Phase-Space Correction for Breit-Wigner Line Shape

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Breit-Wigner Distribution

- Resonances with substantial width follow the relativistic Breit-Wigner distribution:

$$\frac{d\sigma}{dM_{\pi\pi}} \propto \frac{2}{\pi} \frac{\Gamma_\rho M_\rho M_{\pi\pi}}{\left(M_{\pi\pi}^2 - M_\rho^2\right)^2 + \Gamma_\rho^2 M_\rho^2}$$

- Here we have the invariant mass $M_{\pi\pi}$ of the final-state, the resonance mass of the meson M_ρ, and the width Γ_ρ
- But this width is **not** the fixed pole width Γ_0; it must be corrected for

$$\Gamma_\rho = \left(\frac{p_\pi}{p_0}\right)^3 \frac{M_\rho}{M_{\pi\pi}} \times \Gamma_0$$

- Here p_π is the decay momentum in the ρ rest frame, and p_0 is the decay momentum when $M_{\pi\pi} = M_\rho$

Corrected Distribution

- Calculating the decay momenta

$$p_\pi = \sqrt{\frac{M_{\pi\pi}^2}{4} - m_\pi^2} \qquad p_0 = \sqrt{\frac{M_\rho^2}{4} - m_\pi^2}$$

- Plugging this in to the dynamical width:

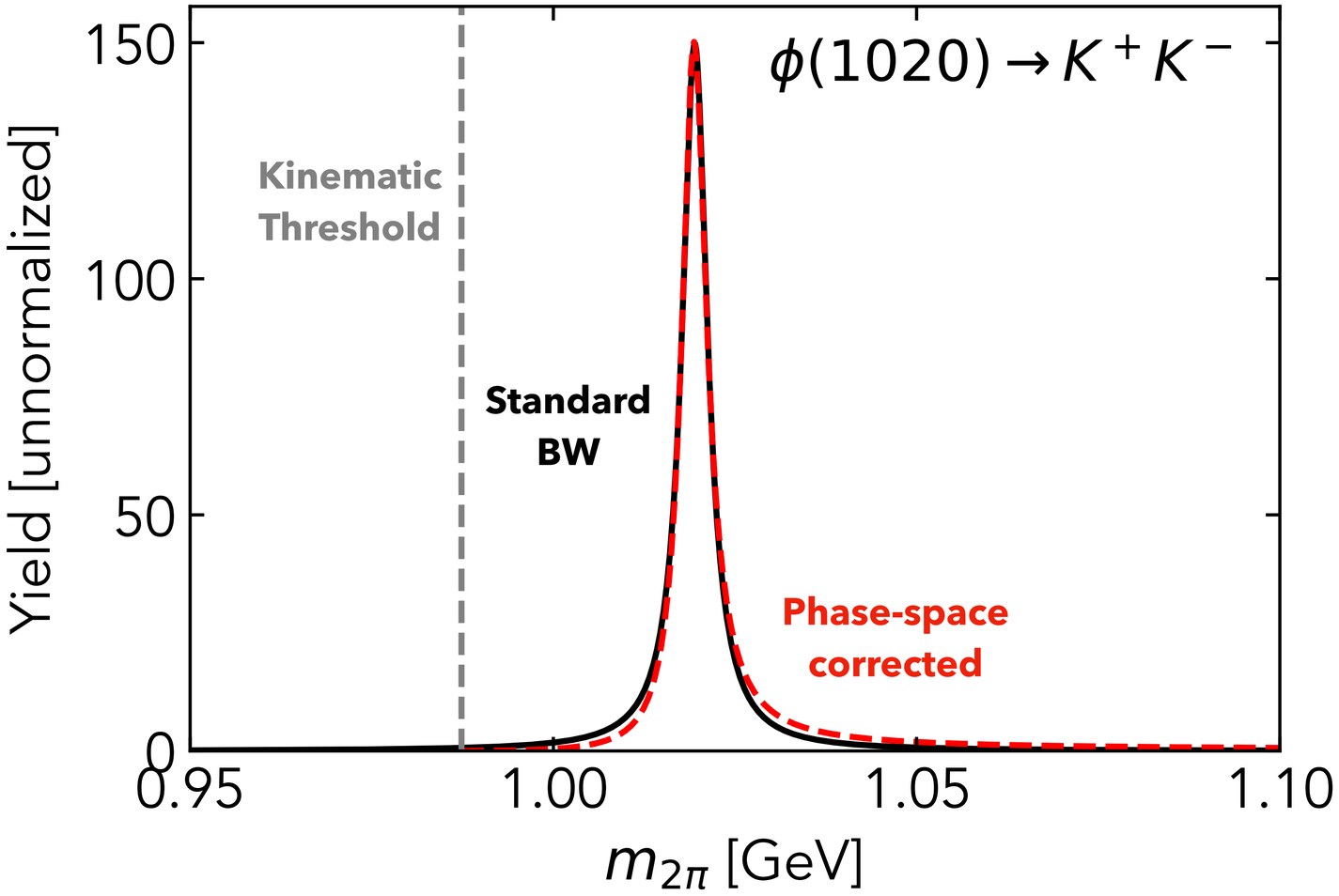
$$\Gamma_\rho = \left(\frac{M_{\pi\pi}^2 - 4m_\pi^2}{M_\rho^2 - 4m_\pi^2} \right)^{3/2} \frac{M_\rho}{M_{\pi\pi}} \times \Gamma_0$$

