Alignment using $K_S \to \pi^+\pi^-$

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Abstract

This note describes the tracking system alignment utilizing $K_S \to \pi^+\pi^-$ events. The alignment procedure is implemented in `plugins/Alignment/MilleKs`.

Contents

1 Introduction

In the first stage of alignment [1], alignment parameters $\boldsymbol{p}^{\text{global}}$ are optimized by minimizing the following "single-track based" χ^2:

$$\chi^2(\boldsymbol{p}^{\text{global}}, \{\boldsymbol{q}_i^{\text{local}}\}) = \sum_{i:\text{tracks}} \sum_{j:\text{hits in track } i} \left(\frac{\text{residual}_j(\boldsymbol{p}^{\text{global}}, \boldsymbol{q}_i^{\text{local}})}{\text{error}_j} \right)^2, \tag{1}$$

where $\boldsymbol{q}_i^{\text{local}}$ represents track parameters for i-th track. The minimization is performed by `Millepede` (see plugins `MilleFieldOn`, `MilleFieldOff`).

Here, we introduce the new parametrization for local parameters $\boldsymbol{q}_i^{\text{local}}$ (defined in Ref. [2]) to leverage the $K_S \to \pi^+\pi^-$ events for the alignment.

The original $\boldsymbol{q}_i^{\text{local}}$ consists of 5 components to represent a single track (10 components to represent π^+ and π^- tracks), while the new parametrization consists of K_S momentum at the decay vertex (3 components), the decay vertex position (3 components), and decay angles (θ, ϕ) in the K_S rest frame (8 components in total). We are saving 2 parameters which means the common vertex and K_S mass information are automatically utilized by using this new parametrization.

2 Derivatives for `Millepede`

To minimize the χ^2 (Eq. (1)), `Millepede` requires the following derivatives:

$$\frac{\partial \text{residual}_j(\boldsymbol{p}^{\text{global}}, \boldsymbol{q}_i^{\text{local}})}{\partial \boldsymbol{p}^{\text{global}}}, \quad \frac{\partial \text{residual}_j(\boldsymbol{p}^{\text{global}}, \boldsymbol{q}_i^{\text{local}})}{\partial \boldsymbol{q}_i^{\text{local}}}. \tag{2}$$

We already have $\frac{\partial \text{residual}_j(\boldsymbol{p}^{\text{global}}, \boldsymbol{q}_i^{\text{local}})}{\partial \boldsymbol{p}^{\text{global}}}$ since this derivative does not change when we use the new parametrization. Therefore, what we should focus on is $\frac{\partial \text{residual}_j(\boldsymbol{p}^{\text{global}}, \boldsymbol{q}_i^{\text{local}})}{\partial \boldsymbol{q}_i^{\text{local}}}$.

From now on, we follow the notations used in Ref. [2]. In this reference, the new parameterization $\boldsymbol{q}_i^{\text{local}}$ is denoted by $(\boldsymbol{v}, \boldsymbol{z}) = (\boldsymbol{v}, p_x, p_y, p_z, \theta, \phi)$ where

1. $\boldsymbol{v} = (v_x, v_y, v_z)^T$ is the position of the decay vertex,
2. $\boldsymbol{p} = (p_x, p_y, p_z)^T$ is the momentum of the primary particle (K_S) at the decay vertex position in the lab-frame, and
3. θ and ϕ are the polar and azimuth angles defining the direction of the secondary particles (π^+ and π^-) in the K_S rest-frame, respectively.

Also, the derivative $\frac{\partial \text{residual}_j(\boldsymbol{p}^{\text{global}}, \boldsymbol{q}_i^{\text{local}})}{\partial \boldsymbol{q}_i^{\text{local}}}$ for the new parametrization is denoted by $\frac{\partial f^\pm}{\partial(\boldsymbol{v},\boldsymbol{z})}$ where f^+ and f^- correspond to residuals which are associated with π^+ and π^- tracks, respectively.

