## Angular distributions

#### Using spin density matrix elements

$$\begin{split} W(\cos\theta,\phi,\Phi) = & \frac{3}{4\pi} \left[ \tfrac{1}{2} (1-\rho_{00}^0) + \tfrac{1}{2} (3\rho_{00}^0 - 1) \cos^2\theta - \sqrt{2} \operatorname{Re} \rho_{10}^0 \sin 2\theta \cos\phi - \rho_{1-1}^0 \sin^2\theta \cos 2\phi \right. \\ & \left. - P_\gamma \cos 2\Phi(\rho_{11}^1 \sin^2\theta + \rho_{00}^1 \cos^2\theta - \sqrt{2} \operatorname{Re} \rho_{10}^1 \sin 2\theta \cos\phi - \rho_{1-1}^1 \sin^2\theta \cos 2\phi \right. \\ & \left. - P_\gamma \sin 2\Phi(\sqrt{2} \operatorname{Im} \rho_{10}^2 \sin 2\theta \sin\phi + \operatorname{Im} \rho_{1-1}^2 \sin^2\theta \sin 2\phi) \right] \end{split}$$

If it is possible to choose the z axis so that s-channel helicity is conserved, W takes a particularly simple form as a function of  $\Psi \equiv \Phi - \phi$ , namely

$$W(\theta, \Psi) \propto (\sin^2 \theta + P_{\gamma} \sin^2 \theta \cos 2\Psi).$$
 (D2a)

This results from the relationships

$$\rho_{1-1}^1 = -\operatorname{Im} \rho_{1-1}^2 = \frac{1}{2} \tag{D2b}$$

with all other  $\rho_{ib}^{\alpha} = 0$  in (D1).

#### Definition of $\Phi$

Here,  $P_{\gamma}$  is the degree of linear polarization of the photon;  $\Phi$  is the angle of the photon electric polarization vector with respect to the production plane measured in the over-all  $(\gamma p)$  c.m. system;  $\theta$  and  $\phi$  are the polar and azimuthal angles of the  $\pi^+$  in the  $\rho^0$  rest frame. (See Fig. 12 and Ref. 36.)

# Definition of z-axis

We consider the angular distribution of  $\rho^0$  decay in three reference systems which differ in the choice of the spin-quantization axis (z axis): the Gottfried-Jackson system, where the z axis is the direction of the incident photon in the  $\rho^0$  rest system; the helicity system, where the z axis is the direction of the  $\rho^0$  in the over-all ( $\gamma p$ ) c.m. system, i. e., opposite to the direction of the outgoing proton in the  $\rho^0$  rest system; and the Adair system, where the z axis is along the direction of the incident photon in the over-all ( $\gamma p$ ) c.m. system. The y axis is always normal to the production plane. The For forward-produced  $\rho^0$  mesons, all three systems coincide.

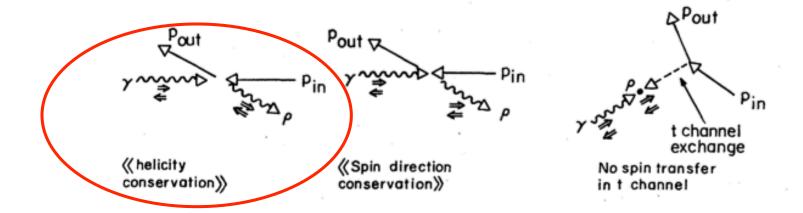
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Depending upon the production mechanism, the  $\rho^0$  may be aligned in one of these three systems. The system which gives the simplest description of the  $\rho^0$  is then: (1) the Gottfried-Jackson system for t-channel helicity conservation (resulting from, for example,  $J^P=0^+$  exchange with no absorption); (2) the helicity system for s-channel c.m. helicity conservation; (3) the Adair system for "spin independence" in the s-channel c.m. system.<sup>37</sup> One

## Bauer, Spital, Yennie, and Pipkin: Hadronic properties of the photon Rev. Mod. Phys., Vol. 50, No. 2, April 1978

### Angular Distributions in $\pi\pi$ rest frame

• Depending upon the production mechanism, the spin of the  $\rho^0$  may be aligned along the z axis in one of these three systems (Gilman et al., 1970). The system which gives the simplest description of the  $\rho^0$  is then: (1) The Gottfried-Jackson system for t-channel helicity conservation; (2) the helicity system for s-channel helicity conservation (SCHC); (3) the Adair system for "spin independence" in the s-channel system. Figure 85 shows a schematic representation of these three coordinate systems.



## Choice of a specifies system used

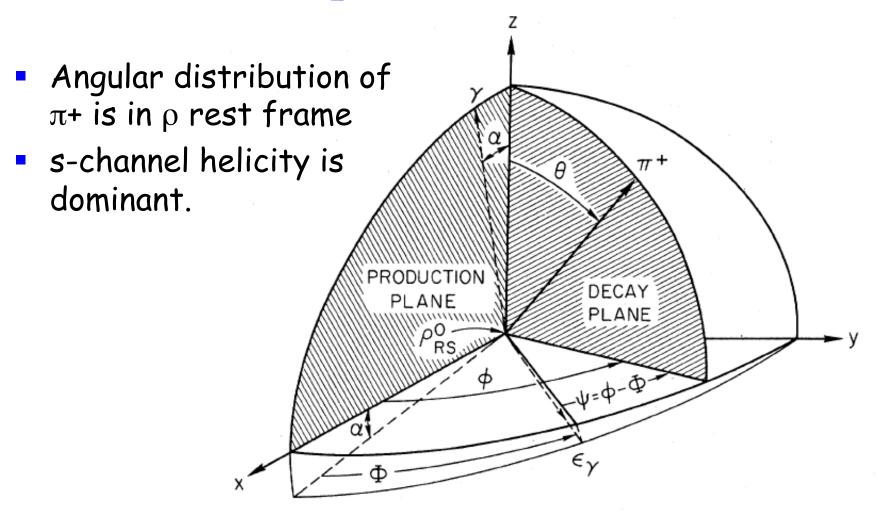


FIG. 12. Angles used in the study of  $\rho^0$  decay. The angle  $\alpha$  is zero in the Gottfried-Jackson system.

#### **SCHC**

The  $\rho^0$  decay distribution may be simplified if we use the angle  $\Psi = \phi - \Phi$  which, in the forward direction, is the angle between the photon polarization and the  $\rho^0$  decay plane. If the  $\rho^0$  production mechanism conserves s-channel helicity, i.e., the  $\rho$  is transverse and linearly polarized like the photon, then in the helicity system

$$\rho_{ik}^{\alpha} = 0, \text{ except}$$

$$\rho_{1-1}^{1} = -Im \, \rho_{1-1}^{2} = \frac{1}{2}$$

$$\rho_{11}^{0} = \frac{1}{2}$$

$$\rho_{1-1}^1 = -\text{Im}\rho_{1-1}^2 = \frac{1}{2} \tag{6}$$

and all other  $\rho_{ik}^{\alpha}$  in Eq. (2) are 0. In these circumstances  $\Psi$  is the azimuthal angle in the helicity system of the decay  $\pi^+$  with respect to the  $\rho^0$  polarization plane and the decay angular distribution is proportional to  $\sin^2\theta\cos^2\Psi$ . The distribution of  $\Psi$  is also related to  $P_{\sigma}$  if the helicity-flip terms are zero: For 100% linear polarization the decay is  $\sin^2\theta\cos^2\Psi$  for  $P_{\sigma}=+1$  while for  $P_{\sigma}=-1$  the decay distribution is  $\sin^2\theta\sin^2\Psi$ .