- We insert this dynamical/phase-space corrected width into the following distribution, though we won't do this here since the resulting expression is complicated:

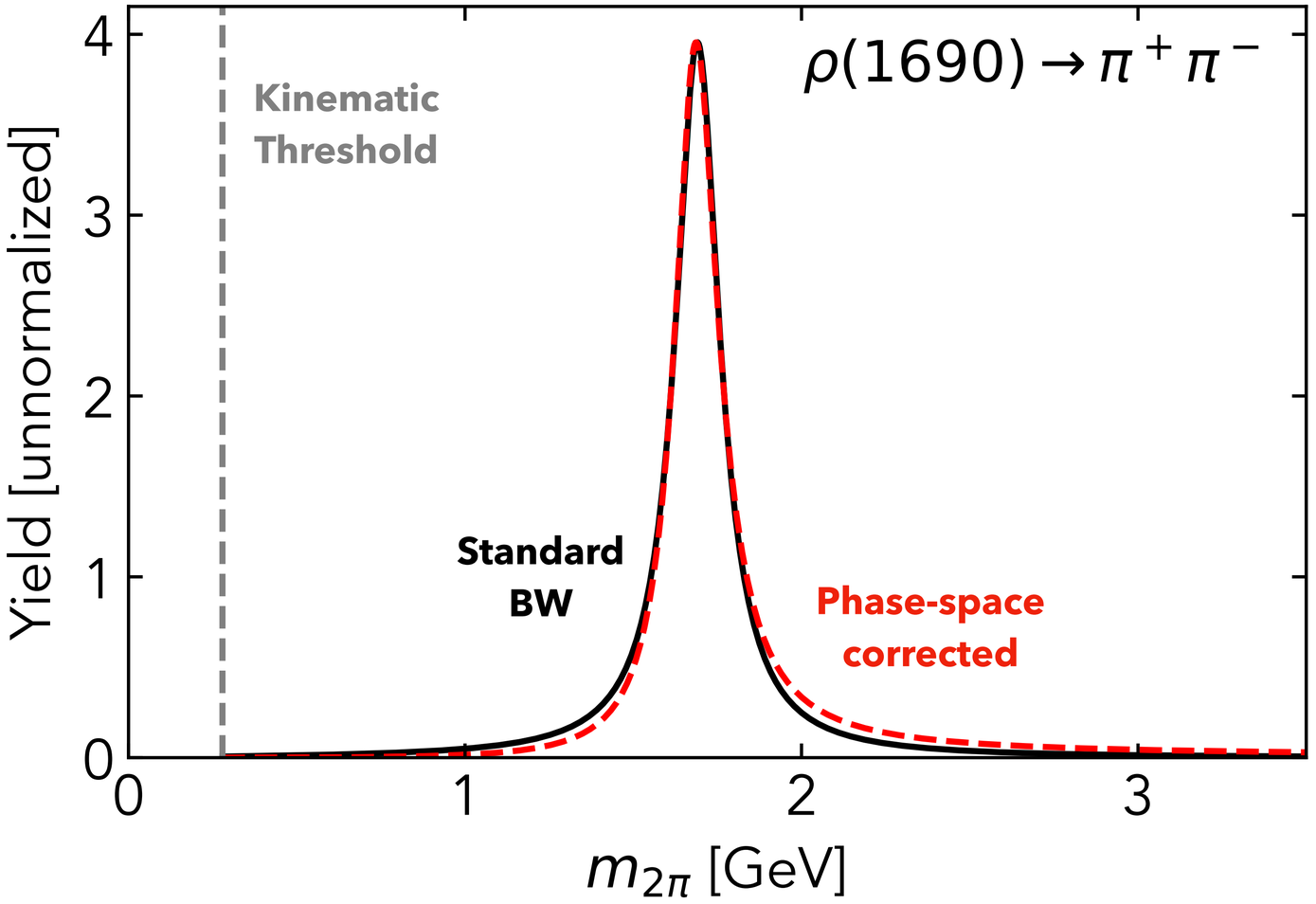
$$P(M_{\pi\pi}) \propto \frac{2}{\pi} \frac{\Gamma_\rho M_\rho M_{\pi\pi}}{\left(M_{\pi\pi}^2 - M_\rho^2 \right)^2 + \Gamma_\rho^2 M_\rho^2}$$