The derivative $\frac{\partial f^\pm}{\partial(\boldsymbol{v},\boldsymbol{z})}$ is decomposed as follows [2]:

$$\frac{\partial f^\pm}{\partial(\boldsymbol{v}, \boldsymbol{z})} = \left(\frac{\partial f^\pm}{\partial \boldsymbol{q}^\pm} \frac{\partial \boldsymbol{q}^\pm}{\partial \boldsymbol{v}} \quad \frac{\partial f^\pm}{\partial \boldsymbol{q}^\pm} \frac{\partial \boldsymbol{q}^\pm}{\partial \boldsymbol{p}^\pm} \frac{\partial \boldsymbol{p}^\pm}{\partial \boldsymbol{z}} \right), \tag{3}$$

where $\boldsymbol{q}^\pm$ are the track parameters for the old parametrization (= state vectors for $\pi^\pm$), and $\boldsymbol{p}^\pm$ represent the $\pi^\pm$ momenta at the decay vertex in the lab-frame. Here, we already have the derivative $\frac{\partial f^{\pm}}{\partial \boldsymbol{q}^{\pm}}$ since it is used in "single-track based" alignment (`MilleFieldOn`). Finally, what we have to newly prepare is a 5×3 matrix $\frac{\partial \boldsymbol{q}^{\pm}}{\partial \boldsymbol{v}}$, a 5×3 matrix $\frac{\partial \boldsymbol{q}^{\pm}}{\partial \boldsymbol{p}^{\pm}}$, and a 3×5 matrix $\frac{\partial \boldsymbol{p}^{\pm}}{\partial \boldsymbol{z}}$.

2.1 $\frac{\partial q^{\pm}}{\partial v}, \frac{\partial q^{\pm}}{\partial p^{\pm}}$

Here, we assume a constant magnetic field whose direction is along the z-axis in the lab-frame, and calculate the derivatives $\frac{\partial \boldsymbol{q}^{\pm}}{\partial \boldsymbol{v}}$ and $\frac{\partial \boldsymbol{q}^{\pm}}{\partial \boldsymbol{p}^{\pm}}$. Note that we are using the tracking state vector $\boldsymbol{q}^{\pm} = (x, y, t_x, t_y, q/p)^T$ where $t_x = \frac{dx}{dz}$ and $t_y = \frac{dy}{dz}$.

The derivative $\frac{\partial \boldsymbol{q}^{\pm}}{\partial \boldsymbol{v}}$ can be approximately calculated as follows:

$$\frac{\partial \boldsymbol{q}^{\pm}}{\partial \boldsymbol{v}} = \begin{pmatrix} \frac{\partial x}{\partial v_x} & \frac{\partial x}{\partial v_y} & \frac{\partial x}{\partial v_z} \\ \frac{\partial y}{\partial v_x} & \frac{\partial y}{\partial v_y} & \frac{\partial y}{\partial v_z} \\ \frac{\partial t_x}{\partial v_x} & \frac{\partial t_x}{\partial v_y} & \frac{\partial t_x}{\partial v_z} \\ \frac{\partial t_y}{\partial v_x} & \frac{\partial t_y}{\partial v_y} & \frac{\partial t_y}{\partial v_z} \\ \frac{\partial (q/p)}{\partial v_x} & \frac{\partial (q/p)}{\partial v_y} & \frac{\partial (q/p)}{\partial v_z} \end{pmatrix} = \begin{pmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{4}$$

To calculate

$$\frac{\partial \boldsymbol{q}^{\pm}}{\partial \boldsymbol{p}^{\pm}} = \begin{pmatrix} \frac{\partial x}{\partial p_x^{\pm}} & \frac{\partial x}{\partial p_y^{\pm}} & \frac{\partial x}{\partial p_z^{\pm}} \\ \frac{\partial y}{\partial p_x^{\pm}} & \frac{\partial y}{\partial p_y^{\pm}} & \frac{\partial y}{\partial p_z^{\pm}} \\ \frac{\partial t_x}{\partial p_x^{\pm}} & \frac{\partial t_x}{\partial p_y^{\pm}} & \frac{\partial t_x}{\partial p_z^{\pm}} \\ \frac{\partial t_y}{\partial p_x^{\pm}} & \frac{\partial t_y}{\partial p_y^{\pm}} & \frac{\partial t_y}{\partial p_z^{\pm}} \\ \frac{\partial (q/p)}{\partial p_x^{\pm}} & \frac{\partial (q/p)}{\partial p_y^{\pm}} & \frac{\partial (q/p)}{\partial p_z^{\pm}} \end{pmatrix}, \tag{5}$$

we solve the equation of motion

$$\frac{d\boldsymbol{p}}{dt} = q\boldsymbol{v} \times \boldsymbol{B}. \tag{6}$$

Using the notations in Table 2 in Ref. [3], this equation can be written as follows:

$$\frac{d\boldsymbol{p}}{dz} = -\frac{\boldsymbol{p}}{p_z} \times \left(a\hat{\boldsymbol{h}}\right), \tag{7}$$

where $\hat{\boldsymbol{h}} = (0, 0, 1)^T$. This equation can be solved as follows:

$$\begin{pmatrix} p_{0x} \\ p_{0y} \\ p_{0z} \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{a(z_0-z)}{p_z}\right) & -\sin\left(\frac{a(z_0-z)}{p_z}\right) & 0 \\ \sin\left(\frac{a(z_0-z)}{p_z}\right) & \cos\left(\frac{a(z_0-z)}{p_z}\right) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}. \tag{8}$$

(See Table 2 in Ref. [3] for definitions of variables.)

