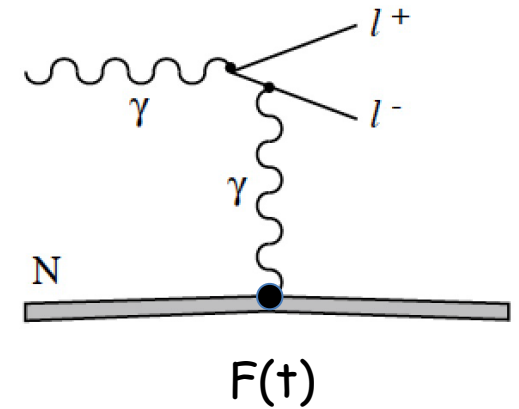


Sensitivity of the Bethe-Heitler cross section to the nuclear RMS charge radius



$$\frac{d\sigma_B^c}{dx d\Omega_1 d\Omega_2} = \frac{2\alpha^3 Z^2 \omega^4 x^2 (1-x)^2}{\pi^2 (\vec{q}^2)^2} \times [W_{\text{unp}} + P_\gamma W_{\text{pol}} \cos(2\phi)]$$

$$\times \left| F_{\text{nuclear}}(\vec{q}^2) - F_{\text{atomic}}(\vec{q}^2) \right|^2$$

$$W_{\text{unp}} = [x^2 + (1-x)^2] |\vec{J}_T|^2 + m^2 |J_S|^2; \quad W_{\text{pol}} = -2x(1-x) |\vec{J}_T|^2.$$

with

$$J_S = \frac{1}{\vec{p}_1^2 + m^2} - \frac{1}{\vec{p}_2^2 + m^2} \quad \vec{J}_T = \frac{\vec{p}_1}{\vec{p}_1^2 + m^2} + \frac{\vec{p}_2}{\vec{p}_2^2 + m^2}$$

Run the Bethe-Heitler event generator for two form factor choices:

1. Standard dipole form factor, modified to give the correct electronic rms proton charge radius:

$$F(Q^2)_{nuclear} = 1 - \frac{1}{6} \frac{\langle r^2 \rangle}{\hbar c^2} Q^2 + O(Q^4)$$

$$F(Q^2)_{nuclear} = \frac{1}{\left(1 + \frac{Q^2}{.71 GeV^2}\right)^2} + 2 \frac{Q^2}{.71 GeV^2} - \frac{1}{6} \frac{\langle r^2 \rangle}{\hbar c^2} Q^2$$

2. Form factor for point particle:

$$F(Q^2)_{nuclear} = 1$$

Run the event generator, bin events as a function of t:

2M events for $F(Q^2)_{F=dipole}$

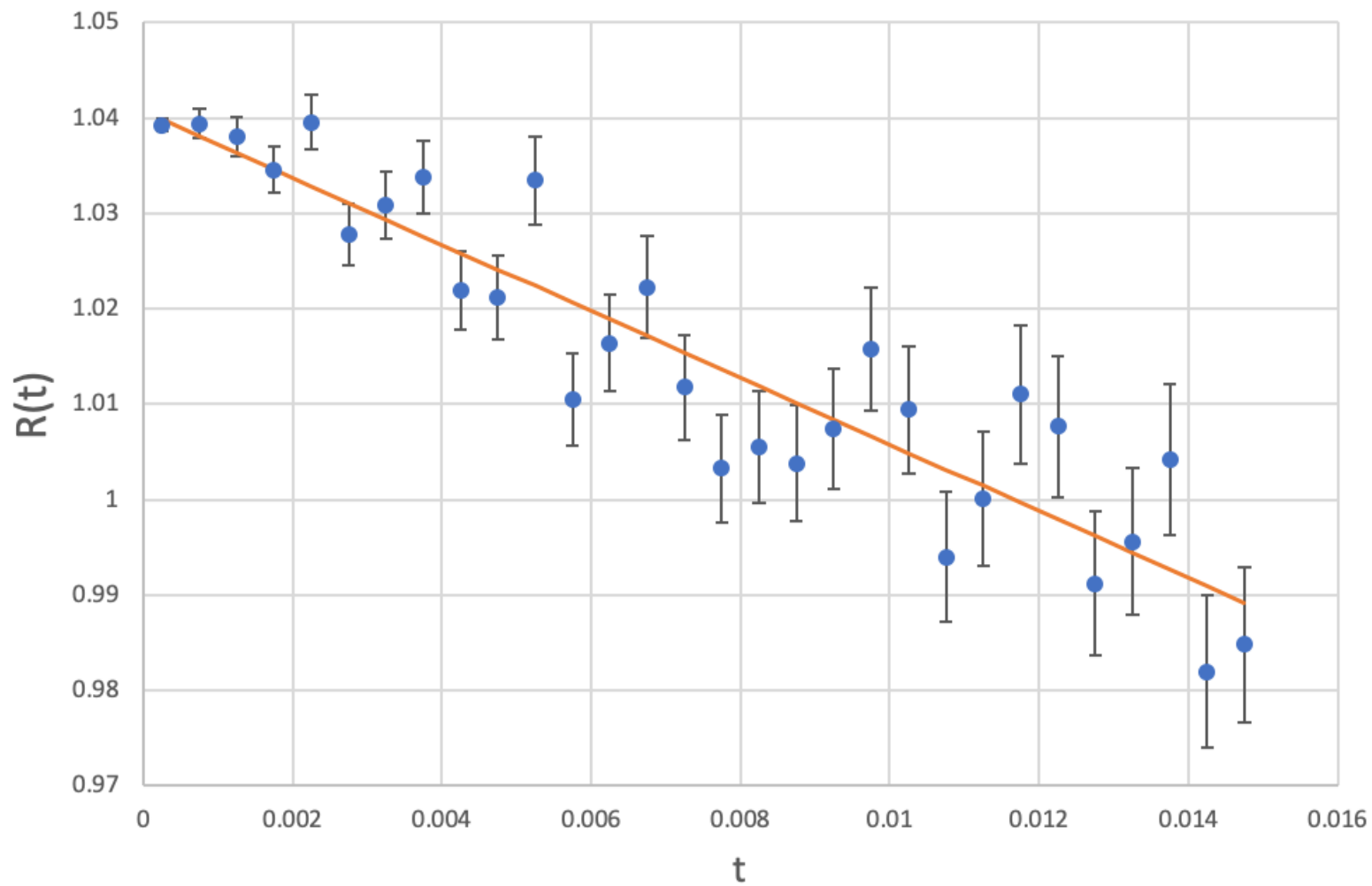
4M events for $F(Q^2)_{F=1}$

Require $W > 0.2 \text{ GeV}$, and $\theta > 0.9^\circ$

Square root of ratio of yields:

$$R(t) = \sqrt{\frac{N(t)_{F=dipole}}{N(t)_{F=1}}} = C \left(1 - \frac{1}{6} \frac{\langle r^2 \rangle}{\hbar c^2} t + O(t^2) \right)$$

Do a linear fit on $R(t)$, varying C and $\langle r^2 \rangle$. Orders $O(t^2)$ and higher are neglected in this fit.



● Data points — Linear fit

Electronic RMS proton charge radius used in event generation: $\sqrt{\langle r^2 \rangle} = .879 \text{ fm}$

Result from the fit: $\sqrt{\langle r^2 \rangle} = .884 \pm .020 \text{ fm}$

$$\frac{\sqrt{\langle r^2 \rangle}_{\text{electronic}} - \sqrt{\langle r^2 \rangle}_{\text{muonic}}}{\sigma_{\text{fit}}} = \frac{.879 - .8409}{.020} \approx 2\sigma_{\text{fit}}$$

Improvements in the toy model:

1. Run event generator to increase statistics in $N(t)_{F=1}$ and $N(t)_{F=dipole}$
2. Adjust t-bin widths so they are consistent with resolution
3. Use a more realistic model for fitting:

$$\sqrt{\frac{N(t)_{F=dipole}}{N(t)_{F=1}}} = C \left(\frac{1}{\left(1 + \frac{t}{.71\text{GeV}^2}\right)^2} + 2\frac{t}{.71\text{GeV}^2} - \frac{1}{6} \frac{\langle r^2 \rangle}{\hbar c^2} t - F(t)_{atomic} \right) (1 - F(t)_{atomic})^{-1}$$

Fit C and $\langle r^2 \rangle$

Analysis of real data:

1. Run the event generator to get high statistics in $N(t)_{F=1}$. Should have $N(t)_{F=1} \gg N(t)_{\text{data}}$
2. Apply realistic fiducial cuts for e^+e^- events. Want $N_{\text{data}} \gg 2M$
3. Find some way to calibrate and test the MC calculation of the shape of the t -acceptance distribution, $\epsilon(t)$.