Run plan for optimizing "Full" versus "Empty" target running

Setting up for data taking:

- Establish a "good" photon beam on the Pb target
- Base instrumentation should be up and running: solenoid, FDCs, TOF, FCAL, DAQ
- Trigger should be NPP trigger
- Be ready to run Andrew's standard Bethe-Heitler e^+e^- data analysis off-line. We need to run the analysis in a relatively short period of time ($\approx$ 12 hours).

Data taking:

Use production of Bethe-Heitler e^+e^-pairs to establish the optimal running fraction for Full target running, f_{full}

- Take runs with the following conditions
  i. 2 hours at nominal CPP/NPP beam current with Full target (Pb frame) in.
  ii. 2 hrs at nominal CPP/NPP beam current with Empty target (empty frame) in.
  iii. 2 hrs at 2 $\times$ nominal CPP/NPP beam current with Empty target in.
- For all three runs record the DAQ live-times, $LT_{full(MT)}$, and the number of pair spectrometer triggers, $N^{PS}_{full(MT)}$.

Off-line data analysis:

- Run Andrew's standard Bethe-Heitler analysis for e^+e^- events. Plot vertex positions for the events using the 2-track vertex without the z=1 constraint. Find yields at the target vertex for full and empty targets: N_{full} and N_{MT}
- Normalize the yields to the number of PS triggers, and correct for live-time:

$$\tilde{N}_{full} = \frac{N_{full}}{LT_{full}N^{PS}_{full}} \qquad \tilde{N}_{MT} = \frac{N_{MT}}{LT_{MT}N^{PS}_{MT}}$$

Optimized running fraction for full-target running:

We assume that Full target data production (not these calibration runs) is at an incident photon rate of R^{γ}_{full}, and Empty target production is at an incident photon rate of R^{γ}_{MT} , with the rates not constrained to be equal. The fractional uncertainty in Pb yield is given by the equation below (see attached notes):

$$\frac{\sigma^2_{Pb}}{N^2_{Pb}} = \frac{1}{TX_{Pb}R^{\gamma}_{full}}\left[\frac{1}{f_{full}}\frac{X_{Pb}+X_{MT}}{X_{Pb}} + \frac{1}{1-f_{full}}\frac{R^{\gamma}_{full}}{R^{\gamma}_{MT}}\frac{X_{MT}}{X_{Pb}}\right]$$

In this equation X_{Pb} and X_{MT} are production cross sections on Pb and empty target. To minimize the fractional uncertainty in Pb yield, differentiate the fractional uncertainty with respect to f_{full} and set the result equal to zero. Use the normalized and live-time corrected yields $\tilde{N}_{full}$ and $\tilde{N}_{MT}$ to evaluate the ratio X_{MT}/X_{full} . This gives the following result for the optimized fraction for Full target running (again, see attached notes).

$$f_{full} = \frac{1}{1 + \sqrt{\frac{R^{\gamma}_{full}}{R^{\gamma}_{MT}}}\sqrt{\frac{\tilde{N}_{MT}}{\tilde{N}_{full}}}}$$

$$f_{MT} = 1 - f_{full}$$

Optimized time for running on full target

T = total running time available for data taking

f = fraction of time running on full target at R^{γ}_{full} photons/s

1-f = fraction of time running on empty target at R^{γ}_{MT} photons/s

counts on full target: $N_{full} = fT(X_{Pb} + X_{MT})R^{\gamma}_{full}$

counts on empty target: $N_{MT} = (1-f)TX_{MT}R^{\gamma}_{MT}$

where X_{Pb} and X_{MT} are cross sections for production on Pb and empty frames

$$N_{Pb} = N_{full} - \frac{f}{1-f}\frac{R^{\gamma}_{full}}{R^{\gamma}_{MT}} N_{MT}$$

$$\sigma^2_{Pb} = N_{full} + \left[\frac{f}{1-f}\frac{R^{\gamma}_{full}}{R^{\gamma}_{MT}}\right]^2 N_{MT}$$

