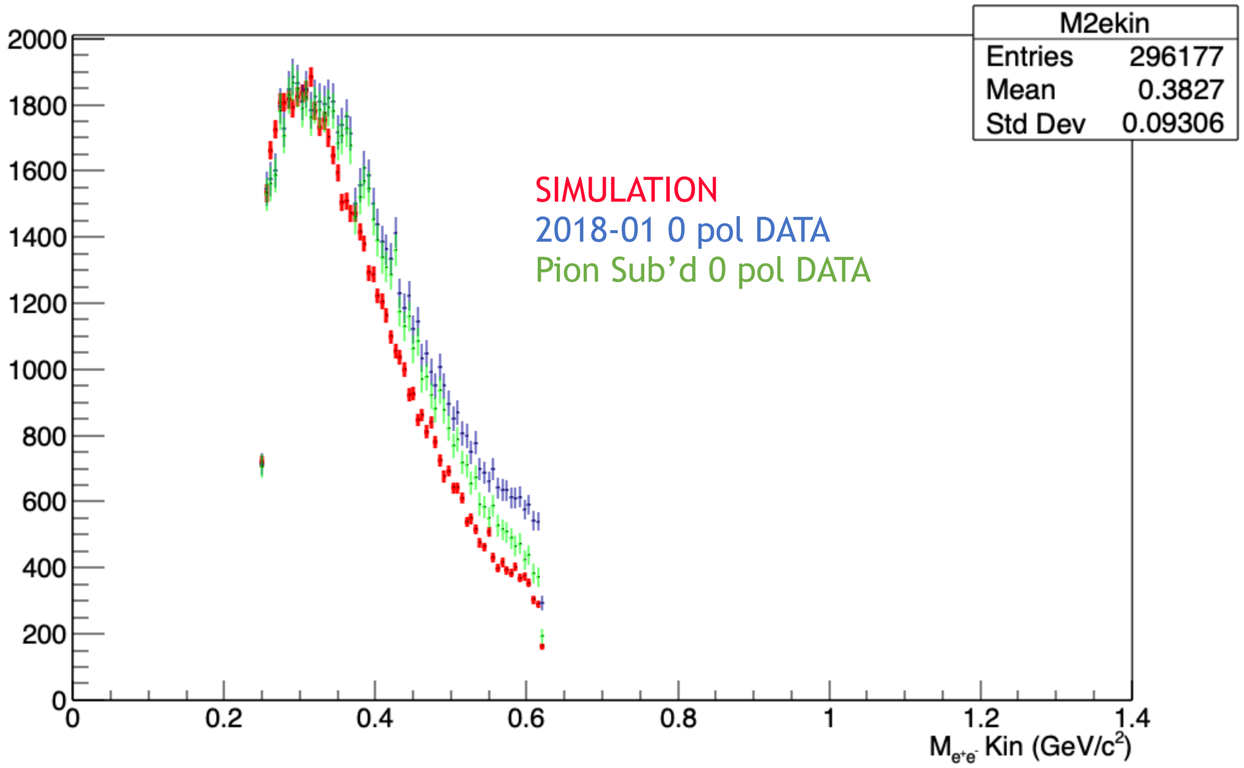
$$N_{\text{corrected}}(\text{MLP}_{\pi^+\pi^-} > 0.8) = N(\text{MLP}_{\pi^+\pi^-} > 0.8) - N_{\rho^0}\, f_{\text{BH}}\, \Gamma_{\rho^0 \to e^+e^-} = 635$$

$$R = \frac{N_{\text{corrected}}(\text{MLP}_{\pi^+\pi^-} > 0.8)}{N(\text{MLP}_{\pi^+\pi^-} < 0.4)} = 0.07317$$

