Understanding the BCAL Energy Deposition per Layer<br>A Progress Report - Installment IV<br>Alex R. Dzierba

Summary of what follows: The goal is to use information about fractional energy deposition per layer in the BCAL test module as a constraint in calibrating the module. We have been using a parameterization of the longitudinal energy deposition in an electromagnetic shower to estimate the fractional energy deposited in each BCAL layer as well as a GEANT-based simulation (carried out by Blake) for photons incident at $90^{\circ}$ and $40^{\circ}$. The analytical parameterization as given in the Particle Data book ${ }^{1}$ depends on the critical energy, $E_{c}$, for which two approximations are given. The radiation length, $X_{0}$, of the lead/scintillating fiber matrix used in the test module is uncertain (Zisis is calculating it) ranging from 1.3 to 1.5 cm . In this note, we assume the results of the simulations, which seems to describe the data ${ }^{2}$, is the Gold Standard and we compare these with the analytic parameterization under difference assumptions about $E_{c}$ and $X_{0}$ to get some feel for how sensitive the analytical calculations are on these parameters.

Conclusions: For $90^{\circ}$ incidence, varying $E_{c}$ and $X_{0}$ can lead to a better agreement between analytic and simulation results for either layer 1 or 2 but not both. The differences between analytic and simulation results increase for $40^{\circ}$ incidence. It is clear that the radial shape of the shower, and its longitudinal dependence, must be taken into account in trying to extend the analytical results to other than non-normal incidence. The radial dependence of the shower must also be taken into account for normal incidence - here we assume full containment transverse to the beam.

Results: The expression for the longitudinal energy deposition in an electromagnetic shower, as given in reference 1 is:

$$
\begin{equation*}
\frac{d E}{d t}=E_{0} b \frac{(b t)^{a-1} e^{-b t}}{\Gamma(a)} \tag{1}
\end{equation*}
$$

where $t$ is thickness in radiation lengths, $E_{0}$ is the energy of the particle initiating the shower, $b \approx 0.5$, and

$$
\begin{equation*}
\frac{a-1}{b}=t_{\max }=\ln \left(\frac{E_{0}}{E_{c}}\right)+0.5 \tag{2}
\end{equation*}
$$

with $E_{c}$ is the critical energy. Reference 1 cites two approximations for $E_{c}$, one $(\approx 610 \mathrm{MeV} /(Z+1.2)$ ) from Rossi ${ }^{3}$ and another $(\approx 800 \mathrm{MeV} /(Z+1.2))$ from Berger and Seltzer ${ }^{4}$. I have been using the latter and Christine has been using the former in understanding test beam data with a pre-radiator.

[^0]Equation 1 is widely used. One relatively recent paper by Grindhammer and Peters ${ }^{5}$ has a nice discussion of the use of the parameterization of longitudinal and radial profiles to describe showers in homogenous and sampling calorimeters.

Figures 1 through 4 show the fractional energy loss per BCAL layer as a function of incident energy $E_{\gamma}$. The solid curves are the results of Blake's simulations and are identical in all the figures. These curves are the results of second-order polynomial fits to Blake's results, as described in reference 2. The dashed curves are the result of using equation 1 with different values of $E_{c}$ and $X_{0}$. In each case the left panel is for $90^{\circ}$ incidence and the right panel is for $40^{\circ}$ incidence.

Figure 5 shows the fraction of energy contained, as a function of $E_{\gamma}$, for different values of the radiation length, estimated by integrating equation 1 over the six layers of the BCAL module and assuming full containment transverse to the beam.


Figure 1: Fractional energy deposition for each BCAL layer, as a function of beam energy, using equation 1 (dashed curves) and simulations (solid curves). The left panel is for $90^{\circ}$ incidence and the right panel is for $40^{\circ}$ incidence.

[^1]

Figure 2: Fractional energy deposition for each BCAL layer, as a function of beam energy, using equation 1 (dashed curves) and simulations (solid curves). The left panel is for $90^{\circ}$ incidence and the right panel is for $40^{\circ}$ incidence.


Figure 3: Fractional energy deposition for each BCAL layer, as a function of beam energy, using equation 1 (dashed curves) and simulations (solid curves). The left panel is for $90^{\circ}$ incidence and the right panel is for $40^{\circ}$ incidence.


Figure 4: Fractional energy deposition for each BCAL layer, as a function of beam energy, using equation 1 (dashed curves) and simulations (solid curves). The left panel is for $90^{\circ}$ incidence and the right panel is for $40^{\circ}$ incidence.


Figure 5: The fraction of energy contained, as a function of $E_{\gamma}$, for different values of the radiation length, estimated by integrating equation 1 over the six layers of the BCAL module and assuming full containment transverse to the beam.


[^0]:    ${ }^{1}$ See the Passage of Charged Particles Through Matter section of the Particle Data Booklet.
    ${ }^{2}$ A. Dzierba, Understanding the BCAL Energy Deposition per Layer - A Progress Report - Installment III, June 7, 2007.
    ${ }^{3}$ B. Rossi, High Energy Particles, Prentice-Hall, Inc., Englewood Cliffs, NJ, 1952.
    ${ }^{4}$ M.J. Berger and S.M. Seltzer, Tables of Energy Losses and Ranges of Electrons and Positrons, National Aeronautics and Space Administration Report NASA-SP-3012 (Washington DC 1964).

[^1]:    ${ }^{5}$ G. Grindhammer and S. Peters, The Parameterized Simulation of Electromagnetic Showers in Homogeneous and Sampling Calorimeters, hep-ex/0001020 (2000).