- The remaining parameters are the amplitude of the distribution, the mass of the ρ meson $M_\rho \approx 775$ MeV, and the pole width of the ρ meson $\Gamma_0 \approx 148$ MeV

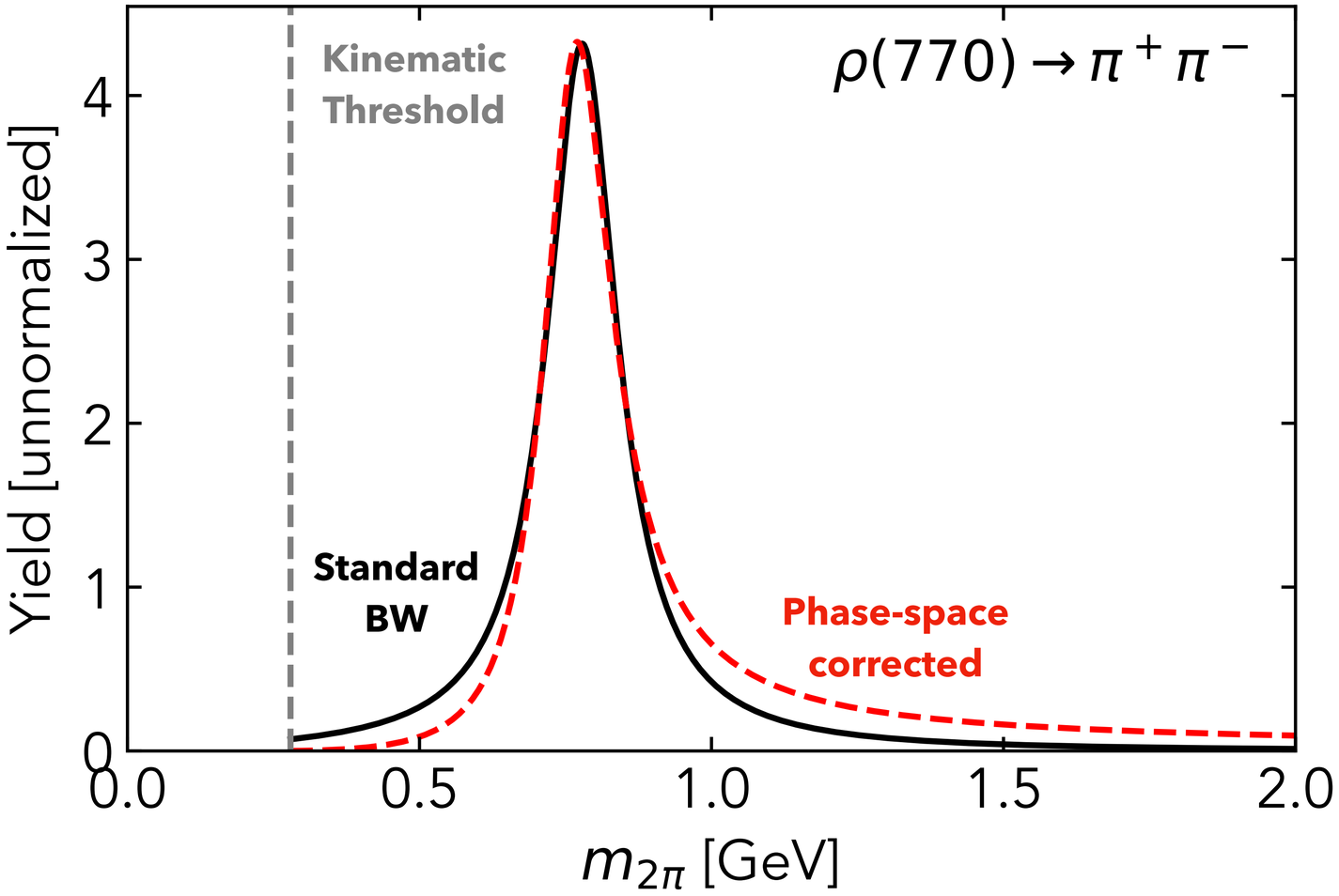
This is a small effect for narrow resonances like the ϕ



The effect is also small for resonances well above mass threshold



For wide resonances near threshold, this is a large distortion; especially big for the $\rho(770)$



References for Phase Space Correction

- https://journals.aps.org/prc/pdf/10.1103/PhysRevC.60.014903
- https://link.springer.com/content/pdf/10.1007/BF02750563.pdf