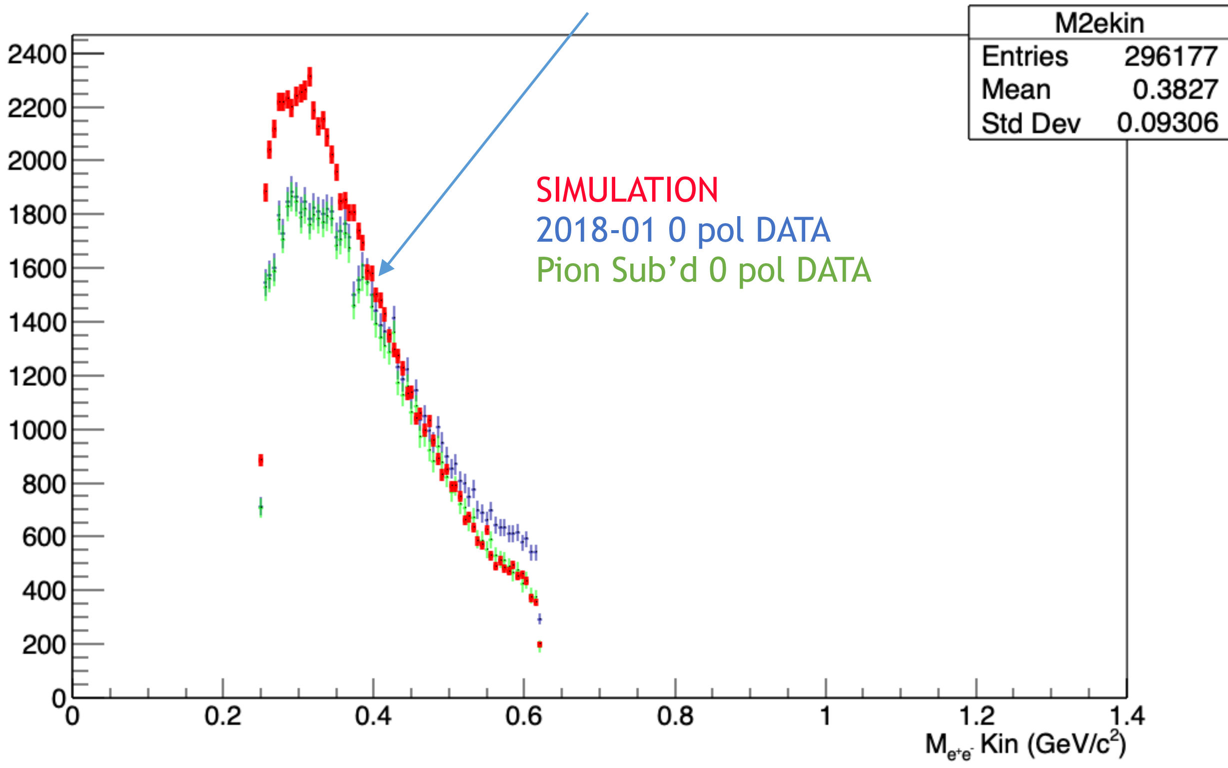
INVARIANT MASS

2018-01 GlueX Data Set, $\gamma p \rightarrow e^+e^-(p)$

MC Scaled to Data Maximum

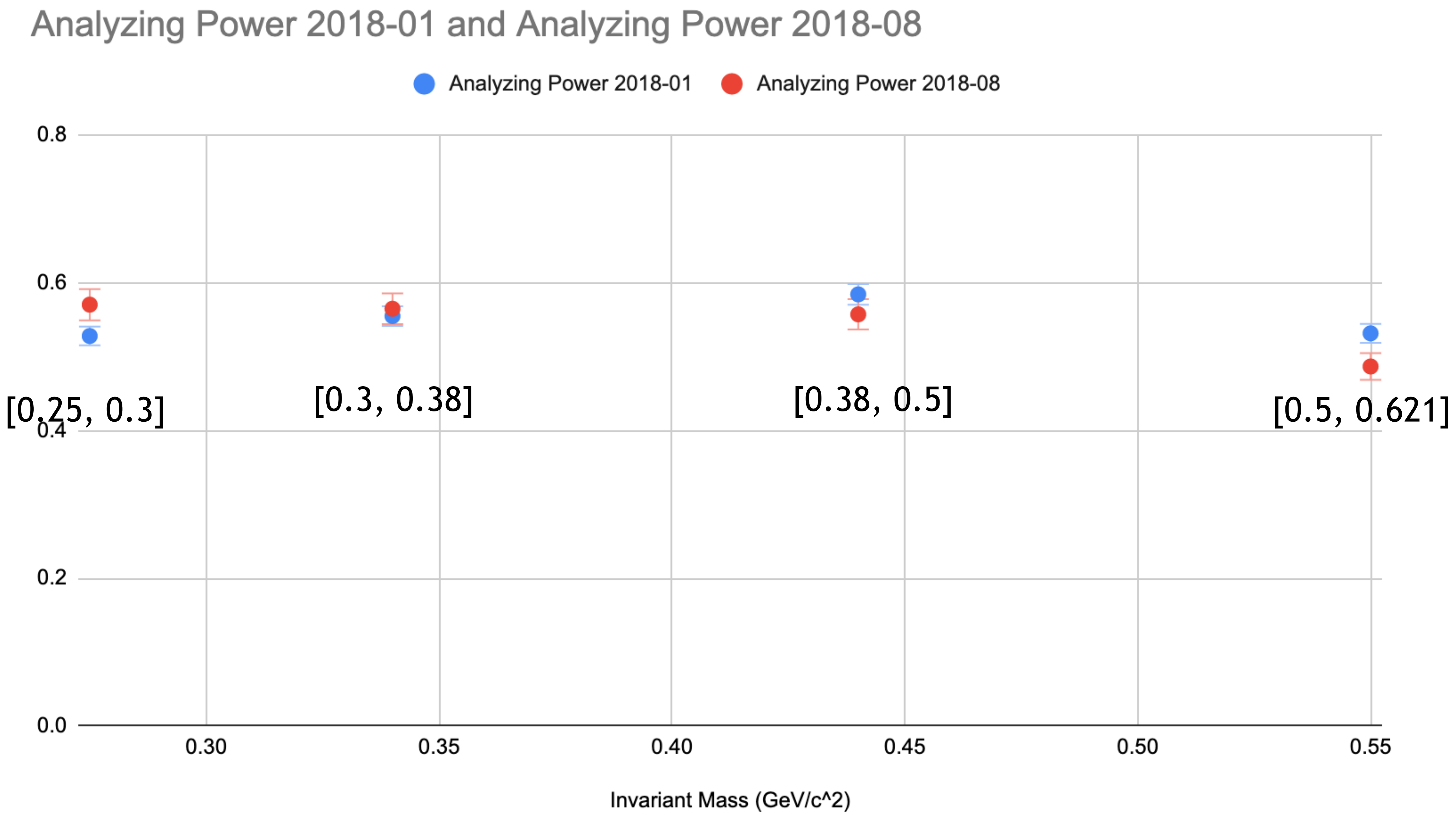


MC Scaled to bin 67



$$\text{Pion contamination} = 1 - \frac{\int W^{BH}_{\pi\text{sub}} dn}{\int W^{BH} dn} = 0.054$$