Using Eq. (32) in Ref. [3]:

$$\boldsymbol{p} = \boldsymbol{p}_0 - (\boldsymbol{x} - \boldsymbol{x}_0) \times \left(a\hat{\boldsymbol{h}}\right), \tag{9}$$

we obtain

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x + \frac{p_{0y}-p_y}{a} \\ y - \frac{p_{0x}-p_x}{a} \end{pmatrix} = \begin{pmatrix} x + \frac{\sin\left(\frac{a(z_0-z)}{p_z}\right)p_x + \left(\cos\left(\frac{a(z_0-z)}{p_z}\right)-1\right)p_y}{a} \\ y - \frac{\left(\cos\left(\frac{a(z_0-z)}{p_z}\right)-1\right)p_x - \sin\left(\frac{a(z_0-z)}{p_z}\right)p_y}{a} \end{pmatrix}. \tag{10}$$

Also,

$$\begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} \frac{p_{0x}}{p_{0z}} \\ \frac{p_{0y}}{p_{0z}} \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{a(z_0-z)}{p_z}\right)\frac{p_x}{p_z} - \sin\left(\frac{a(z_0-z)}{p_z}\right)\frac{p_y}{p_z} \\ \sin\left(\frac{a(z_0-z)}{p_z}\right)\frac{p_x}{p_z} + \cos\left(\frac{a(z_0-z)}{p_z}\right)\frac{p_y}{p_z} \end{pmatrix}. \tag{11}$$

Then, we can calculate necessary derivatives as follows:

$$\frac{\partial x}{\partial p_x^\pm} = \frac{S}{a}, \quad \frac{\partial x}{\partial p_y^\pm} = \frac{C-1}{a} \tag{12}$$

$$\frac{\partial x}{\partial p_z^\pm} = -\frac{z_0-z}{p_z^{\pm 2}}\left(p_x^\pm C - p_y^\pm S\right) \tag{13}$$

$$\frac{\partial y}{\partial p_x^\pm} = -\frac{C-1}{a}, \quad \frac{\partial y}{\partial p_y^\pm} = \frac{S}{a} \tag{14}$$

$$\frac{\partial y}{\partial p_z^\pm} = -\frac{z_0-z}{p_z^{\pm 2}}\left(p_x^\pm S + p_y^\pm C\right) \tag{15}$$

$$\frac{\partial t_x}{\partial p_x^\pm} = \frac{C}{p_z^\pm}, \quad \frac{\partial t_x}{\partial p_y^\pm} = -\frac{S}{p_z^\pm} \tag{16}$$

$$\frac{\partial t_x}{\partial p_z^\pm} = \left(\frac{a(z_0-z)p_y^\pm}{p_z^{\pm 3}} - \frac{p_x^\pm}{p_z^{\pm 2}}\right)C + \left(\frac{a(z_0-z)p_x^\pm}{p_z^{\pm 3}} + \frac{p_y^\pm}{p_z^{\pm 2}}\right)S \tag{17}$$

$$\frac{\partial t_y}{\partial p_x^\pm} = \frac{S}{p_z^\pm}, \quad \frac{\partial t_y}{\partial p_y^\pm} = \frac{C}{p_z^\pm} \tag{18}$$

$$\frac{\partial t_y}{\partial p_z^\pm} = -\left(\frac{a(z_0-z)p_x^\pm}{p_z^{\pm 3}} + \frac{p_y^\pm}{p_z^{\pm 2}}\right)C + \left(\frac{a(z_0-z)p_y^\pm}{p_z^{\pm 3}} - \frac{p_x^\pm}{p_z^{\pm 2}}\right)S \tag{19}$$

$$\frac{\partial (q/p)}{\partial p_i^\pm} = -\frac{q p_i^\pm}{p^{\pm 3}} \;\; (i = x, y, z) \tag{20}$$

where

$$C = \cos\left(\frac{a(z_0-z)}{p_z^\pm}\right), \quad S = \sin\left(\frac{a(z_0-z)}{p_z^\pm}\right). \tag{21}$$

2.2 $\frac{\partial p^\pm}{\partial z}$

Here, we calculate the 3×5 matrix $\frac{\partial \boldsymbol{p}^\pm}{\partial \boldsymbol{z}}$. We follow the notations in Ref. [2].

