$$
P_{\sigma}=\frac{\sigma^{N}-\sigma^{U}}{\sigma^{N}+\sigma^{U}} .
$$

At high energies

$$
\begin{equation*}
P_{\sigma}=2 \rho_{1-1}^{1}-\rho_{00}^{1} . \tag{4}
\end{equation*}
$$

Note that $P_{\sigma}$ is invariant under rotations around the normal to the production plane; e.g., it is the same in the three systems described above. We also point out that $P_{\sigma}$ is sensitive to possible $\rho^{0}$ helicity or spin-flip terms (contributing to $\rho_{00}^{1}$ ) which are not usually measured in counter experiments. Counter experiments of the type of Refs. 40 and 41 measure the asymmetry $\Sigma$ defined as

$$
\begin{equation*}
\Sigma=\frac{\sigma_{\| 1}-\sigma_{\perp}}{\sigma_{11}+\sigma_{\perp}}=\frac{\rho_{11}^{1}+\rho_{1-1}^{1}}{\rho_{11}^{0}+\rho_{1-1}^{0}} . \tag{5}
\end{equation*}
$$

Here $\sigma_{\|}$and $\sigma_{\perp}$ are the cross sections for the pions from symmetric $\rho$ decay ( $\theta=\frac{1}{2} \pi, \quad \phi=\frac{1}{2} \pi$ ) to emerge in the plane of the photon polarization ( $\Phi=\frac{1}{2} \pi$ ) and perpendicular to it ( $\Phi=0$ ). When the helicity-flip terms, $\rho_{00}^{1}, \rho_{11}^{1}, \rho_{00}^{0}, \rho_{1-1}^{0}$ are zero, $\Sigma$ is equal to $P_{\sigma}$.
The $\rho^{0}$ decay distribution may be simplified if we use the angle $\Psi=\phi-\Phi$ which, in the forward direction, is the angle between the photon polarization and the $\rho^{0}$ decay plane. If the $\rho^{0}$ production mechanism conserves $s$-channel helicity, i.e., the $\rho$ is transverse and linearly polarized like the photon, then in the helicity system

$$
\begin{equation*}
\rho_{1-1}^{1}=-\operatorname{Im} \rho_{1-1}^{2}=\frac{1}{2} \tag{6}
\end{equation*}
$$

and all other $\rho_{i k}^{\alpha}$ in Eq. (2) are 0 . In these circumstances $\Psi$ is the azimuthal angle in the helicity system of the decay $\pi^{+}$with respect to the $\rho^{0}$ polarization plane and the decay angular distribution is proportional to $\sin ^{2} \theta \cos ^{2} \Psi$. The distribution of $\Psi$ is also related to $P_{\sigma}$ if the helicity-flip terms are zero: For $100 \%$ linear polarization the decay is $\sin ^{2} \theta \cos ^{2} \Psi$ for $P_{\sigma}=+1$ while for $P_{\sigma}=-1$ the decay distribution is $\sin ^{2} \theta \sin ^{2} \Psi$.

## 4. The Moments, $Y_{l}^{m}$, of the Dipion System

Figure 13 shows the distributions of the polar angle $\theta$ and the angle $\Psi$ in the helicity system for events in the $\rho^{0}$ mass region ( $0.60-0.85 \mathrm{GeV}$ ) with $|t|<0.4 \mathrm{GeV}^{2}$. This figure shows that the $\rho^{0}$ decay has a simple description in terms of $\theta$ and $\Psi$ in the helicity system, viz., the $\rho^{0}$ is well described by a $\sin ^{2} \theta \cos ^{2} \Psi$ angular distribution for $|t|<0.4 \mathrm{GeV}^{2}$. Consequently, in order to give an over-all description of the characteristics of the decay angular distribution of the $\pi^{+} \pi^{-}$system, we present in Fig. 14 the moment sums, $\sum \operatorname{Re} Y_{l}^{m}(\theta, \Psi)$, of the $\pi^{+} \pi^{-}$ system in the helicity frame as a function of $\pi^{+} \pi^{-}$ mass for $|t|<0.4 \mathrm{GeV}^{2}$. Only those moments are

(a) $\gamma p-p \pi^{+} \pi^{-}$
$\mathrm{E}_{\boldsymbol{\gamma}}=2.8 \mathrm{GeV}$
$0.60<M_{\pi \pi}<0.85 \mathrm{GeV}$
$0.02<1+1<0.4 \mathrm{GeV}^{2}$ 1236 EVENTS




> (b) $\gamma \mathrm{p} \rightarrow \mathrm{p} \pi^{+} \pi^{-}$ $\mathrm{E}_{\gamma}=4.7 \mathrm{GeV}$
> $0.60<\mathrm{M}_{\pi} \pi^{<}<0.85 \mathrm{GeV}$ $0.02<1 \mid 1<0.4 \mathrm{GeV}^{2}$ 1457 EVENTS


FIG. 13. Reaction $\gamma p \rightarrow p \rho^{0}$ at (a) 2.8 GeV and (b) 4.7 GeV , respectively. $\rho$-decay angular distributions in the helicity system without background subtraction. The curves are proportional to $\sin ^{2} \theta_{H}$ and $\left(1+P_{\gamma} \cos 2 \Psi_{H}\right)$.
shown which have a significant deviation from zero in either the 2.8 - or $4.7-\mathrm{GeV}$ data; other moments can be found in Ref. 17. From the moments we conclude that:
(a) Strong $Y_{2}^{0}$ and $Y_{2}^{2}$ moments are present in the $\rho^{0}$ region which follow the asymmetric $\rho^{0}$ shape. This and the small values of higher even moments demonstrates that it is the $p$-wave part of the mass