5.4% contamination



[0.25, 0.3]

[0.3, 0.38]

[0.38, 0.5]

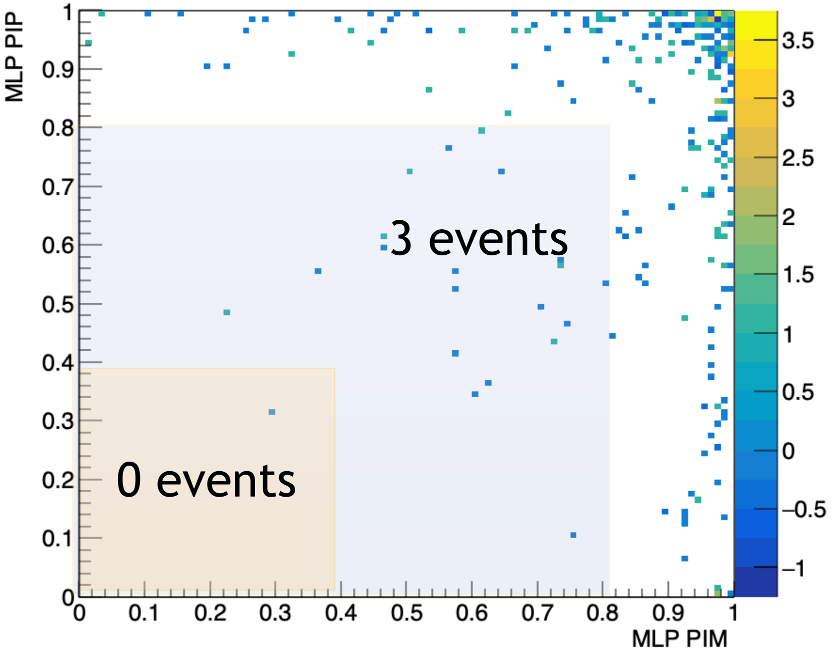
[0.5, 0.621]

Want to take the ratio of Pion Sub ratio

Three 0-polarization orientation runs

ρ^0 exclusive channel
FCAL Energy > 0
TOF dE/dx > 0
52cm < Vertex Z < 78 cm
Lab theta > 1.5 deg
700 MeV < W < 770 MeV

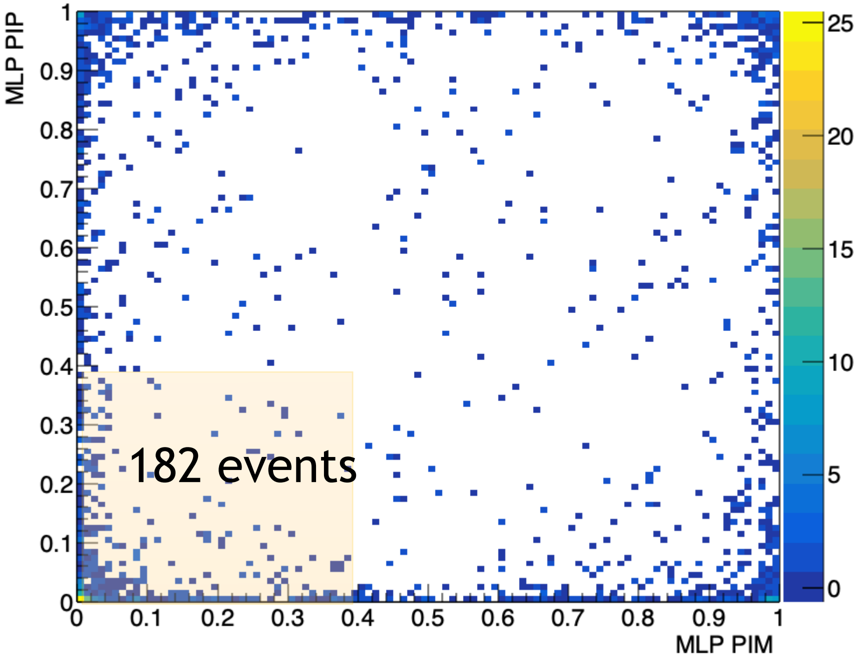
$$\frac{E_{\mathrm{FCAL}}}{p_{\mathrm{meas}}} > 0.7$$