First, $\boldsymbol{p}^\pm$ can be calculated as follows:

$$\boldsymbol{p}^\pm(p_x, p_y, p_z, \theta, \phi) = R\boldsymbol{p}_0^\pm, \tag{22}$$

where

$$R = \begin{pmatrix} \frac{p_x p_z}{p_T p} & -\frac{p_y}{p_T} & \frac{p_x}{p} \\ \frac{p_y p_z}{p_T p} & \frac{p_x}{p_T} & \frac{p_y}{p} \\ -\frac{p_T}{p} & 0 & \frac{p_z}{p} \end{pmatrix}, \quad \boldsymbol{p}_0^\pm = \begin{pmatrix} \pm m\sqrt{\alpha^2-1}\sin\theta\cos\phi \\ \pm m\sqrt{\alpha^2-1}\sin\theta\sin\phi \\ \frac{p}{2} \pm \frac{1}{2}\sqrt{\frac{\alpha^2-1}{\alpha^2}\left(p^2+M^2\right)}\cos\theta \end{pmatrix}. \tag{23}$$

Using the following properties

$$\frac{\partial p_T}{\partial p_i} = \frac{p_i}{p_T}, \quad \frac{\partial p}{\partial p_i} = \frac{p_i}{p} \;\; (i = x, y, z) \tag{24}$$

derivatives of the matrix R and the vector $\boldsymbol{p}_0^{\pm}$ are calculated as follows:

$$\frac{\partial R}{\partial p_x} = \begin{pmatrix} \frac{p_z}{p_T p} - \frac{p_x^2 p_z (p^2+p_T^2)}{p_T^3 p^3} & \frac{p_x p_y}{p_T^3} & \frac{p^2-p_x^2}{p^3} \\ -\frac{p_x p_y p_z (p^2+p_T^2)}{p_T^3 p^3} & \frac{p_y^2}{p_T^3} & -\frac{p_x p_y}{p^3} \\ -\frac{p_x p_z^2}{p_T p^3} & 0 & -\frac{p_x p_z}{p^3} \end{pmatrix} \tag{25}$$

$$\frac{\partial R}{\partial p_y} = \begin{pmatrix} -\frac{p_x p_y p_z (p^2+p_T^2)}{p_T^3 p^3} & -\frac{p_x^2}{p_T^3} & -\frac{p_x p_y}{p^3} \\ \frac{p_z}{p_T p} - \frac{p_y^2 p_z (p^2+p_T^2)}{p_T^3 p^3} & -\frac{p_x p_y}{p_T^3} & \frac{p^2-p_y^2}{p^3} \\ -\frac{p_y p_z^2}{p_T p^3} & 0 & -\frac{p_y p_z}{p^3} \end{pmatrix} \tag{26}$$

$$\frac{\partial R}{\partial p_z} = \begin{pmatrix} \frac{p_x p_T}{p^3} & 0 & -\frac{p_x p_z}{p^3} \\ \frac{p_y p_T}{p^3} & 0 & -\frac{p_y p_z}{p^3} \\ \frac{p_T p_z}{p^3} & 0 & \frac{p_T^2}{p^3} \end{pmatrix} \tag{27}$$

$$\frac{\partial \boldsymbol{p}_0^{\pm}}{\partial p_i} = p_i \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2p} \pm \frac{\sqrt{\alpha^2-1}\cos\theta}{2\alpha\sqrt{p^2+M^2}} \end{pmatrix} \quad (i = x, y, z) \tag{28}$$

Then, $\frac{\partial \boldsymbol{p}^{\pm}}{\partial \boldsymbol{z}}$ will be obtained using the chain rule.

References

[1] Michael Staib and Alex Barnes, GlueX-doc-3739-v1 (Oct-2017).

[2] E. Widl and R. Fruhwirth, CMS Note **CMS NOTE-2007/032** (5-Oct-2007).

[3] Paul Mattione, GlueX-doc-2112-v5 (16-Apr-2016).