$$\frac{\sigma^2_{Pb}}{N^2_{Pb}} = \frac{1}{(fTX_{Pb}R^{\gamma}_{full})^2} fT(X_{Pb} + X_{MT})R^{\gamma}_{full} +$$

$$\frac{1}{(fTX_{Pb}R^{\gamma}_{full})^2}\left[\frac{f}{1-f}\frac{R^{\gamma}_{full}}{R^{\gamma}_{MT}}\right]^2 (1-f)TX_{MT}R^{\gamma}_{MT}$$

$$\frac{\sigma^2_{Pb}}{N^2_{Pb}} = \frac{1}{TX_{Pb}R^{\gamma}_{full}}\left[\frac{1}{f}\frac{X_{Pb} + X_{MT}}{X_{Pb}} + \frac{1}{1-f}\frac{R^{\gamma}_{full}}{R^{\gamma}_{MT}}\frac{X_{MT}}{X_{Pb}}\right]$$

Let $\alpha = \frac{X_{MT}}{X_{Pb}}$ $s = \frac{R^{\gamma}_{full}}{R^{\gamma}_{MT}}$ $\frac{\sigma^2_{Pb}}{N^2_{Pb}} = \frac{1}{TX_{Pb}R^{\gamma}_{full}}\left[\frac{1+\alpha}{f} + \frac{s\alpha}{1-f}\right]$

Minimize $\frac{\sigma_{Pb}^2}{N_{Pb}^2}$ wrt f: $\frac{d}{df}\left(\frac{\sigma_{Pb}^2}{N_{Pb}^2}\right) = \frac{1}{TX_{Pb}R_{full}^{\gamma}}\frac{d}{df}\left(\frac{1+\alpha}{f} + \frac{s\alpha}{1-f}\right) = 0$

$$-\frac{1+\alpha}{f^2} + \frac{s\alpha}{(1-f)^2} = \left(\frac{\sqrt{s\alpha}}{1-f} + \frac{\sqrt{1+\alpha}}{f}\right)\left(\frac{\sqrt{s\alpha}}{1-f} - \frac{\sqrt{1+\alpha}}{f}\right) = 0$$

2nd root is the physical solution: $\frac{\sqrt{s\alpha}}{1-f} - \frac{\sqrt{1+\alpha}}{f} = 0$ $\quad f = \frac{\sqrt{1+\alpha}}{\sqrt{s\alpha}+\sqrt{1+\alpha}}$

$$f = \frac{\sqrt{X_{Pb}+X_{MT}}}{\sqrt{sX_{MT}}+\sqrt{X_{Pb}+X_{MT}}} = \frac{\sqrt{X_{full}}}{\sqrt{sX_{MT}}+\sqrt{X_{full}}}$$

$$f = \frac{1}{1+\sqrt{\frac{R_{full}^{\gamma}}{R_{MT}^{\gamma}}}\sqrt{\frac{X_{MT}}{X_{full}}}}$$

Use runs on full and empty targets to establish $\dfrac{X_{MT}}{X_{full}}$

$$N_{full} = LT_{full} X_{full} N_{full}^{PS} \qquad N_{MT} = LT_{MT} X_{MT} N_{MT}^{PS}$$

$$\tilde{N}_{full} = X_{full} = \frac{N_{full}}{LT_{full} N_{full}^{PS}} \qquad \tilde{N}_{MT} = X_{MT} = \frac{N_{MT}}{LT_{MT} N_{MT}^{PS}}$$

where $LT_{full(MT)}$ are DAQ lifetimes, $N_{full(MT)}^{PS}$ are pair spectrometer triggers, and $\tilde{N}_{full(MT)}$ are normalized yields corrected for dead-time. Substituting into the equation for f gives:

$$f = \frac{1}{1 + \sqrt{\dfrac{R_{full}^{\gamma}}{R_{MT}^{\gamma}}} \sqrt{\dfrac{\tilde{N}_{MT}}{\tilde{N}_{full}}}}$$