3 events

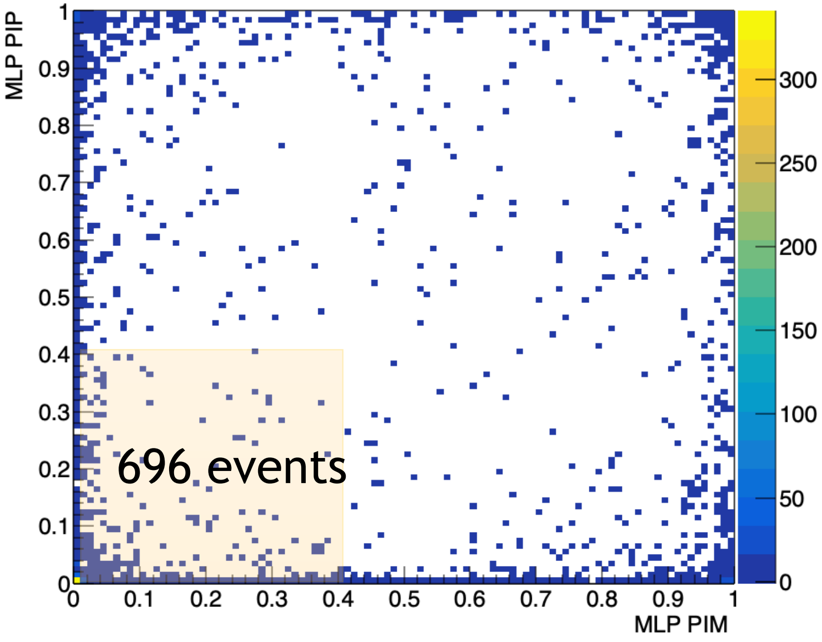
0 events

$$\frac{E_{\mathrm{FCAL}}}{p_{\mathrm{meas}}} > 0.5$$



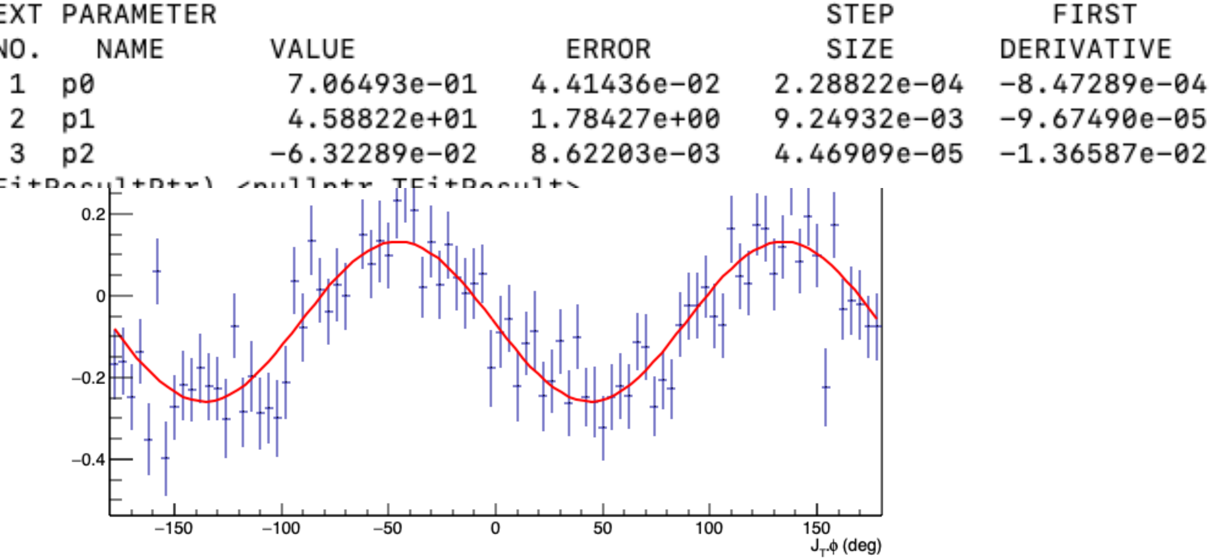
182 events

$$\frac{E_{\mathrm{FCAL}}}{p_{\mathrm{meas}}} > 0.4$$

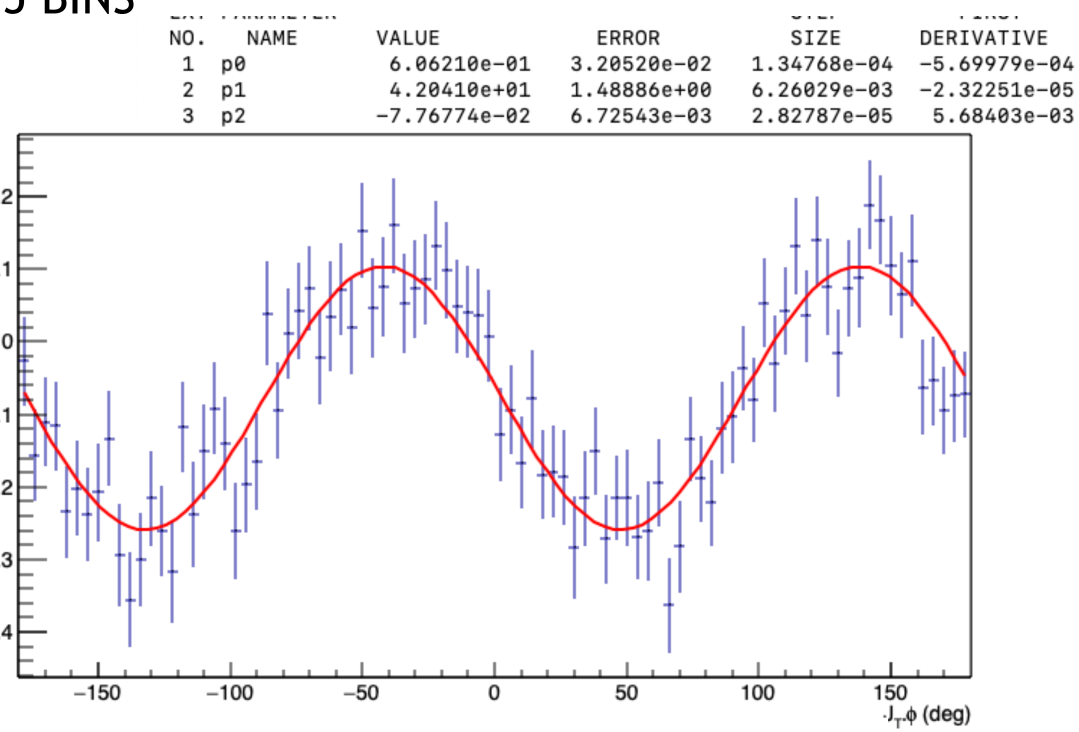


696 events

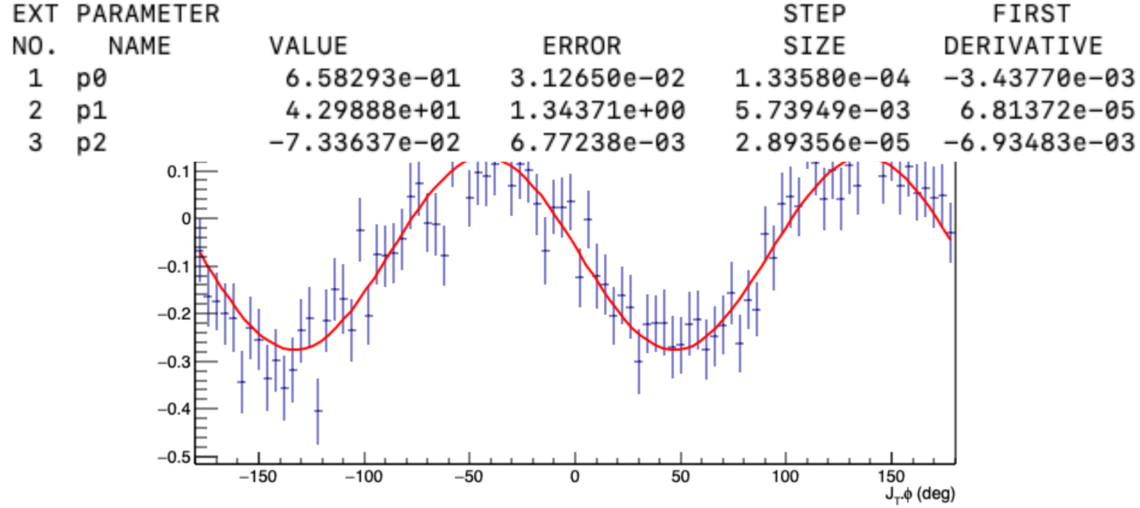
45 and 135 BINS



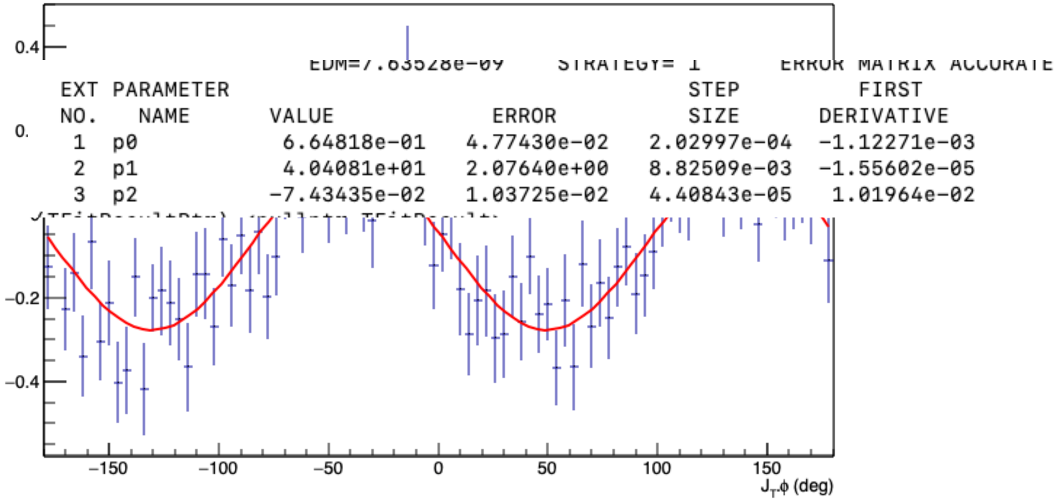
```
EXT PARAMETER                                    STEP         FIRST
NO.   NAME        VALUE           ERROR          SIZE         DERIVATIVE
 1  p0            7.06493e-01   4.41436e-02   2.28822e-04   -8.47289e-04
 2  p1            4.58822e+01   1.78427e+00   9.24932e-03   -9.67490e-05
 3  p2           -6.32289e-02   8.62203e-03   4.46909e-05   -1.36587e-02
```



```
NO.   NAME       VALUE          ERROR          SIZE           DERIVATIVE
 1  p0          6.06210e-01   3.20520e-02   1.34768e-04   -5.69979e-04
 2  p1          4.20410e+01   1.48886e+00   6.26029e-03   -2.32251e-05
 3  p2         -7.76774e-02   6.72543e-03   2.82787e-05    5.68403e-03
```



```
EXT PARAMETER                                 STEP          FIRST
NO.   NAME        VALUE          ERROR          SIZE          DERIVATIVE
 1  p0            6.58293e-01  3.12650e-02  1.33580e-04  -3.43770e-03
 2  p1            4.29888e+01  1.34371e+00  5.73949e-03   6.81372e-05
 3  p2           -7.33637e-02  6.77238e-03  2.89356e-05  -6.93483e-03
```

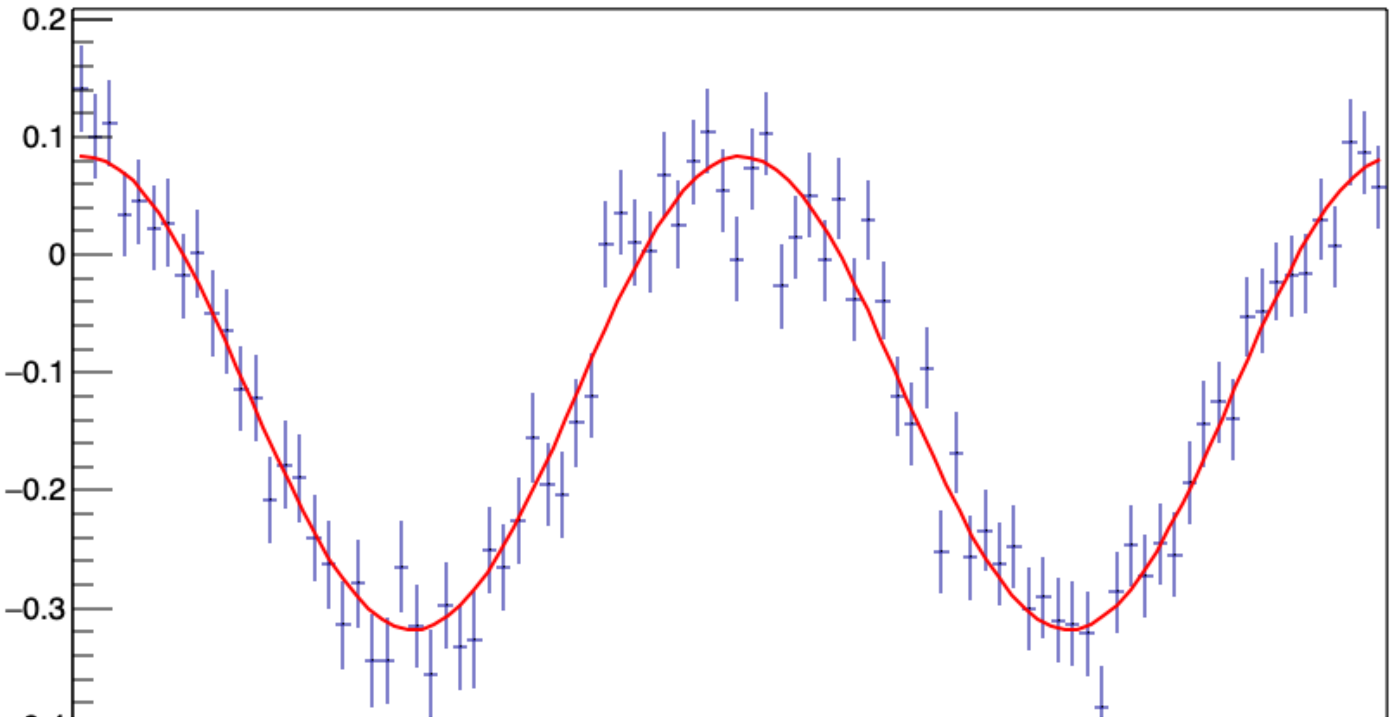


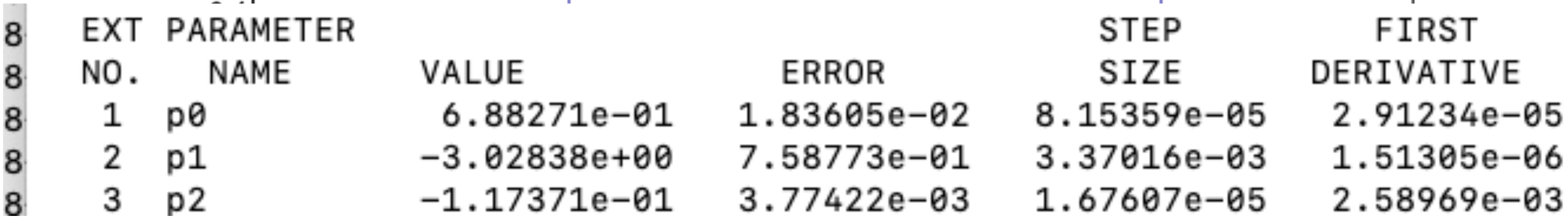
```
EDM=7.63528e-09    STRATEGY= 1      ERROR MATRIX ACCURATE
EXT PARAMETER                                  STEP          FIRST
NO.   NAME       VALUE          ERROR          SIZE          DERIVATIVE
 1  p0           6.64818e-01   4.77430e-02   2.02997e-04   -1.12271e-03
 2  p1           4.04081e+01   2.07640e+00   8.82509e-03   -1.55602e-05
 3  p2          -7.43435e-02   1.03725e-02   4.40843e-05    1.01964e-02
```

```
0.34413550      0.0091802500
```

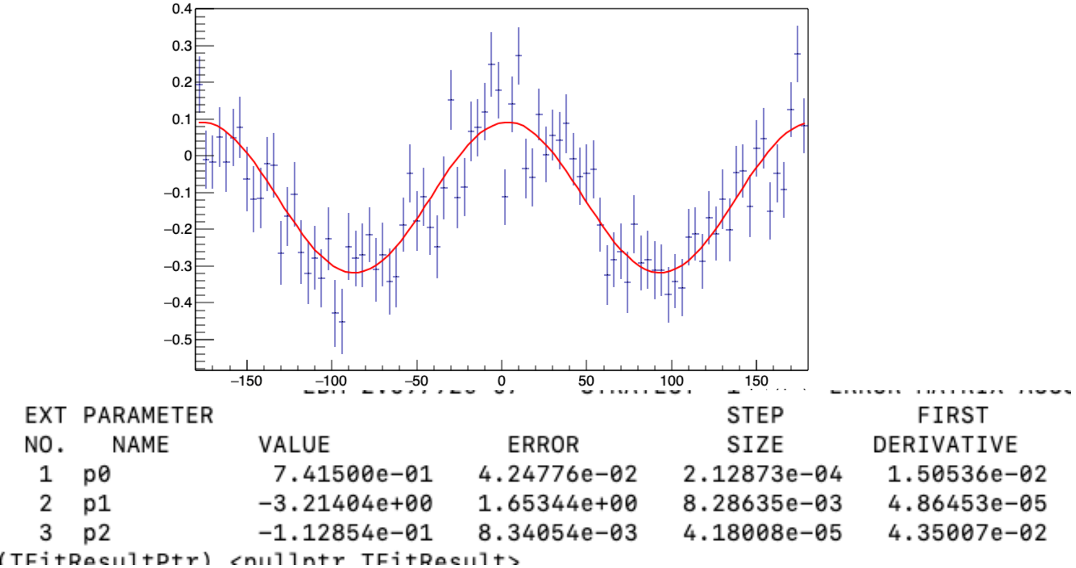
Integrated Result

0 and 90 2018-01 runs

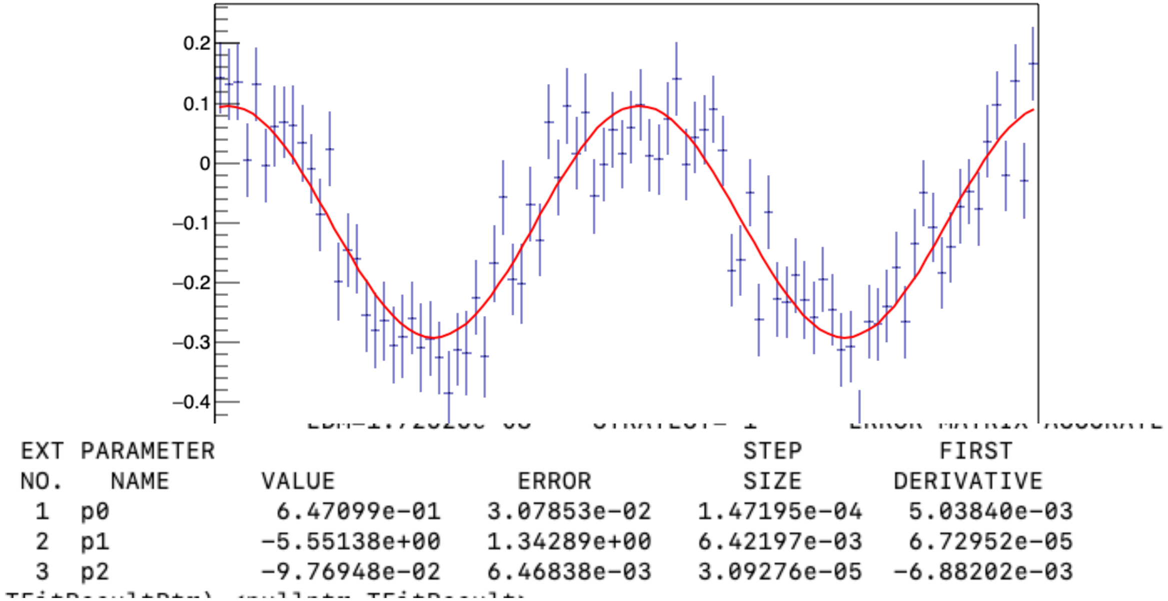




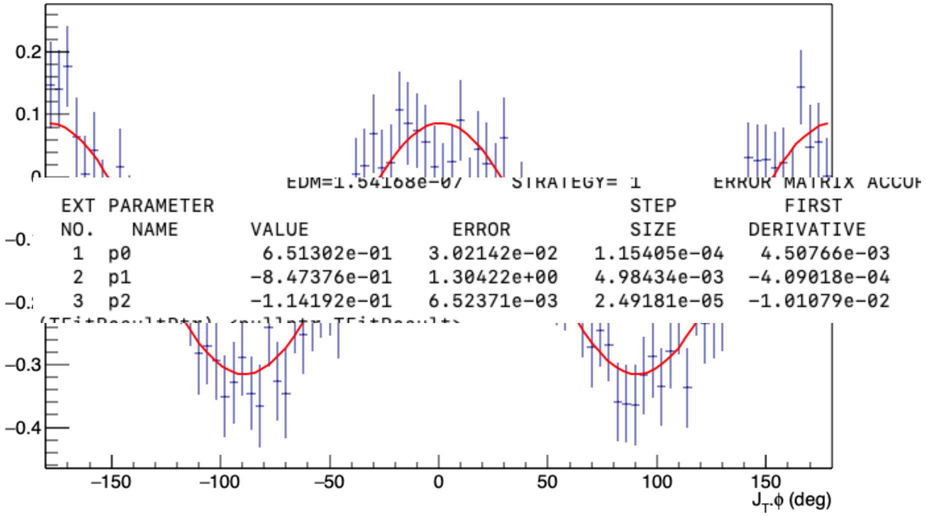
```
8   EXT PARAMETER                                   STEP         FIRST
8   NO.   NAME      VALUE            ERROR          SIZE      DERIVATIVE
8    1   p0          6.88271e-01   1.83605e-02   8.15359e-05   2.91234e-05
8    2   p1         -3.02838e+00   7.58773e-01   3.37016e-03   1.51305e-06
8    3   p2         -1.17371e-01   3.77422e-03   1.67607e-05   2.58969e-03
```



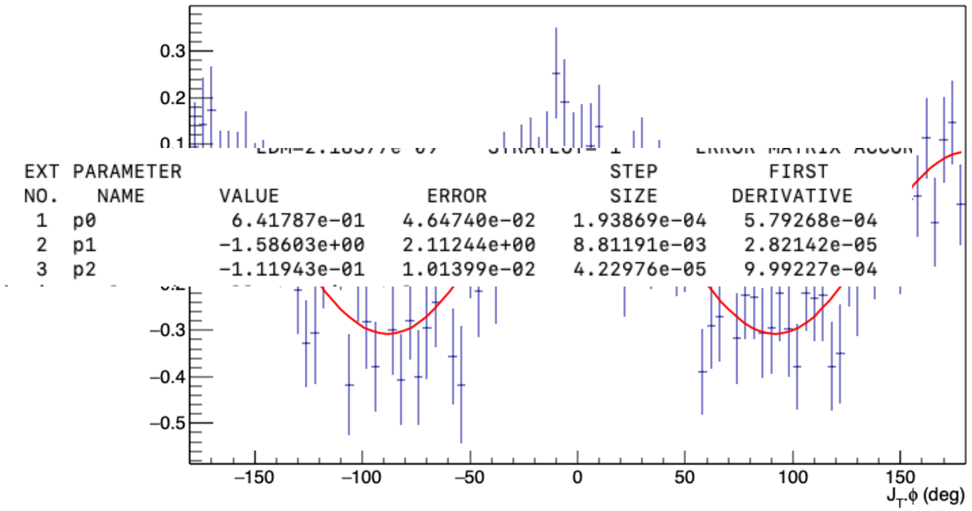
```
EXT PARAMETER                                   STEP         FIRST
NO.   NAME      VALUE            ERROR          SIZE         DERIVATIVE
 1  p0           7.41500e-01   4.24776e-02   2.12873e-04   1.50536e-02
 2  p1          -3.21404e+00   1.65344e+00   8.28635e-03   4.86453e-05
 3  p2          -1.12854e-01   8.34054e-03   4.18008e-05   4.35007e-02
(TFitResultPtr) <nullptr TFitResult>
```



```
EXT PARAMETER                                   STEP         FIRST
NO.   NAME      VALUE            ERROR          SIZE         DERIVATIVE
 1  p0           6.47099e-01   3.07853e-02   1.47195e-04   5.03840e-03
 2  p1          -5.55138e+00   1.34289e+00   6.42197e-03   6.72952e-05
 3  p2          -9.76948e-02   6.46838e-03   3.09276e-05  -6.88202e-03
```



```
                EDM=1.54168e-07    STRATEGY= 1      ERROR MATRIX ACCUR
EXT PARAMETER                                   STEP         FIRST
NO.   NAME      VALUE            ERROR          SIZE         DERIVATIVE
 1  p0           6.51302e-01   3.02142e-02   1.15405e-04   4.50766e-03
 2  p1          -8.47376e-01   1.30422e+00   4.98434e-03  -4.09018e-04
 3  p2          -1.14192e-01   6.52371e-03   2.49181e-05  -1.01079e-02
```



```
EXT PARAMETER                                   STEP         FIRST
NO.   NAME      VALUE            ERROR          SIZE         DERIVATIVE
 1  p0           6.41787e-01   4.64740e-02   1.93869e-04   5.79268e-04
 2  p1          -1.58603e+00   2.11244e+00   8.81191e-03   2.82142e-05
 3  p2          -1.11943e-01   1.01399e-02   4.22976e-05   9.99227e-04
```